

Chapter 1

Concept Application Exercises

Exercise 1.1

1. (a) True, radian is SI unit of angle.

$$\therefore \theta = \frac{1}{r} \frac{\text{meter}}{\text{meter}} = \text{dimensionless}$$

Hence, radian is supplementary unit.

- (b) True, it is basic rule for writing unit.

- (c) True, it is basic rule for writing unit.

- (d) True, same type of quantity are added or subtracted.

2. (a) Same work = $F \cdot s$

$$\therefore \text{The unit of work} = \text{N m or kg m}^2 \text{ s}^{-2} \text{ or J}$$

$$\text{Kinetic energy} = \frac{1}{2} mv^2$$

$$\text{Thus, the unit of energy} = \text{kg(m/s)}^2 = \text{kg m}^2 \text{ s}^{-2}$$

$$(b) \text{ Power} = \frac{\text{Work}}{\text{Time}} = \text{J s}^{-1} = \text{kg m}^2 \text{ s}^{-3} = \text{W}$$

- (c) The unit of energy per unit volume

$$\frac{\text{Energy}}{\text{Volume}} = \text{J m}^{-3} = \frac{\text{kg m}^2}{\text{s}^2 \text{ m}^3} = \text{kg m}^{-1} \text{ s}^{-2}$$

3. (a) The volume of a cube of side 1 cm is given by $V = (1 \text{ cm})^3$

$$\text{or } V = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$$

- (b) The surface area of a solid cylinder of radius r and height h is given by

$$A = \text{Area of two caps} + \text{Curved surface area} \\ = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$$

$$\text{Here } r = 2 \text{ cm} = 20 \text{ mm}, h = 10 \text{ cm} = 100 \text{ mm}$$

$$A = 2 \times \frac{22}{7} \times 20(20 + 100) \text{ mm}^2 \\ = 15086 \text{ mm}^2 = 1.5086 \times 10^4 \text{ mm}^2 = 1.5 \times 10^4 \text{ mm}^2$$

$$(c) \text{ Here } v = 18 \text{ km h}^{-1} = \frac{18 \times 1000 \text{ m}}{3600 \text{ s}} = 5 \text{ m s}^{-1}$$

$$t = 1 \text{ s}$$

$$x = vt = 5 \times 1 = 5 \text{ m}$$

- (d) Relative density of lead = 11.3, density of water = 1 g cm^{-3}

$$\text{We know that relative density of lead} = \frac{\text{Density of lead}}{\text{Density of water}}$$

$$\therefore \text{Density of lead} = \text{Relative density of lead} \times \text{Density of water} \\ = 11.3 \times 1 \text{ g cm}^{-3} = 11.3 \text{ g cm}^{-3}$$

$$\text{Also in S.I. system, density of water} = 10^3 \text{ kg m}^{-3}$$

$$\therefore \text{Density of lead} = 11.3 \times 10^3 \text{ kg m}^{-3} = 1.13 \times 10^4 \text{ kg m}^{-3}$$

$$4. (a) 1 \text{ kg m}^2 \text{ s}^{-2} = 1 \times 10^{-3} \text{ g (10}^2 \text{ cm)}^2 \text{ s}^{-2} = 10^7 \text{ g cm}^2 \text{ s}^{-2}$$

$$(b) 3 \text{ m s}^{-2} = 3 \times 10^{-3} \text{ km} \left(\frac{1}{60 \times 60} \text{ h} \right)^{-2}$$

$$(c) G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \\ = 6.67 \times 10^{-11} (\text{kg m s}^{-2}) \text{ m}^2 \text{ kg}^{-2} = 6.67 \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1} \\ = 6.67 \times 10^{-11} (100 \text{ cm})^3 \text{ s}^{-2} (1000 \text{ g})^{-1} = 6.67 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}$$

Exercise 1.2

1. From the principle of dimensional homogeneity, $[x] = [br^2]$

$$\Rightarrow [b] = \left[\frac{x}{r^2} \right]$$

$$\text{Therefore, unit of } b = \text{km s}^{-2}.$$

2. From the principle of dimensional homogeneity, $[F] = [at]$

$$\therefore [a] = \left[\frac{F}{t} \right] = \left[\frac{MLT^{-2}}{T} \right] = [MLT^{-3}]$$

$$\text{Similarly, } [F] = [bt^2]$$

$$\therefore [b] = \left[\frac{F}{t^2} \right] = \left[\frac{MLT^{-2}}{T^2} \right] = [MLT^{-4}]$$

3. From the principle of dimensional homogeneity, $[\alpha] = \text{dimensionless}$

$$\therefore [\alpha] = \left[\frac{1}{t} \right] = [T^{-1}]$$

$$\text{Similarly, } [x] = \frac{[v_0]}{[\alpha]}$$

$$\therefore [v_0] = [x][\alpha] = [L][T^{-1}] = [LT^{-1}]$$

4. $[X] = [\text{Force}] \times [\text{Density}] = [MLT^{-2}] \times [ML^{-3}] = [M^2L^{-2}T^{-2}]$

5. (n) = Number of particle passing from unit area in unit time

$$= \frac{\text{Number of particles}}{A \times t} = \frac{[M^0L^0T^0]}{[L^2][T]} = [L^{-2}T^{-1}]$$

$$[n_1] = [n_2] = \text{Number of particles in unit volume} = [L^{-3}]$$

Now from the given formula,

$$[D] = \frac{[n][x_2 - x_1]}{[n_2 - n_1]} = \frac{[L^{-2}T^{-1}][L]}{[L^{-3}]} = [L^2T^{-1}]$$

6. According to principle of dimensional homogeneity

$$[k] = \left[\frac{x}{v} \right] = \left[\frac{L}{LT^{-1}} \right] = [T]$$

7. From the principle of dimensional homogeneity,

$$[x^2] = [B]$$

$$\therefore [B] = [L^2]$$

$$\text{As well as } [U] = \frac{[A][x^{1/2}]}{[x^2] + [B]} \Rightarrow [ML^2T^{-2}] = \frac{[A][L^{1/2}]}{[L^2]}$$

$$\therefore [A] = [ML^{7/2}T^{-2}]$$

$$\text{Now } [AB] = [ML^{7/2}T^{-2}] \times [L^2] = [ML^{11/2}T^{-2}]$$

8. Let x = length. Therefore, $[X] = [L]$ and $[dx] = [L]$.

By the principle of dimensional homogeneity, $\left[\frac{x}{a} \right] = \text{dimensionless}.$

$$\therefore [a] = [x] = [L]$$

By substituting the dimension of each quantity in both sides,

$$\frac{[L]}{[L^2 - L^2]^{1/2}} = [L^n] \Rightarrow n = 0$$

$$9. [P] = [ML^2T^{-3}]$$

Using the relation,

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^x \left[\frac{L_1}{L_2} \right]^y \left[\frac{T_1}{T_2} \right]^z$$

$$= 1 \times 10^6 \left[\frac{1 \text{ kg}}{10 \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{1 \text{ dm}} \right]^2 \left[\frac{1 \text{ s}}{1 \text{ min}} \right]^{-3}$$

[As 1 MW = 10^6 W]

$$= 10^6 \left[\frac{1 \text{ kg}}{10 \text{ kg}} \right] \left[\frac{10 \text{ dm}}{1 \text{ dm}} \right]^2 \left[\frac{1 \text{ s}}{60 \text{ s}} \right]^{-3} = 2.16 \times 10^{12} \text{ unit}$$

10. $[E] = [ML^2T^{-2}]$

$$= [100 \text{ kg}] \times [1 \text{ km}]^2 \times [100 \text{ s}]^{-2}$$

$$= 100 \text{ kg} \times 10^6 \text{ m}^2 \times 10^{-4} \text{ s}^{-2}$$

$$= 10^4 \text{ kg} \times \text{m}^2 \text{ s}^{-2} = 10^4 \text{ J}$$

11. $\text{g cm s}^{-1} = 10^{-3} \text{ kg} \times 10^{-2} \text{ m} \times \text{s}^{-1} = 10^{-5} \text{ kg} \times \text{m} \times \text{s}^{-1} = 10^{-5} \text{ N s}$

12. Given S_t = distance traveled by the body in t th second. = $[LT^{-1}]$,

$$a = \text{Acceleration} = [LT^{-2}],$$

$$v = \text{Velocity} = [LT^{-1}],$$

$$t = \text{Time} = [T]$$

By substituting the dimension of each quantity, we can check the accuracy of the formula

$$S_t = u + \frac{1}{2} a(2t-1) \quad \therefore [LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$

$$\Rightarrow [LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

Since the dimensions of each term are equal, therefore, this equation is dimensionally correct. And after deriving this equation from kinematics, we can also prove that this equation is correct numerically also.

13. Let $T \propto S^x r^y \rho^z$ or $T = KS^x r^y \rho^z$

By substituting the dimension of each quantity in both sides,

$$[M^0 L^0 T^4] = K [MT^{-2}]^x [L]^y [ML^{-3}]^z = [M^{x+z} L^{y-3z} T^{-2x}]$$

By equating the power of M , L , and T in both sides,

$$x + z = 0, y - 3z = 0, -2x = 1$$

By solving above three equations, we get

$$x = -1/2, y = 3/2, z = 1/2$$

So the time period can be given as

$$T = KS^{-1/2} r^{3/2} \rho^{1/2} = K \sqrt{\frac{\rho r^3}{S}}$$

14. $[P^x Q^y C^z] = M^0 L^0 T^0$

By substituting the dimension of each quantity in the given expression,

$$[ML^{-1} T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = [M^{x+y} L^{-x+z} T^{-2x-3y-z}]$$

$$= M^0 L^0 T^0$$

By equating the power of M , L , and T on both sides, we get

$$x + y = 0, -x + z = 0, \text{ and } -2x - 3y - z = 0$$

By solving, we get $x = 1$, $y = -1$, and $z = 1$.

15. Let $M = V^a F^b E^c$

Putting dimensions of each quantities on both sides

$$[M] = [LT^{-1}]^a [MLT^{-2}]^b [ML^2 T^{-2}]^c$$

Equating powers of dimensions, we have

$$b + c = 1, a + b + 2c = 0, \text{ and } -a - 2b - 2c = 0$$

Solving these equations, $a = -2$, $b = 0$, and $c = 1$.

So $M = [V^{-2} F^0 E^1]$

3. Total surface area = $6 \times (5.402)^2 = 175.089 \text{ cm}^2 = 175.1 \text{ cm}^2$ (up to correct number of significant figure)
Total volume = $(5.402)^3 = 157.639 \text{ cm}^3 = 157.6 \text{ cm}^3$ (up to correct number of significant figure).

4. $9.99 \text{ m} + 0.0099 \text{ m} = 9.999 \text{ m} = 10.00 \text{ m}$ (In proper significant figures).

5. $3.124 \times 4.576 = 14.295 = 14.3$ (Correct to three significant figures).

6. Value of current (3.23 A) has minimum significant figure (3). So the value of potential difference $V (= IR)$ has only three significant figures. Hence, its value is 35.0 V.

7. (a) 953 (b) 954 (c) 953.3 (d) 953.4

8. (a) 7 (b) 6 (c) 6.6 (d) 6.4

9. $3.71 \text{ g} = 0.00371 \text{ kg}$

$$1.4 + 0.00371 = 1.40371 \text{ kg} \sim 1.4 \text{ kg}$$

[correct up to one place of decimal]

10. $A = \pi r^2 = \frac{22}{7} (0.56)^2 = 0.9856 \text{ m}^2 \sim 0.99 \text{ m}^2$

(two significant figures)

11. (a) $3.8 \times 10^{-6} + 4.2 \times 10^{-5} = 0.38 \times 10^{-5} + 4.2 \times 10^{-5}$
 $= 4.58 \times 10^{-5} \sim 4.6 \times 10^{-5}$

[up to one place of decimal]

(b) $4.7 \times 10^{-4} - 3.2 \times 10^{-6} = 4.7 \times 10^{-4} - 0.032 \times 10^{-4}$
 $= 4.668 \times 10^{-4} \sim 4.7 \times 10^{-4}$

[one decimal place]

(c) $4.8 \times 10^4 - 1.5 \times 10^3 = 4.8 \times 10^4 - 0.15 \times 10^4$
 $= 4.65 \times 10^4 \sim 4.6 \times 10^4$

[one decimal place]

12. $l = 4.234 \text{ m}$, $b = 1.005 \text{ m}$, $t = 2.01 \text{ cm} = 0.0201 \text{ m}$

$$\text{Area} = 2(lb + bt + tl)$$

$$= 2 [4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234]$$

$$= 8.7209478 \text{ m}^2 \sim 8.72 \text{ m}^2 \quad (\text{three significant figures})$$

$$\text{Volume} = lbt = 4.234 \times 1.005 \times 0.0201$$

$$= 0.0855289 \text{ m}^3 = 0.0855 \text{ m}^3$$

(three significant figures)

Exercise 1.4

1. Option (c) 2.000 cm, because it contains maximum number of significant figures, 4.

2. Not significant. In a number without decimal, the zeros on the right of non-zero digits are not significant.

3. Probable error reduces to 1/5 as the number of observations is made 5 times.

4. Quantity having higher powers, because errors get multiplied by powers.

5. Here $S = (13.8 \pm 0.2) \text{ cm}$; $t = (4.0 \pm 0.3) \text{ s}$

$$V = \frac{13.8}{4.0} = 3.45 \text{ ms}^{-1}$$

$$\frac{\Delta V}{V} = \pm \left(\frac{\Delta S}{S} + \frac{\Delta t}{t} \right) = \pm \left(\frac{0.2}{13.8} + \frac{0.3}{4.0} \right) = \pm 0.0895$$

$$\Delta V = \pm 0.0895 \times 3.45 = \pm 0.3$$

(rounded off to one place of decimal)

$$V = (3.45 \pm 0.3) \text{ ms}^{-1}$$

$$\text{Percentage error} = \frac{\Delta V}{V} \times 100 = 0.0895 \times 100 = 8.95\%$$

6. Volume $V = \frac{4}{3} \pi r^3 \Rightarrow \Delta V = \frac{4}{3} \pi 3r^2 \Delta r$

$$\frac{\Delta V}{V} \times 100 = 3 \frac{\Delta r}{r} \times 100 = 3 \times 1\% = 3\%$$

7. In parallel, $R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{5.0 \times 10.0}{5.0 + 10.0} = \frac{50}{15} = 3.3 \Omega$

Exercise 1.3

1. Thickness is measured up to second place of decimal.

2. Area = $1.5 \times 1.203 = 1.8045 \text{ cm}^2 = 1.8 \text{ cm}^2$ (up to correct number of significant figure).

Also $\frac{\Delta R_p}{R_p} \times 100 = \frac{\Delta R_1}{R_1} \times 100 + \frac{\Delta R_2}{R_2} \times 100 + \frac{\Delta(R_1 + R_2)}{R_1 + R_2} \times 100$
 $= \frac{0.2}{5.0} \times 100 + \frac{0.1}{10.0} \times 100 + \frac{0.3}{15} \times 100 = 7\%$
 $R_p = 3.3 \Omega \pm 7\%$

8. 35.0 V, the final result should contain three significant figures.

9. Length $l = 2.53 + 1.27 = 3.80$ cm, $\Delta l = 0.01 + 0.01 = 0.02$

(Most probable errors of both the rods are added)

Hence true value $= (3.80 \pm 0.02)$ cm

10. Maximum percentage error in $P = 4\% + 2 \times 2\% = 8\%$

11. Maximum error in density $= 3 + 3 \times 2 = 9\%$.

12. $R = \frac{V}{I}$

Hence $\left(\frac{\Delta R}{R} \times 100\right)_{\max} = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$
 $= \frac{5}{100} \times 100 + \frac{0.2}{10} \times 100$
 $= (5 + 2)\% = 7\%$

13. Volume of cylinder $V = \pi r^2 l$

Percentage error in volume

$\frac{\Delta V}{V} \times 100 = \frac{2\Delta r}{r} \times 100 + \frac{\Delta l}{l} \times 100$
 $= \left(2 \times \frac{0.01}{2.0} \times 100 + \frac{0.1}{5.0} \times 100\right)$
 $= (1 + 2)\% = 3\%$

14. $H = I^2 R t$

$\therefore \frac{\Delta H}{H} \times 100 = \left(\frac{2\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}\right) \times 100$
 $= (2 \times 3 + 4 + 6)\% = 16\%$

15. Quantity C has maximum power. So it brings maximum error in P .

Exercises

Single Correct Answer Type

1. (2) If a quantity depends upon more than three factors, each having dimensions, then the method of dimensional analysis cannot be applied. It is because applying the principle of homogeneity will give only three equations.

2. (4) The formula can be written as

$$\frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}} = 1$$

This formula is dimensionally correct as both the sides are dimensionless. Numerically, this ratio is equal to refractive index which is greater than 1. Hence, the equation is numerically incorrect.

3. (3) The correct relation for time period of simple pendulum is $T = 2\pi(l/g)^{1/2}$. So the given relation is numerically incorrect as the factor of 2π is missing. But it is correct dimensionally.

4. (4) As $\mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$, hence μ is dimensionless. Thus, each term on the RHS of given equation should be dimensionless, i.e., B/λ^2 is dimensionless, i.e., B should have dimension of λ^2 , i.e., cm^2 , i.e., area.

5. (3) Random error is reduced by making a large number of observations and taking mean of all the results.

6. (4) Energy density $= \frac{\text{Energy}}{\text{Volume}} = [ML^{-1}T^{-2}]$

Force/Area $= ML^{-1}T^{-2}$ [Charge/Volume] $\times$ [Voltage]

$= \frac{Q}{\text{vol.}} \times \frac{W}{Q} = \frac{\text{Work}}{\text{Volume}} = ML^{-1}T^{-2}$

The dimensions of (4) are different, i.e., $\frac{ML^2T^{-1}}{M} = M^0L^2T^{-1}$

Hence, correct choice is (4).

7. (4) $\frac{A}{B} = m$, $B = \frac{A}{m} = \frac{\text{Force}}{\text{Linear density}} = \frac{MLT^{-2}}{ML^{-1}}$

$\therefore B = [M^0L^2T^{-2}]$

Latent heat $= \frac{\text{Heat energy}}{\text{Mass}} = \frac{ML^2T^{-2}}{M} = [M^0L^2T^{-2}]$

Thus, B has same dimensions as that of latent heat.

8. (3) Momentum = Force $\times$ Time = N s

9. (3) Screw gauge has minimum least count of 0.001 cm; hence, it is the most precise instrument.

10. (4) Here $(2\pi ct/\lambda)$ as well as $(2\pi x/\lambda)$ are dimensionless. So unit of ct is same as that of λ . Unit of x is same as that of λ . Also,

$\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$

Hence, $\left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$. In option (d), x/λ is unitless. This is not the case with c/λ .

11. (2) Here units of y and A will be same and that of x and λ will be same. $\frac{2\pi}{\lambda}(ct - x)$ is dimensionless. Here ct/λ and x/λ are dimensionless. Unit of ct is same as that of λ or x .

12. (4) Here $(\omega t + \phi_0)$ is dimensionless because it is an argument of a trigonometric function.

13. (2) Percentage error in volume is

$\frac{0.01}{15.12} \times 100 + \frac{0.01}{10.15} \times 100 + \frac{0.01}{5.28} \times 100 = 0.35\%$

14. (4) $[x] = [Ay] = [B] \Rightarrow [y] = [B/A]$

Also, $[x] \neq [A]$ and $[Cz] = \text{dimensionless}$

$\Rightarrow [C] = [z^{-1}]$

15. (4) Relative density $\rho_r = \frac{W_1}{W_1 - W_2} = \frac{8.00}{8.00 - 6.00} = 4.00$

$\frac{\Delta \rho_r}{\rho_r} \times 100 = \frac{\Delta W_1}{W_1} \times 100 + \frac{\Delta(W_1 - W_2)}{W_1 - W_2} \times 100$
 $= \frac{0.05}{8.00} \times 100 + \frac{0.05 + 0.05}{2} \times 100 = 5.62\%$

$\therefore \rho_r = 4.00 \pm 5.62\%$

16. (3) $\frac{A}{B} = \frac{\text{Force}}{\text{Force}} = [M^0L^0T^0]$

$Ct = \text{angle} \Rightarrow C = \frac{\text{Angle}}{\text{Time}} = \frac{1}{T} = T^{-1}$

$Dx = \text{angle} \Rightarrow D = \frac{\text{Angle}}{\text{Distance}} = \frac{1}{L} = L^{-1}$

$\therefore \frac{C}{D} = \frac{T^{-1}}{L^{-1}} = [M^0LT^{-1}]$

17. (2) $y = r \sin(\omega t - kx)$

Here $\omega t = \text{angle} \Rightarrow \omega = \frac{1}{T} = T^{-1}$

Similarly, $kx = \text{angle} \Rightarrow k = \frac{1}{\lambda} = L^{-1} \therefore \frac{\omega}{k} = \frac{T^{-1}}{L^{-1}} = LT^{-1}$

Or simply ω/k represents wave velocity.

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = v, \text{ where } f \text{ is frequency.}$$

18. (4) $y = a \sin \omega t + bt + ct^2 \cos \omega t$

Here $a = y; b = y/t; c = y/t^2$

$$\therefore a \times b \times c = y \times y/t \times y/t^2 = (y/t)^3$$

19. (3) $n = \frac{1}{2l} \sqrt{\frac{T}{m}} \Rightarrow n^2 = \frac{1}{4l^2} \frac{T}{m}$

$$m = \frac{T}{4l^2 n^2} = \left[\frac{MLT^{-2}}{L^2 \times T^{-2}} \right] = [ML^{-1}]$$

20. (4) Momentum, $p = mv = MLT^{-1} = ML^{-3}L^4T^{-4}T^3 = DV^4F^{-3}$

21. (2) Here x^2 has the dimensions of L^2 , $B = [L^2]$

$$\text{Also } ML^2T^{-2} = \frac{AL^{1/2}}{L^2} \quad \text{or} \quad A = ML^{7/2}T^{-2}$$

$$\therefore A \times B = ML^{11/2}T^{-2}$$

22. (4) $AT^2 = LT^{-2} \times T^2 = [M^0L^0T^0]$

23. (1) Here at is dimensionless. So

$$a = \frac{1}{t} = \left[\frac{1}{T} \right] = [T^{-1}]$$

$$x = \frac{V_0}{a} \text{ and } V_0 = xa = [LT^{-1}] = [M^0LT^{-1}]$$

24. (2) Here x^n has the same dimensions as a^2 . Thus, $n = 2$ will make the expression dimensionally correct.

25. (2) Here αt^2 is a dimensionless. Therefore, $\alpha = 1/t^2$ and has the dimensions of T^{-2} .

26. (4) $f = Cm^xk^y$. Writing dimensions on both sides, $[M^0L^0T^{-1}] = M^x[ML^0T^{-2}]^y = [M^{x+y}T^{-2y}]$

Comparing dimensions on both sides, we have

$$0 = x + y \text{ and } -1 = -2y \Rightarrow y = \frac{1}{2}, x = -\frac{1}{2}$$

Aliter. Remembering that the frequency of oscillation of loaded spring is

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} (k)^{1/2} m^{-1/2}$$

$$\text{which gives } x = -\frac{1}{2} \text{ and } y = \frac{1}{2}$$

27. (4) Since LHS is displacement, so RHS should have dimensions of displacement. In (1), aT does not have the dimensions of displacement. Also the argument of a trigonometric function should be dimensionless. In (2), argument is not dimensionless and in (3), a/T does not have the dimensions of displacement. Hence, the correct choice is (4).

28. (4) $\frac{C^2}{g} = \frac{L^2T^{-2}}{LT^{-2}} = [L]$

29. (1) Here, $[\tan \theta] = \left[\frac{v^2}{rg} \right] = M^0L^0T^0$. Also, in actual expression

for the angle of banking of a road, there is no numerical factor involved. Therefore, the relation is both numerically and dimensionally correct.

30. (3) $X = M^x L^y T^{-z}$

$$\therefore \frac{\Delta X}{X} \times 100 = x \left(\frac{\Delta M}{M} \times 100 \right) + y \left(\frac{\Delta L}{L} \times 100 \right) + z \left(\frac{\Delta T}{T} \times 100 \right)$$

(errors are always added)

$$= (ax + by + cz) \text{ percent}$$

31. (4) $v = \sqrt{\frac{T}{m}} = \left[\frac{\frac{m'g}{M}}{l} \right]^{1/2} = \left[\frac{m'lg}{M} \right]^{1/2}$

It follows from here,

$$\frac{\Delta v}{v} = \frac{1}{2} \left[\frac{\Delta m'}{m'} + \frac{\Delta l}{l} + \frac{\Delta M}{M} \right] = \frac{1}{2} \left[\frac{0.1}{3.0} + \frac{0.001}{1.000} + \frac{0.1}{2.5} \right]$$

$$= \frac{1}{2} [0.03 + 0.001 + 0.04] = 0.036$$

$$\text{Percentage error in the measurement} = 0.036 \times 100 = 3.6$$

32. (2) Here a has the same dimensions as r^2 .

$$a = [T^2], b = \frac{T^2}{P \cdot x} = \frac{T^2}{ML^{-1}T^{-2}L^1} = \frac{T^4}{M} = [M^{-1}T^4]$$

$$\text{Now } \frac{a}{b} = \frac{T^2}{M^{-1}T^4} = ML^0T^{-2}$$

33. (2) Here, b and $x^2 = L^2$ have same dimensions.

$$\text{Also, } a = \frac{L^2}{E \times t} = \frac{L^2}{(ML^2T^{-2})T} = M^{-1}T^1 \Rightarrow a \times b = [M^{-1}L^2T^1]$$

34. (1) $h = [ML^2T^{-1}], G = [M^{-1}L^3T^{-2}], C = [LT^{-1}]$

$$\therefore h^{1/2}G^{-1/2}C^{1/2} = M^{1/2}L^{3/2}T^{-1/2} \times M^{1/2}L^{-3/2}T^1 \times L^{1/2}T^{-1/2}$$

$$= ML^0T^0 = \text{Mass}$$

35. (2) $(92.0 + 2.15) \text{ cm} = 94.15 \text{ cm}$. Rounding off to first decimal place, we get 94.2 cm.

36. (4) $n_2 = n_1 \left(\frac{M_1}{M_2} \right)^a \left(\frac{L_1}{L_2} \right)^b \left(\frac{T_1}{T_2} \right)^c$

$$\text{Dimensional formula of moment of inertia} = [ML^2T^0]$$

$$\therefore a = 1, b = 2, c = 0$$

$$n_1 = 12.0, M_1 = 1 \text{ kg}, M_2 = 10 \text{ g}$$

$$L_1 = 1 \text{ m}, L_2 = 5 \text{ cm}, T_1 = 1 \text{ s}, T_2 = 1 \text{ s}$$

$$n_2 = 12.0 \left(\frac{1 \text{ kg}}{10 \text{ g}} \right)^1 \left(\frac{1 \text{ m}}{5 \text{ cm}} \right)^2 \left(\frac{1 \text{ s}}{1 \text{ s}} \right)^0$$

$$= 12 \times \left(\frac{1000 \text{ g}}{10 \text{ g}} \right)^1 \left(\frac{100 \text{ cm}}{5 \text{ cm}} \right)^2 \times 1$$

$$= 12 \times 100 \times 400 = 4.8 \times 10^5$$

37. (1) Let $Y = [V^a A^b F^c]$

$$[ML^{-1}T^{-2}] = [LT^{-1}]^a [LT^{-2}]^b [MLT^{-2}]^c$$

$$ML^{-1}T^{-2} = M^c L^{a+b+c} T^{-a-2b-2c}$$

$$\therefore c = 1, a + b + c = -1, -a - 2b - 2c = -2$$

$$\text{On solving, we get } a = -4, b = 2, \text{ and } c = 1.$$

38. (3) Kinetic energy, $E = \frac{1}{2}mv^2$

$$\frac{\Delta E}{E} \times 100 = \frac{\Delta m}{m} \times 100 + 2 \frac{\Delta v}{v} \times 100 = 2 + 2 \times 3 = 8\%$$

$$\frac{\Delta V}{V} \times 100 = \frac{1}{2} \frac{\Delta a}{a} \times 100 + 2 \frac{\Delta b}{b} \times 100 + 3 \frac{\Delta c}{c} \times 100$$

$$= \frac{1}{2} \times 1 + 2 \times 3 + 3 \times 2 = 12.5\%$$

$$R = \frac{V}{I} = \frac{8}{4} = 2 \Omega$$

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$= \frac{0.5}{8} \times 100 + \frac{0.2}{4} \times 100 = 11.25\%$$

$$\Rightarrow R = (2 \pm 11.25\%) \Omega$$

$$\frac{M}{At} \propto P^x V^y$$

$$\Rightarrow ML^{-2}T^{-1} = [ML^{-1}T^{-2}]^x [L^3T^{-1}]^y = M^x L^{-x+y} T^{-2x-y}$$

$x = 1, -x + y = -2$, and $-2x - y = -1$

From here, we get $y = -1$. Thus, $x = -y$.

$$\frac{\Delta x}{x} \times 100 = \frac{\Delta a}{a} \times 100 + 2 \frac{\Delta b}{b} \times 100 + \frac{1}{2} \frac{\Delta c}{c} \times 100$$

$$= [4 + 2 \times 2 + 1/2 \times 3]\% = 9.5\%$$

$$\text{Required percentage} = 2 \times \frac{0.02}{0.24} \times 100 + \frac{1}{30} \times 100 + \frac{0.01}{4.80} \times 100$$

$$= 16.7 + 3.3 + 0.2 = 20\%$$

$$\text{By Coulomb's laws, } F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$\epsilon_0 = \frac{q_1 q_2}{4\pi \times F \times r^2}$$

$$\text{Taking dimensions, } \epsilon_0 = \frac{(AT)(AT)}{ML^1T^{-2} \times L^2} = [M^{-1}L^{-3}T^4A^2]$$

$$m \propto v^a p^b g^c \quad ML^0T^0 \propto (LT^{-1})^a (ML^{-3})^b (LT^{-2})^c$$

Comparing the powers of M, L , and T and solving, we get

$b = 1, c = -3, a = 6 \Rightarrow m \propto v^6$

$$\left[\frac{mr^2}{6\pi\eta} \right] = \left[\frac{ML^2}{ML^{-1}T^{-1}} \right] = [L^3T]$$

$$\text{As we have } [\eta] = [ML^{-1}T^{-1}]$$

$$\left[\left(\frac{6\pi m r \eta}{g^2} \right)^{1/2} \right] = \left[\left(\frac{MLML^{-1}T^{-1}}{L^2T^{-4}} \right)^{1/2} \right]$$

$$\left[\frac{m}{6\pi\eta r v} \right] = \left[\frac{M}{ML^{-1}T^{-1}LLT^{-1}} \right] = [L^{-1}T^2]$$

Thus, none of the given expressions have the dimensions of time constant.

$$\text{Let } T^2 = \rho^a r^b \sigma^c$$

$$T^2 = (ML^{-3})^a (L)^b (MT^{-2})^c = M^{a+c} L^{-3a+b} T^{-2c}$$

$a + c = 0, -3a + b = 0$, and $-2c = 2$

Hence, $a = 1, b = 3$, and $c = -1 \quad T^2 = \rho r^3 \sigma^{-1} = \frac{\rho r^3}{\sigma}$

$$[\eta] = [ML^{-1}T^{-2}]$$

$$\text{Hence, } \left[\sqrt{\frac{M}{\eta L}} \right] = \sqrt{\frac{[M]}{[ML^{-1}T^{-2}][L]}} = [T]$$

49. (4) Maximum error in measuring mass = 0.001 g, because least count is 0.001 g. Similarly, maximum error in measuring volume is 0.01 cm³.

$$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{0.001}{20.000} + \frac{0.01}{10.00}$$

$$= (5 \times 10^{-5}) + (1 \times 10^{-3}) = 1.05 \times 10^{-3}$$

$$\Delta \rho = (1.05 \times 10^{-3}) \times \rho$$

$$= 1.05 \times 10^{-3} \times \frac{20.000}{10.00} = 0.002 \text{ g cm}^{-3}$$

$$\Delta A = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} \right) A = \pm \left(\frac{0.1}{10.0} + \frac{0.01}{1.00} \right) \times (10.0 \times 1.00) \text{ cm}^2$$

$$= \pm 0.02 \times 10 = \pm 0.2 \text{ cm}^2$$

$$\text{Relative density} = \frac{W_a}{W_a - W_w}, \quad \rho = \frac{W_a}{w}$$

where ρ is relative density, W_a is weight in air, and w is loss in weight.

$$\frac{\Delta \rho}{\rho} = \frac{\Delta W_a}{W_a} + \frac{\Delta w}{w}$$

$$\text{For maximum error, } \frac{\Delta \rho}{\rho} = \frac{\Delta W_a}{W_a} + \frac{\Delta w}{w}$$

For maximum percentage error,

$$\frac{\Delta \rho}{\rho} \times 100 = \frac{\Delta W_a}{W_a} \times 100 + \frac{\Delta w}{w} \times 100$$

$$\text{Given } \Delta W_a = 0.1 \text{ gf and } W_a = 10.0 \text{ gf}$$

$$w = 10.0 - 5.0 = 5.0 \text{ gf}$$

$$\Delta w = \Delta W_a + \Delta W_w = 0.1 + 0.1 = 0.2 \text{ gf}$$

$$\frac{\Delta \rho}{\rho} \times 100 = \left(\frac{0.1}{10.0} \right) \times 100 + \left(\frac{0.2}{5.0} \right) \times 100 = 1 + 4 = 5$$

$$52. (4) \quad X = M^{-1}L^3T^{-2}$$

$$\frac{\Delta X}{X} = \frac{\Delta M}{M} + 3 \frac{\Delta L}{L} + 2 \frac{\Delta T}{T} = 2 + 3 \times 3 + 2 \times 4 = 19$$

$$53. (4) \quad \text{Required error is } 2 \times 2\% + 1\% + 1\%, \text{ i.e., } 6\%.$$

$$54. (2) \quad \text{Subtract 3.87 from 4.23 and then divide by 2.}$$

$$55. (3) \quad Y = \frac{F}{A} \cdot \frac{L}{\Delta L} = \frac{\text{dyne}}{\text{cm}^2} = \frac{10^{-5} \text{ N}}{10^{-4} \text{ m}^2} = 0.1 \text{ N/m}^2$$

$$56. (2) \quad F = \frac{\text{Newton}}{\text{m}^2}$$

$$\frac{hc}{d^2} = \text{joule-sec} \left(\frac{\text{metre}}{\text{sec}} \right) \frac{1}{(\text{metre})^2}$$

$$= \frac{\text{joule}}{\text{metre}} = \text{Newton}$$

$$\frac{hc}{d^4} = \left(\frac{hc}{d^2} \right) \frac{1}{d^2} = \frac{\text{Newton}}{\text{m}^2}. \text{ Hence } F = hc/d^4$$

$$\frac{hd}{c} = \frac{\text{Joule} \cdot \text{sec} \cdot (\text{metre})^2}{\left(\frac{\text{metre}}{\text{sec}} \right)} = \text{Newton} \cdot \text{metre}^2 \cdot \text{sec}^2$$

$$\frac{d^4}{hc} = \frac{1}{\left(\frac{hc}{d^4} \right)} = \frac{\text{m}^2}{\text{Newton}}$$

57. (3) Since for 50.14 cm, significant number = 4 and for 0.00025, significant numbers = 2

58. (1) Weight in air $(5.00 \pm 0.05)N$ Weight in water $= (4.00 \pm 0.05)N$ Loss of weight in water $= (1.00 \pm 0.1)N$ Now relative density $= \frac{\text{weight in air}}{\text{weight loss in water}}$

$$\text{i.e., } R.D = \frac{5.00 \pm 0.05}{1.00 \pm 0.1}$$

Now relative density with max permissible error

$$= \frac{5.00}{1.00} \pm \left(\frac{0.05}{5.00} + \frac{0.1}{1.00} \right) \times 100 = 5.0 \pm (1 + 10)\%$$

$$= 5.0 \pm 11\%$$

59. (4) $f = \frac{uv}{u+v}$

$$\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta(u+v)}{u+v} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u}{u+v} + \frac{\Delta v}{u+v}$$

60. (3) $D = (4.23 \pm 0.01) \text{ cm}$

$d = (3.89 \pm 0.01) \text{ cm}$

$$\Delta t = (D - d)/2 = (4.23 \pm 0.01) - (3.89 \pm 0.01)$$

$$= (4.23 - 3.89) \pm (0.01 + 0.01)$$

$$= (0.34 \pm 0.02)/2 \text{ cm} = (0.17 \pm 0.01) \text{ cm}$$

61. (4) $T \propto p^a d^b E^c$

$$[T] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b [ML^2T^{-2}]^c$$

Equating the exponent, $a + b + c = 0$

$$-a - 3b + 2c = 0 \text{ and } -a - 2c = 1$$

$$\text{By solving } a = -\frac{5}{6}$$

62. (1) $\left[\frac{\alpha Z}{K\theta} \right] = [M^0 L^0 T^0]$

$$[\alpha] = \left[\frac{K\theta}{Z} \right]$$

$$\text{But } [P] = \left[\frac{\alpha}{\beta} \right] \text{ So, } [\beta] = \left[\frac{\alpha}{P} \right] = \left[\frac{K\theta}{ZP} \right]$$

$$= \left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}} \right] = [M^0 L^2 T^0]$$

(Since dimensions of K' is of energy)

63. (1) $LHS = [x_1 + x_2] = [x_1] = [x_2] = [M^0 L T^0]$

$$RHS = \left[\frac{a_1 a_2 t^2}{2(a_1 + a_2)} \right] = \frac{[LT^{-2}][LT^{-2}][T^2]}{[LT^{-2}]} = [M^0 L T^0]$$

$$\therefore LHS = RHS$$

According to homogeneity principle, equation is dimensionally correct.

Hence, option (a) is correct.

64. (1) $V = \frac{S}{t}$

$$\Delta V = \frac{1}{t} \cdot \frac{\partial S}{\partial S} \cdot \Delta S + \frac{S}{t^2} \Delta t = \left(\frac{\Delta S}{t} + \frac{S \Delta t}{t^2} \right)$$

$$\text{So } \frac{\Delta S}{t} + \frac{S \Delta t}{t^2} = \frac{\Delta S}{\Delta t} \cdot \frac{\Delta t}{t} + \frac{(\Delta S) \cdot S(\Delta t)^2}{(\Delta S) \cdot t^2 (\Delta t)}$$

$$\text{or } \frac{\Delta S}{\Delta t} \left[\frac{\Delta t}{t} + \frac{S(\Delta t)^2}{(\Delta S)t^2} \right] = \pm \frac{\Delta S}{\Delta t} \text{ (given)}$$

$$\text{So, } \frac{\Delta t}{t} + \frac{S(\Delta t)^2}{(\Delta S)t^2} = \pm 1$$

Multiple Correct Answers Type

1. (1), (2), (4)

$$\text{Torque} = \vec{r} \times \vec{F}, \text{ Work} = \vec{r} \cdot \vec{F}$$

Both have dimensions $[ML^2T^{-2}]$.Angular momentum and Planck's constant have same dimensions $[ML^2T^{-1}]$.Light year and wavelength have the same dimension $[L]$.

2. (2), (3), (4)

Frequency and angular velocity have the same dimension $[T^{-1}]$.Tension has dimensions of force and surface tension has dimensions of force/length. Density has dimensions of mass/volume and energy density has dimensions of energy/volume. Angular momentum has dimensions of linear momentum $\times$ distance.

3. (1), (2), (3)

Pressure has dimensions $[ML^{-1}T^{-2}]$, force per area, energy per unit volume and momentum per unit area per second have same dimensions $[ML^{-1}T^{-2}]$.

4. (2), (4)

A dimensionally correct equation may or may not be correct. For example, $s = ut + at^2$ is dimensionally correct, but not correct actually. A dimensionally incorrect equation may be correct also. For example,

$$s = u + \frac{a}{2} (2n - 1) \text{ is a correct equation, but not correct dimensionally.}$$

5. (1), (2), (4)

$$\text{Mean value} = \frac{1.51 + 1.53 + 1.53 + 1.52 + 1.54}{5} = 1.53$$

Absolute errors are $(1.53 - 1.51 = 0.02)$,

$$(1.53 - 1.53 = 0.00), (1.53 - 1.53 = 0.00),$$

$$(1.53 - 1.52 = 0.01) \text{ and } (1.54 - 1.53 = 0.01)$$

Mean absolute error is

$$\frac{0.02 + 0.00 + 0.00 + 0.01 + 0.01}{5} = \frac{0.04}{5} = 0.008 \approx 0.01$$

So choice (1) is correct.

$$\text{Relative error} = \frac{0.01}{1.53} = 0.00653 \approx 0.01\% \text{ error} = \frac{0.01}{1.53} \times 100 = 1\%$$

6. (1), (2), (3)

Least count = 1 MSD - 1 VSD

$$= S - V = S - \left(\frac{n-1}{n} \right) S = \frac{S}{n} \quad [\because nV = (n-1)S]$$

It is also called vernier constant.

So choices (1) and (2) are correct.

Choice (4) is wrong and choice (3) is correct, since for all vernier scales similar approach can be used.

7. (1), (3)

 L/R , CR and $\sqrt{LC}$ all have dimensions of time $[T]$.

8. (1), (3)

Dimensions of magnetic flux = magnetic field $\times$ area

$$= \left[\frac{F}{iL} \right] \times [\text{Area}] = \frac{MLT^{-2}}{AL} L^2 = [ML^2T^{-2}A^{-1}]$$

$$\text{Dimensions of } \frac{h}{a} = \left[\frac{\text{Planck's constant}}{\text{Charge}} \right] = \left[\frac{ML^2T^{-1}}{AT} \right] = [ML^2T^{-2}A^{-1}]$$

$$\text{From Gauss theorem, electric flux} = \frac{q}{\epsilon_0}$$

9. (1), (2), (3)

$$\text{Unit of } x = \frac{\text{Unit of } E}{\text{Unit of } B} = \text{Unit of velocity}$$

Because $E = vB$

$$y = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \rightarrow \text{Velocity of light}$$

$$\text{Unit of } z = \frac{\text{Unit of } l}{\text{Unit of } RC} = \frac{\text{Length}}{\text{Time}} = \text{Velocity}$$

10. (1), (2)

$$\text{We know that pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\text{Pressure} = \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Distance}} = \frac{\text{Work}}{\text{Volume}} = \frac{\text{Energy}}{\text{Volume}}$$

11. (1), (2), (3), (4)

$$\text{Velocity} = \text{length/time, acceleration} = \text{length}/(\text{time})^2$$

$$\Rightarrow \text{Length} = \frac{(\text{velocity})^2}{\text{acceleration}} \quad \text{i.e., } L' = \frac{v'^2}{a'}, L = \frac{v^2}{a}$$

$$\Rightarrow \frac{L'}{L} = \left(\frac{v'}{v}\right)^2 \left(\frac{a}{a'}\right) = \left(\frac{\alpha^2}{\beta}\right)^2 \frac{1}{\alpha\beta} = \alpha^3/\beta^3$$

$$\text{Now, } m' = \frac{F'}{a'}, m = \frac{F}{a}$$

$$\frac{m'}{m} = \frac{F' a}{F a'} = \frac{1}{\alpha\beta} \times \frac{1}{\alpha\beta} = \frac{1}{\alpha^2\beta^2}$$

$$\text{Time} = \text{velocity/acceleration, i.e.,}$$

$$T' = \frac{v'}{a'} \text{ and } T = \frac{v}{a}$$

$$\frac{T'}{T} = \frac{v' a}{v a'} = \frac{\alpha^2}{\beta} \frac{1}{\alpha\beta} = \frac{\alpha}{\beta^2}$$

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

$$p' = m'v', p = mv, \frac{p'}{p} = \frac{m' v'}{m v} = \frac{1}{\alpha^2\beta^2} \frac{\alpha^2}{\beta} \frac{\alpha^2}{\beta} = \frac{1}{\beta^3}$$

12. (1), (2), (3), (4)

$$[P] = \left[\frac{a}{V^2}\right] \Rightarrow [PV] = \left[\frac{a}{V}\right]$$

$$\text{also } [PV] = \left[\frac{a}{V}\right] = \left[\frac{ab}{V^2}\right] \quad \therefore [b] = [v]$$

$$\text{as } [b] = [V] \text{ So } [Pb] = [PV]$$

13. (1), (2), (3)

$$(1) \text{ Here, } \sin^{-1}\left(\frac{x}{b} - 1\right) \text{ is dimensionless.}$$

$$(2) \text{ Let } \sin^{-1}\left(\frac{x}{b} - 1\right) = \theta$$

$$\therefore \sin \theta = \left(\frac{x}{b} - 1\right)$$

Here, θ should be in radian.

So, the unit of $\sin^{-1}\left(\frac{x}{b} - 1\right)$ is radian.

$$(3) \therefore \text{Dimensions of } \frac{x}{b} = \text{Dimensions of } 1 = \text{Dimensionless.}$$

$$\therefore \text{Dimension of } x = \text{Dimensions of } b.$$

14. (1), (2), (3), (4)

$$(ii) [v] = [Bt]$$

$$\therefore [B] = \frac{[v]}{[t]} = [LT^{-2}]$$

It is dimension of acceleration.

$$\therefore [V] = [\sqrt{AB}] \text{ or } \frac{[v]^2}{[t]} = [A]$$

$$\therefore [A] = \frac{[L^2 T^{-2}]}{[L T^{-2}]} = [L]$$

= Dimensions of distance. $[D] = [l]$

$$\therefore \text{Dimension of time.}$$

$$\therefore [V] = \left[\frac{C}{t}\right]$$

$$\therefore [C] = [L] = \text{Dimensions of distance.}$$

15. (1), (2), (3), (4)

$$[P] = [A] = [B]$$

$$\text{and } [bt] = [ct]$$

$$\therefore [b] = [c]$$

16. (2), (3), (4)

$$[D] = [x] \text{ and } [A] = \left[\frac{B}{C^2}\right]$$

The dimensions of A and B may be same if C is dimensionless. Hence option (2) may be correct.

$$\left(A + \frac{B}{C^2}\right)(D - x) = y$$

$$\Rightarrow AD - Ax + \frac{BD}{C^2} - \frac{Bx}{C^2} = y$$

$$\Rightarrow [y] = [AD] = [Ax] = \left[\frac{BD}{C^2}\right] = \left[\frac{Bx}{C^2}\right]$$

17. (1), (3)

$$\therefore V = \frac{4}{3}\pi R^3$$

$$\therefore \frac{\Delta V}{V} = \frac{3\Delta R}{R}$$

$$\therefore \frac{\Delta V}{V} \times 100 = 3 \frac{\Delta R}{R} \times 100 = 3\%$$

$$(3) A = 4\pi R^2$$

$$\therefore \frac{\Delta A}{A} = \frac{2\Delta R}{R}$$

$$\therefore \frac{\Delta A}{A} \times 100 = 2 \left(\frac{\Delta R}{R} \times 100\right) = 2 \times 1\% = 2\%$$

18. (2), (4)

$$\therefore V = \frac{4}{3}\pi R^3$$

$$\therefore \frac{\Delta V}{V} = 3 \frac{\Delta R}{R}$$

$$\text{or } \frac{\Delta V}{V} \times 100 = \frac{\Delta R}{R} \times 100$$

$$\text{or } 9\% = 3 \frac{\Delta R}{R} \times 100$$

$$\therefore \frac{\Delta R}{R} \times 100 = 3\%$$

$$(4) \therefore A = 4\pi R^3$$

$$\therefore \frac{\Delta A}{A} = \frac{3\Delta R}{R}$$

$$\therefore \frac{\Delta A}{A} \times 100 = 3 \frac{\Delta R}{R} \times 100 = 3 \times 3\% = 9\%$$

19. (2), (3)

$$\text{Dimensions of } \left[\frac{p}{\rho g}\right] = \text{dimensions of } \left[\frac{v^2}{g}\right]$$

= dimensions of h = dimensions of u_0 .

$$\therefore \text{Dimensions of } \mu_0 = [M^0 L T^0]$$

Hence, options (2) and (3) are correct.

20. (1), (2), (3)

- (1) Strain and coefficient of friction both are dimensionless
- (2) Disintegration constant and frequency of light both have dimensions $[T^{-1}]$
- (3) Heat capacity has unit cal/kg while gravitational potential has unit J/kg i.e. both have dimensions $[L^2T^{-2}]$.

21. (1), (4)

According to the problem, P , Q and R are having different dimensions, since sum and difference of physical dimensions are meaningless.

i.e., $(P - Q)$ and $(R + Q)$ are not meaningful.

So, in option (2) PQ may have the same dimensions are those of R and in option (3) PR and Q^2 may have same dimensions as those of R .

Hence, (1) and (4) are not meaningful.

22. (2), (4)

We know that energy of radiation, $E = hv$. So, we have to compare h with dimensional formula of each option.

$$[h] = \frac{[E]}{[v]} = \frac{\text{Force} \times \text{Displacement}}{\text{Frequency}} = \frac{[M^1L^2T^{-2}]}{[T^{-1}]} = [M^1L^2T^{-1}]$$

(1) Dimension of linear impulse

$$[I] = [Ft] = [MLT^{-2}][T] = [MLT^{-1}]$$

Where t is the time interval.

(2) Dimension of angular impulse

$$[J] = [I\omega] = [ML^2][T^{-1}] = [ML^2T^{-1}]$$

(3) Dimension of linear momentum

$$[P] = [mv] = [M][LT^{-1}] = [MLT^{-1}]$$

(4) Dimension of angular momentum

$$[L] = [mvr] = [M][LT^{-1}][L] = [ML^2T^{-1}]$$

Hence, dimension of angular impulse and angular momentum is same as Planck's constant (h).

23. (1), (2)

$$(1) \text{ Let } 1 \text{ m/s} = x \left(\frac{10 \text{ m}}{20 \text{ s}} \right) \Rightarrow x = 2$$

$$(2) 1 \text{ W} = x \text{ W}'$$

$$x = 1 \frac{\text{W}}{\text{W}'} = 1 \left(\frac{M_1 L_1^2 T_1^{-3}}{M_2 L_2^2 T_2^{-3}} \right) = 1 \left(\frac{1 \text{ kg}}{5 \text{ kg}} \right) \left(\frac{1 \text{ m}}{10 \text{ m}} \right)^2 \left(\frac{1 \text{ s}}{20 \text{ s}} \right)^2 = 16$$

$$(3) 1 \text{ J} = x \text{ J}'$$

$$x = 1 \frac{\text{J}}{\text{J}'} = 1 \left(\frac{M_1 L_1^2 T_1^{-2}}{M_2 L_2^2 T_2^{-2}} \right) = 1 \left(\frac{1 \text{ kg}}{5 \text{ kg}} \right) \left(\frac{1 \text{ m}}{10 \text{ m}} \right)^2 \left(\frac{1 \text{ s}}{20 \text{ s}} \right)^2 = \frac{4}{5}$$

$$(4) 1 \text{ N} = x \text{ N}'$$

$$x = 1 \frac{\text{N}}{\text{N}'} = 1 \left(\frac{M_1 L_1 T_1^{-2}}{M_2 L_2 T_2^{-2}} \right) = 1 \left(\frac{1 \text{ kg}}{5 \text{ kg}} \right) \left(\frac{1 \text{ m}}{10 \text{ m}} \right) \left(\frac{1 \text{ s}}{20 \text{ s}} \right)^{-2} = 8$$

24. (1), (2), (3), (4)

$$\text{Unit of mass} = \frac{\text{Force}}{\text{Acceleration}} = \frac{6}{24} = \frac{1}{4}$$

$$\text{Unit of time} = \frac{\text{Velocity}}{\text{Acceleration}} = \frac{4}{24} = \frac{1}{6}$$

$$\text{Unit of energy} = (\text{Mass})(\text{Velocity})^2 = \left(\frac{1}{4} \right) (4)^2 = 4 \text{ J}$$

$$\text{Unit of momentum} = \text{Force} \times \text{Time} = 6 \times 24 \text{ W}$$

$$\text{Unit of power} = \frac{\text{Energy}}{\text{Time}} = \frac{4}{1/6} = 24 \text{ W}$$

Linked Comprehension Type

For Problems 1-5

$$1. (1) [a] = [PV^2] = \left[\frac{FV^2}{A} \right] = \frac{[MLT^{-2}L^6]}{[L^2]} = [ML^5T^{-2}]$$

$$2. (3) [b] = [V] = [L^3]$$

$$3. (3) \text{ According to given equation, } PV + \frac{a}{V} - Pb - \frac{ab}{V^2} = RT$$

As dimensions of all the terms on LHS must be equal to dimensions of term RT on RHS, a/V^2 does not have the same dimensional formula as that of RT .

$$4. (4) \left[\frac{ab}{V^2} \right] = [RT] \Rightarrow \left[\frac{ab}{RT} \right] = [V^2] = [L^6]$$

$$5. (1) [RT] = [PV] = \left[\frac{F}{A} \cdot V \right] = [ML^2T^{-2}]$$

So the dimensional formula of RT is same as that of energy.

For Problems 6-8

$$6. (4) 1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kg m}^2 \text{ s}^{-2} = n_2 (\alpha \text{ kg}) (\beta \text{ m})^2 (\gamma \text{ s})^{-2}$$

$$\Rightarrow n_2 = 4.2 \alpha^{-1} \beta^2 \gamma^2$$

$$\text{So, } 1 \text{ cal} = (4.2 \alpha^{-1} \beta^2 \gamma^2) \text{ new units}$$

$$7. (3) \text{ Let } T \propto \sigma^a \rho^b r^c$$

$$M^0 L^0 T = (MT^{-2})^a (ML^{-3})^b L^c$$

$$\text{Equating the powers of } M: 0 = a + b \Rightarrow b = -a$$

$$L: 0 = -3b + c \Rightarrow c = 3b$$

$$T: 1 = -2a \Rightarrow a = -\frac{1}{2}, b = \frac{1}{2}, c = \frac{3}{2} \Rightarrow T = k \sqrt{\frac{r^3 \rho}{\sigma}}$$

$$8. (4) E \propto m^a f^b A^c$$

$$ML^2T^{-2} = m^a T^{-b} L^c$$

$$\therefore a = 1, b = 2, c = 2$$

For Problems 9-11

$$9. (2) A = lb = 4.4457 \text{ m}^2 = 4.45 \text{ m}^2$$

One should retain only three significant figures.

$$10. (2) \text{ For } x \text{ to be order of magnitude, } 0.5 < \frac{147}{10^x} \leq 5 \text{ should be satisfied}$$

$$11. (2) \text{ To find say difference between 20.17 and 20.15, we get 0.02 (i.e., one significant figure only).}$$

For Problems 12-14

$$12. (2), 13. (3), 14. (2)$$

$$v = k p^x d^y$$

$$LT^{-1} = [ML^{-1}T^{-2}]^x [ML^{-3}]^y$$

$$LT^{-1} = M^{x+y} L^{-x-3y} T^{-2x}$$

$$\Rightarrow x + y = 0, -x - 3y = 1, -2x = -1$$

$$\Rightarrow x = \frac{1}{2}, y = -\frac{1}{2}$$

$$\text{So } v = k p^{1/2} d^{-1/2} = k \sqrt{\frac{p}{d}}$$

Obviously if d increases, v decreases.

For Problems 15-17

$$15. (1)$$

$$\text{Given } X = YFe^{-\beta r^2} + ZW \sin(\alpha r)$$

$$\text{Here } \text{Dim}(\beta) = L^{-2}, \text{Dim}(\alpha) = L^{-1}$$

$$\text{Also } \text{Dim}(X) = \text{Dim}(YF)$$

and
Now
So

$$\text{Dim}(X) = \text{Dim}(ZW)$$

$$\text{if } \text{Dim}(Y) = L$$

$$\text{Dim}(X) = [LMLT^{-2}] = [ML^2T^{-2}] \text{ from equation (i)}$$

and

$$\text{Dim}(Z) = \text{Dim}\left(\frac{X}{W}\right) = \left[\frac{ML^2T^{-2}}{ML^2T^{-2}}\right] = [M^0L^0T^0]$$

From equation (ii)

$$\text{Thus, } \text{Dim}\left(\frac{\alpha YZ}{\beta F}\right) = \left[\frac{L^{-1}L}{L^{-2}MLT^{-2}}\right] = [M^{-1}LT^2]$$

$$16. (1) \text{ If } D(Y) = [LT^{-1}], \text{ then } D(X) = [LT^{-1}MLT^{-2}]$$

$$= [ML^2T^{-3}] \text{ from equation (i).}$$

$$17. (2) \text{ If } D(Z) = T^{-1}, \text{ then } D(Y) = ?$$

$$\text{From equation (ii): } D(X) = [T^{-1}ML^2T^{-2}] = [ML^2T^{-3}]$$

So from Equation (i):

$$D(Y) = \frac{D(X)}{D(F)} = \left[\frac{ML^2T^{-3}}{MLT^{-2}}\right] = [LT^{-1}]$$

For Problems 18 and 19

18. (1)

The dimensions of resistance are $[I^{-2}][ML^2T^{-3}]$, where I is electric current.

$$\therefore Q = It$$

where Q is electric charge.

$$\therefore [Q] = [IT] \Rightarrow [I] = [QT^{-1}]$$

$$\text{Dimensions of resistance are } [Q^{-2}ML^2T^{-1}]$$

$$\therefore n_1v_1 = n_2v_2$$

$$\text{or } 1[Q_1^2 M_1 L_1^2 T_1^{-1}] = n_2[Q_2^2 M_2 L_2^2 T_2^{-1}]$$

$$\therefore n_2 = \left[\frac{Q_1}{Q_2}\right]^{-2} \left[\frac{M_1}{M_2}\right] \left[\frac{L_1}{L_2}\right]^2 \left[\frac{T_1}{T_2}\right]^{-1}$$

$$= \left[\frac{Q_1}{Q_1 d}\right]^{-2} \left[\frac{M_1}{AM_1}\right] \left[\frac{L_1}{BL_1}\right]^2 \left[\frac{T_1}{CT_1}\right]^{-1}$$

$$= [A^{-1}B^{-2}Cd^{-2}]$$

19. (2)

Farad is SI unit of capacitance of capacitor, its dimensions are $[I^2M^{-1}L^{-2}T^4]$

$$= [Q^2M^{-1}L^{-2}T^2]$$

$$\therefore n_1v_1 = n_2v_2$$

$$\text{or } 1[Q_1^2 M_1^{-1} L_1^{-2} T_1^2] = n_2[Q_2^2 M_2^{-1} L_2^{-2} T_2^2]$$

$$\therefore n_2 = \left[\frac{Q_1}{Q_2}\right]^2 \left[\frac{M_1}{M_2}\right]^{-1} \left[\frac{L_1}{L_2}\right]^{-2} \left[\frac{T_1}{T_2}\right]^2$$

$$= \left[\frac{Q_1}{Q_1 d}\right]^2 \left[\frac{M_1}{AM_1}\right]^{-1} \left[\frac{L_1}{BL_1}\right]^{-2} \left[\frac{T_1}{CT_1}\right]^2$$

$$= [d^{-2}AB^2C^{-2}]$$

Matrix Match Type

$$1. \text{ i} \rightarrow \text{c}; \text{ ii} \rightarrow \text{d}; \text{ iii} \rightarrow \text{a}; \text{ iv} \rightarrow \text{b}$$

$$(i) [LC] = \left[\frac{L}{R}RC\right] = T^2$$

...(ii)

$$(ii) [LR] = \left[\frac{L}{R}R^2\right] = T[R^2] = T\left[\frac{V}{I}\right]^2 = T\left[\frac{W}{QI}\right]^2$$

$$= T\left(\frac{W}{I^2t}\right)^2 = T\left(\frac{ML^2T^{-2}}{A^2T}\right)^2 = M^2L^4T^{-5}A^{-4}$$

$$(iii) [H] = \left[\frac{\text{Heat}}{\text{Mass}}\right] = \frac{ML^2T^{-2}}{M} = L^2T^{-2}$$

$$(iv) [s] = \left[\frac{\text{Heat}}{\text{Mass} \times \text{Temperature}}\right] = \frac{ML^2T^{-2}}{MK} = L^2T^{-2}K^{-1}$$

$$2. \text{ i} \rightarrow \text{c}; \text{ ii} \rightarrow \text{a}; \text{ iii} \rightarrow \text{d}; \text{ iv} \rightarrow \text{b}$$

We know that least count is given by s/n , so

$$(i) \text{ least count} = s/n = 1/10 = 0.1 \text{ mm}$$

$$(ii) \text{ least count} = s/n = 0.5/10 = 0.05 \text{ mm}$$

$$(iii) \text{ least count} = s/n = 0.5/20 = 0.025 \text{ mm}$$

$$(iv) \text{ least count} = s/n = 1/100 = 0.01 \text{ mm}$$

$$3. \text{ i} \rightarrow \text{d}; \text{ ii} \rightarrow \text{a}, \text{ c}; \text{ iii} \rightarrow \text{b}; \text{ iv} \rightarrow \text{c}, \text{ d}$$

Backlash error is caused by loose fittings, wear and tear, etc. in the screw mechanisms.

Zero error may be positive or negative but will always be subtracted.

In vernier callipers, least count is the difference between 1 MSD and 1 VSD.

Error in screw gauge may be positive or negative and may be due to loose fitting of the circular scale.

$$4. \text{ i} \rightarrow \text{a}; \text{ ii} \rightarrow \text{a}; \text{ iii} \rightarrow \text{d}; \text{ iv} \rightarrow \text{c}$$

All digits are significant after decimal

$$47.23 \div (2.3) = 20.5 \approx 21 \text{ (should have only two significant digits.)}$$

3 is a number with one significant digit.

$$5. (1)$$

$$(i) 1 \text{ N} = \frac{1 \text{ kg } 1 \text{ m}}{1 \text{ sec}^2} = \frac{\left(\frac{1 \text{ unit mass}}{\alpha}\right)\left(\frac{1 \text{ unit length}}{\beta}\right)}{\left(\frac{1 \text{ unit time}}{\gamma}\right)^2} = \frac{\gamma^2}{\alpha\beta}$$

unit force in new system.

$$(ii) 1 \text{ J} = \frac{1 \text{ kg}(1 \text{ m})^2}{(1 \text{ sec})^2} = \frac{1\left(\frac{1}{\alpha}\right)^2}{\left(\frac{1}{\gamma}\right)^2} \text{ unit energy} = \alpha^{-1}\beta^{-2}\gamma^2$$

$$(iii) 1 \text{ pascal} = \frac{1 \text{ kg}}{(1 \text{ m})(1 \text{ sec})^2} = \frac{\frac{1}{\alpha}}{\frac{1}{\beta}\left(\frac{1}{\gamma}\right)^2} \text{ unit pressure} = \alpha^{-1}\beta\gamma^2$$

$$(iv) \alpha_{ACME} = \alpha \frac{(1 \text{ unit mass})(1 \text{ unit length})^2}{(1 \text{ unit time})^3}$$

$$= \alpha \frac{\alpha \text{ kg} (\beta \text{ m})^2}{(\gamma \text{ sec})^3} = \alpha^2 \beta^2 \gamma^{-3} \text{ watt}$$

$$6. (3)$$

$$(i) \text{ Given } I = \frac{GM_e}{R_e^2}$$

The unit of G can be found from newton gravitational law

$$F = \frac{GM_1M_2}{r^2} \Rightarrow G = \frac{Fr^2}{M_1M_2}$$

$$\text{Hence unit of } G \text{ is } \frac{\text{Nm}^2}{\text{kg}^2}$$

Hence unit of $I = \left(\frac{\text{Nm}^2}{\text{kg}^2} \right) \cdot \text{kg} = \text{newton/kg}$

(ii) Given $E = \frac{F}{Q}$

Hence the unit of E should be newton/coulomb.

(iii) Given $R = \frac{V}{I}$

The SI unit of V is volt and that of I is ampere. Hence unit of R should be volt/ampere, this is called ohm (Ω).

(iv) Given $R = \frac{pV}{nT} = \frac{(\text{N/m}^2) \times \text{m}^3}{\text{mol} \times \text{kelvin}}$

$$= \frac{\text{Nm}}{\text{mol-kelvin}} = \frac{\text{joule}}{\text{mol-kelvin}}$$

7. (1)

(i) Modulus of rigidity = $\frac{\text{Shear stress}}{\text{Shear strain}}$

Shear strain has no unit,

But stress = $\frac{F}{A} = \text{Nm}^{-2}$

$\therefore$ The unit of modulus of rigidity is Nm^{-2}
 Thus, (i) $\rightarrow$ (d)

(ii) $\Delta H = mL$

where ΔH = heat supplied, m = mass, L = latent heat

$\therefore L = \frac{\Delta H}{m} = \text{Jkg}^{-1}$

Thus, (ii) $\rightarrow$ (c)

(iii) According to Ohm's law, $V = IR$

where V = potential difference. Its unit is volt.

I = electric current. Its unit is ampere.

$\therefore$ Electric resistance, $R = \frac{V}{I} = \text{Volt per ampere or ohm}$

$\therefore R = \rho \frac{l}{A}$, where, l = length of rod

and A = area of cross-section of rod

$\therefore$ Electric resistivity = $\rho = \frac{RA}{l} = \frac{\text{ohm m}^2}{\text{m}}$
 $= \text{ohm-metre}$
 $= \text{volt-metre per ampere}$

Thus, (iii) $\rightarrow$ (a, b)

(iv) Gravitational potential is gravitational potential energy per unit mass. Thus, its unit is Joule per kilogram.

Thus, (iv) $\rightarrow$ (c)

8. (2)

(i) Pressure amplitude in pressure wave = βAk

$\therefore \beta Ak = \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \text{Nm}^{-2}$

(ii) Intensity of wave = $\frac{1}{2} \rho A^2 \omega^2 c = \frac{\text{Power}}{\text{Area}} = \text{Wm}^{-2}$

Thus, (ii) $\rightarrow$ (a)

(iii) $\frac{F^2 l}{SY} = \frac{\text{N}^2 \text{m}}{\text{m}^2 \text{N/m}^2} = \text{Nm} = \text{J}$

Thus, (iii) $\rightarrow$ (b)

$$\sqrt{\frac{r^3}{GM}} = \sqrt{\frac{m^3}{\frac{\text{Nm}^2}{\text{kg}^2} \times \text{kg}}}$$

$$= \sqrt{\frac{\text{kg m}}{\text{N}}} = \sqrt{\frac{\text{kg m}}{\text{kg m/s}^2}} = \text{second}$$

Thus, (iv) $\rightarrow$ (d)

Numerical Value Type

1. (0)

$$\left(\frac{t^2}{a^2} - 1 \right) \text{ is dimensionless}$$

$$\pm t = \pm a$$

$$[a] = [t]$$

$$\sqrt{(3at - 2t^2)} = [t]$$

$$\left[\frac{dt}{\sqrt{3at - t^2}} \right] = \frac{[t]}{[t]} = [M^0 L^0 T^0]$$

a^x should be dimensionless, so $x = 0$.

2. (8)

The unit of Planck's constant is joule second or $\text{kgm}^2 \text{ per second}$

$$\frac{(\text{unit of mass})(\text{unit of length})^2}{(\text{unit of time})} = \frac{2 \times 4^2}{4} = 8 \text{ times}$$

$$\therefore n = 8$$

3. (6)

The unit of LHS is kg. The unit of RHS should be kg.

$$\text{Here, } k\rho v^x g^{-3} = \left(\frac{\text{kg}}{\text{m}^3} \right) \left(\frac{\text{m}}{\text{s}} \right)^x \left(\frac{\text{m}}{\text{s}^2} \right)^{-3}$$

$$= (\text{kg})(\text{m})^{-3+x-3}(\text{s})^{+6-x}$$

$$\therefore -6 + x = 0$$

$$\Rightarrow x = 6$$

4. (1)

$$\therefore F = ma$$

$$\Rightarrow m = \frac{F}{a} = (\text{Force}) + \frac{\text{Change in velocity}}{\text{Time}}$$

$$= \frac{\text{Force} \times \text{Time}}{\text{Change in velocity}}$$

$$= (\text{strength})(\text{sec})(\text{run})^{-1}$$

Thus, $x = 1, y = 1$ and $x = -1$

$$\therefore \frac{y}{x} = 1$$

5. (1)

$$[F] = [MLT^{-2}]$$

$$\Rightarrow [V] = [LT]^{-1}$$

$$\therefore [L] = [vT]$$

$$\therefore [M] = \frac{[F]}{[LT^{-2}]} = \frac{[F]}{[vT] \times [T^{-2}]} = [Fv^{-1}T]$$

$$\therefore \text{Pressure} = [ML^{-1}T^{-2}]$$

$$[Fv^{-1}T][vT]^{-1}[T^{-2}] = [Fv^{-2}T^{-2}]$$

Thus, in the dimensional formula of pressure, the dimension of force is 1.

6. (1)

If x is length, then the dimensions of LHS $\frac{[L]}{[L^2]} = [L^{-1}]$

Also, dimensions of $\frac{1}{a} = \text{dimensions of } x = [L]$

The dimensions of RHS $= [L^{-1}]^n = L^{-n}$

According to homogeneity principle,
Dimensions of LHS = Dimensions of RHS

$$[L^{-1}] = [L^{-n}] \Rightarrow n = 1$$

(1) $LHS = [L]^3$

$$RHS = [M^{-1}L^3T^{-2}]^x [LT^{-1}]^y [ML^2T^{-1}]^z$$

$$= [M^{-x+z}L^{3x+y+2z}T^{-2x-y-z}]$$

According to homogeneity principle,

$$LHS = RHS$$

$$-x + z = 0$$

$$z = x$$

$$3x + y + 2z = 3$$

$$5x + y = 3$$

and $-2x - y - z = 0$ or $-2x - y - x = 0$

or $-3x - y = 0$

or $3x + y = 0$

From Eqs. (ii) and (iii), we get

$$2x = 3 \Rightarrow x = 3/2$$

$$\therefore z = x = 3/2 \Rightarrow 3x + y = 0$$

or $\frac{9}{2} + y = 0 \Rightarrow y = -\frac{9}{2}$

$$\therefore \frac{x}{z} = \frac{3/2}{3/2} = 1$$

(5)

$$\therefore g = \frac{GM}{R^2}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta M}{M} + \frac{2\Delta R}{R}$$

Percentage error in g is

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{2\Delta R}{R} \times 100$$

$$= 1 + 4 = 5\%$$

(7)

$$\therefore T = \frac{p^2}{2m} \text{ or } \frac{\Delta T}{T} = 2 \frac{\Delta p}{p} + \frac{\Delta m}{m}$$

Percentage error in kinetic energy is

$$\frac{\Delta T}{T} \times 100 = 2 \frac{\Delta p}{p} \times 100 + \frac{\Delta m}{m} \times 100$$

$$= 2 \times 2 + 3 = 7\%$$

$$\therefore n = 7$$

(1)

$$\therefore y = 2.21 \times 0.3 = 0.663 = 0.7$$

The number of significant digit is 1.

(2)

$$x = 0.72 + 0.8 + 3.87 - 1.089 = 0.7 + 0.8 + 3.9 - 1.1 = 4.3$$

(2)

Thus, the number of significant digit is 2.

The volume of wire

$$x = 0.72 + 0.8 + 3.87 - 1.089 = 0.7 + 0.8 + 3.9 - 1.1 = 4.3$$

$$= 1.4418 \text{ cm}^3 = 1.4 \text{ cm}^3$$

Thus, number of significant digits in 1.4 cm^3 is 2.

13. (2)

$$[\text{Density}] = [ML^{-3}]$$

$$\therefore n_2 = n_1 \left(\frac{M_1}{M_2} \right) \left(\frac{L_1}{L_2} \right)^{-3}$$

$$n_2 = (4) \left(\frac{1 \text{ g}}{16 \text{ g}} \right) \left(\frac{1 \text{ cm}}{2 \text{ cm}} \right)^{-3} = 4 \left(\frac{1}{16} \right) (8) = 2$$

14. (4)

$$\text{Energy} \propto m^x \alpha^y f^z$$

$$[ML^2T^{-2}] = [M]^x [LT^{-2}]^y [T^{-1}]^z$$

$$1 = x$$

$$2 = y$$

$$-2 = -2y - z \Rightarrow z = -2$$

$$\text{Energy} \propto \frac{ma^2}{f^2}$$

$$0.1 \text{ J} = xJ'$$

$$x = (0.1) \frac{J}{J'} = (0.1) \left(\frac{M_1}{M_2} \right)^1 \left(\frac{a_1}{a_2} \right)^2 \left(\frac{f_1}{f_2} \right)^{-2}$$

$$= (0.1) \left(\frac{1 \text{ kg}}{100 \text{ g}} \right) \left(\frac{1 \text{ m/s}^2}{2 \text{ m/s}^2} \right)^2 \left(\frac{1 \text{ sec}^{-1}}{4 \text{ sec}^{-1}} \right)^{-2}$$

$$= (0.1)(10) \left(\frac{1}{4} \right) (16) = 4$$

15. (1000)

Let in new system, dimensions be M' , L' and T'

$$\therefore \text{Given } 100 M'L'T'^{-2} = 100 MLT^{-2}$$

...(i)

$$20 M'L'^2T'^{-2} = 20 \times 100 ML^2T^{-2}$$

...(ii)

Solving (i) & (ii); $L' = 100L$

$$\therefore M'T'^{-2} = \frac{1}{100} MT^{-2}$$

$$\therefore 10 \text{ surface tension } (MT^{-2}) \text{ in SI unit} = 10^3 M'T'^{-2}$$

16. (0.50)

$$n_2 = n_1 \left(\frac{M_1}{M_2} \right)^1 \left(\frac{L_1}{L_2} \right)^2 \left(\frac{T_1}{T_2} \right)^{-2}$$

$$= 1 \cdot \left(\frac{1 \text{ kg}}{2 \text{ kg}} \right)^1 \left(\frac{1 \text{ m}}{1/2 \text{ m}} \right)^2 \left(\frac{1 \text{ s}}{1/2 \text{ s}} \right)^{-2}$$

$$n_2 = 1 \cdot \left(\frac{1}{2} \right) = 0.50$$

17. (1000)

$$[P] = [M^1L^2T^{-3}]; n_2 = n_1 \left(\frac{M_1}{M_2} \right)^1 \left(\frac{L_1}{L_2} \right)^2 \left(\frac{T_1}{T_2} \right)^{-3}$$

$$1 \text{ unit} = xW \left(\frac{1 \text{ kg}}{2 \text{ kg}} \right)^1 \left(\frac{10 \text{ m}}{10 \text{ m}} \right)^2 \left(\frac{1 \text{ s}}{1 \text{ s}} \right)^{-3}$$

$$x = 1000$$

18. (3)

From dimensional analysis, velocity of water splash $v \propto kF^a P^b f^c$

$$\Rightarrow M^0L^1T^{-1} = (MLT^{-2})^a (ML^2T^{-3})^b L^c$$

$$\therefore a + b = 0$$

$$a + 2b + c = 1$$

$$-2a - 3b = -1$$

$$\therefore a = -1, b = 1 \text{ and } c = 0$$

Hence $v \propto \frac{P}{F}$

$$\therefore \frac{v_1}{v_2} = \frac{F_1^{-1} P_1}{F_2^{-1} P_2} \Rightarrow v_2 = \frac{v_1 P_2 F_1}{P_1 F_2} = \frac{3 \times 400 \times 1000}{800 \times 500} = 3 \text{ m/s}$$

19. (25)

$$\begin{aligned} n_2 &= n_1 \left[\frac{M_2}{M_1} \right]^1 \left[\frac{L_2}{L_1} \right]^2 \left[\frac{T_1}{T_2} \right]^{-2} \\ &= 10 \left[\frac{1 \text{ kg}}{100 \times 1000 \text{ kg}} \right]^1 \left[\frac{1 \text{ m}}{4 \text{ m}} \right]^2 \left[\frac{1 \text{ s}}{2 \text{ s}} \right]^{-2} \\ &= 10[10] \times \left[\frac{1}{16} \right] \times [4] = 25 \text{ units} \end{aligned}$$

20. (6.9)

Least count

$$= \frac{1 \text{ mm}}{10} = 0.1 \text{ mm}; \text{ Zero error} = -(10 - 6) \times 0.1 = -0.4 \text{ mm}$$

$$\text{Reading} = 6 + 5 \times (0.1) - (0.4) = 6.9 \text{ mm}$$

Archives

Single Correct Answer Type

1. (1) Least count = $\frac{\text{Value of main scale division}}{\text{Number of divisions on vernier scale}}$

$$= \frac{1}{30} \text{ MSD} = \frac{1}{30} \times \left(\frac{1}{2} \right)^\circ = \left(\frac{1}{60} \right)^\circ = 1 \text{ minute}$$

2. (3) Resistance of a given wire, $R = \frac{V}{I}$

$$\therefore \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

The percentage error in R is:

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100 = 3\% + 3\% = 6\%$$

3. (1) According to Coulomb's law,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\therefore \epsilon_0 = \frac{1}{4\pi} \frac{q_1 q_2}{F r^2}$$

$$[\epsilon_0] = \frac{[AT][AT]}{[MLT^{-2}][L]^2} = [M^{-1}L^{-3}T^4A^2]$$

4. (1) Measured time period of 100 oscillations are 90 sec, 91 sec, 95 sec and 92 sec.

$$\text{Mean value of time} = t_m = \frac{90 + 91 + 95 + 92}{4} = 92 \text{ sec}$$

Absolute error in measurement

$$|\Delta t_1| = |t_m - t_1| = 2 \text{ sec}$$

$$|\Delta t_2| = |t_m - t_2| = 1 \text{ sec}$$

$$|\Delta t_3| = |t_m - t_3| = 3 \text{ sec}$$

$$|\Delta t_4| = |t_m - t_4| = 0 \text{ sec}$$

$$\text{Mean absolute error, } \Delta t_{\text{mean}} = \frac{2+1+3+0}{4} = 1.5 \text{ sec}$$

But the least count of the measuring clock is 1 second, so it cannot measure up to 0.5 second, so we have to round it off.

So mean error will be 2 seconds.

Hence, mean time is (92 ± 2) seconds.

5. (2) Least count, $LC = \frac{\text{Pitch}}{\text{No. of div. on circular scale}}$

$$\Rightarrow LC = \frac{0.5 \text{ mm}}{50} = 0.01 \text{ mm}$$

When jaws are closed, the zero error will be

$$= \text{Main scale reading} + (\text{Circular scale reading}) (\text{Least count})$$

$$= -0.5 \text{ mm} + (45)(0.01)$$

Hence, zero error, $e = -0.05 \text{ mm}$

When the sheet is placed between the jaws:

$$\text{Measured thickness} = 0.5 \text{ mm} + (25)(0.01) = 0.75 \text{ mm}$$

$$\text{Hence actual thickness or true reading} = \text{Observed reading} + \text{Zero error}$$

$$= 0.75 \text{ mm} - (-0.05) = 0.80 \text{ mm}$$

6. (4) Density (ρ) = $\frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3}$

$$\frac{\Delta \rho}{\rho} \% = \frac{\Delta M}{M} \% + 3 \cdot \frac{\Delta L}{L} \% = 1.5 + 3(1) = 4.5\%$$

Single Correct Answer Type

1. (4) 16 MSD = 20 VSD

$$\Rightarrow 1 \text{ VSD} = 4/5 \text{ MSD}$$

$$LC = 1 \text{ MSD} - 1 \text{ VSD}$$

$$= \left(1 - \frac{4}{5} \right) \text{ MSD} = \left(1 - \frac{4}{5} \right) (1 \text{ mm}) = 0.2 \text{ mm}$$

2. (3) Least count = $\frac{0.5}{50} = 0.01 \text{ mm}$

$$\text{Diameter of ball } D = 2.5 \text{ mm} + (20)(0.01) = 2.7 \text{ mm}$$

$$\rho = \frac{M}{\text{Vol}} = \frac{M}{\frac{4}{3} \pi \left(\frac{D}{2} \right)^3}$$

$$\left(\frac{\Delta \rho}{\rho} \right) = \frac{\Delta m}{m} + 3 \frac{\Delta D}{D} = 2\% + 3 \left(\frac{0.01}{2.7} \right) \times 100\% = 3.1\%$$

3. (1) $\Delta d = \Delta l = \frac{0.5}{100} \text{ mm}$ $y = \frac{4MLg}{\pi l d^2} \left(\frac{\Delta y}{y} \right)_{\text{max}} = \frac{\Delta l}{l} + 2 \frac{\Delta d}{d}$

$$\text{Error due to } l \text{ measurement, } \frac{\Delta l}{l} = \frac{0.5/100 \text{ mm}}{0.25 \text{ mm}}$$

$$\text{Error due to } d \text{ measurement, } 2 \left(\frac{\Delta d}{d} \right) = \frac{2 \times \frac{0.5}{100}}{0.5 \text{ mm}} = \frac{0.5/100}{0.25}$$

So error in y due to l measurement = error in y due to d measurement

4. (2) Main scale division (s) = 0.05 cm

$$\text{Vernier scale division } (v) = \frac{49}{1000} = 0.049$$

$$\text{Least count} = 0.05 - 0.049 = 0.001 \text{ cm}$$

$$\text{Diameter: } 5.10 + 24 \times 0.001 = 5.124 \text{ cm}$$

5. (4) $d = \frac{\lambda}{2 \sin \theta} \Rightarrow \ln d = \ln \left(\frac{\lambda}{2} \right) - \ln \sin \theta; \frac{\Delta d}{d} = 0 - \frac{\cos \theta \Delta \theta}{\sin \theta}$

$$\Rightarrow \left(\frac{\Delta d}{d} \right)_{\text{max}} = \pm \cot \theta \Delta \theta$$

$$\text{Also } (\Delta d)_{\text{max}} = d \cot \theta \Delta \theta$$

$$\frac{\lambda}{2 \sin \theta} \cot \theta \Delta \theta = \frac{\lambda \cot \theta}{2 \sin^2 \theta} \Delta \theta$$

As θ increases, $\cot \theta$ decreases and $\frac{\cot \theta}{\sin^2 \theta}$ also decreases.

6. (1) For Vernier C_1

$$10 \text{ VSD} = 9 \text{ MSD} = 9 \text{ mm}$$

$$1 \text{ VSD} = 0.9 \text{ mm}$$

$$\Rightarrow \text{LC} = 1 \text{ MSD} - 1 \text{ VSD} = 1 \text{ mm} - 0.9 \text{ mm} = 0.1 \text{ mm}$$

$$\text{Reading of } C_1 = \text{MSR} + (\text{VSR}) (\text{L.C.})$$

$$= 28 \text{ mm} + (7) (0.1)$$

$$\text{Reading of } C_1 = 28.7 \text{ mm} = 2.87 \text{ cm}$$

For Vernier C_2 , the Vernier C_2 is abnormal.

So we have to find the reading from basics.

The point where both of the marks are matching:

Distance measured from main scale = Distance measured from Vernier scale

$$28 \text{ mm} + (1 \text{ mm}) (8) = (28 \text{ mm} + x) + (1.1 \text{ mm}) (7)$$

$$\text{Solving } x = 0.3 \text{ mm}$$

$$\text{So reading of } C_2 = 28 \text{ mm} + 0.3 \text{ mm} = 2.83 \text{ cm}$$

$$7. (4) \text{ Total time taken, } T = \sqrt{\frac{2L}{g}} + \frac{L}{c}$$

Now, for an error δL in L , we have an error δT in T .

$$\begin{aligned} \text{So, } T + \delta T &= \sqrt{\frac{2(L + \delta L)}{g}} + \frac{(L + \delta L)}{c} \\ &= \sqrt{\frac{2L}{g} \left(1 + \frac{\delta L}{L}\right)} + \frac{L}{c} \left(1 + \frac{\delta L}{L}\right) \end{aligned}$$

Since, $\delta T/T$ is very small, hence $\delta L/L$ is also small. So taking binomial approximation, we get

$$T + \delta T = \sqrt{\frac{2L}{g}} \left(1 + \frac{1}{2} \frac{\delta L}{L}\right) + \frac{L}{c} \left(1 + \frac{\delta L}{L}\right)$$

$$T + \delta T = \left(\sqrt{\frac{2L}{g}}\right) + \sqrt{\frac{2L}{g}} \left(\frac{1}{2} \frac{\delta L}{L}\right) + \left(\frac{L}{c}\right) + \frac{L}{c} \left(\frac{\delta L}{L}\right)$$

$$= \left(\sqrt{\frac{2L}{g}} + \frac{L}{c}\right) + \left(\frac{1}{2} \sqrt{\frac{2L}{g}} + \frac{L}{c}\right) \left(\frac{\delta L}{L}\right)$$

$$= T + \left(\frac{1}{2} \sqrt{\frac{2 \times 20}{10}} + \frac{20}{300}\right) \frac{\delta L}{L}$$

$$\Rightarrow \delta T = \left(1 + \frac{1}{15}\right) \frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta L}{L} = \left(\frac{15}{16}\right) \delta T = \left(\frac{15}{16}\right) \left(\frac{1}{100}\right)$$

$$\% \text{ error } \left(\frac{\delta L}{L}\right) \times 100\% = \frac{15}{16} \% \approx 1\%$$

Multiple Correct Answers Type

1. (1), (3)

$$T = 40/20 = 2 \text{ s}$$

$$\frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{1}{40} \Rightarrow \Delta T = \frac{T}{40} = \frac{2}{40} = 0.05 \text{ s}$$

$$g = \frac{4\pi^2 L n^2}{t^2}$$

where $t = nT$

$$\frac{\Delta g}{g} = \frac{2\Delta t}{t} \Rightarrow \% \text{ error} = \frac{2\Delta t}{t} \times 100 = 5\%$$

2. (1), (3), (4)

For M

$$[M] = [h]^p [c]^q [G]^r$$

$$[M] = [M^1 L^2 T^{-1}]^p [L^1 T^{-1}]^q [L^1 T^{-1}]^r [m^{-1} L^3 T^{-2}]^r$$

$$[M] = [M]^p [L]^{2p+q+3r} [T]^{-p-q-2r}$$

$$\therefore p - r = 1 \quad \dots(i)$$

$$2p + q + 3r = 0 \quad \dots(ii)$$

$$-p - q - 2r = 0 \quad \dots(iii)$$

On solving (i), (ii) and (iii), we get

$$p = \frac{1}{2}, r = \frac{-1}{2} \text{ and } q = \frac{1}{2} \Rightarrow [M] \propto \sqrt{h}$$

$$[M] \propto \sqrt{c}$$

$$[M] \propto \frac{1}{\sqrt{G}}$$

Similarly for $[L]$,

$$p - r = 0 \quad \dots(iv)$$

$$2p + q + 3r = 1 \quad \dots(v)$$

$$-p - q - 2r = 0 \quad \dots(vi)$$

On solving (iv), (v) and (vi), we get

$$p = \frac{1}{2}, q = \frac{-3}{2}, r = \frac{1}{2} \Rightarrow [L] \propto \sqrt{h}$$

$$[L] \propto c^{3/2}$$

$$[L] \propto \sqrt{G}$$

3. (1), (3)

$$[V] = [M^1 L^2 T^{-3} A^{-1}]$$

$$[I] = [A]$$

$$[c] = [L^1 T^{-1}]$$

$$[e_0] = [M^{-1} L^{-3} T^4 A^2]$$

$$[\mu_0] = [M^1 L^1 T^{-2} A^{-2}]$$

Option (1)

$$[\mu_0 I^2] = [M^1 L^1 T^{-2}]; [E_0 V^2] = [M^1 L^1 T^{-1}]$$

Option (2)

$$[e_0 I] = [M^1 L^{-3} T^4 A^3]$$

$$[\mu_0 V] = [M^2 L^3 T^{-5} A^{-3}]$$

Option (3)

$$[I] = [A]$$

$$[e_0 c V] = [A]$$

Option (4)

$$[\mu_0 c I] = [M^1 L^2 T^{-3} A^{-1}]$$

$$[e_0 V] = [L^{-1} T A]$$

Options (1) and (3) correct.

4. (2), (4)

We know,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \Rightarrow \frac{q^2}{\epsilon_0} = (Fr^2) 4\pi$$

So, dimension

$$\frac{q^2}{\epsilon_0} = \dim(Fr^2) = MLT^{-2} \times L^2 ML^3 T^{-2}$$

Similarly,

$$E = \frac{3}{2} K_B T \Rightarrow \dim(K_B T) = \dim(\text{Energy}) = ML^2 T^{-2}$$

$$1. \sqrt{\frac{nq^2}{\epsilon k_B T}} = \sqrt{\frac{L^{-3} \times ML^3 T^{-2}}{ML^2 T^{-2}}} = \frac{1}{L}$$

$$2. \sqrt{\frac{(E) \times \text{vol}}{Fr^2}} = \sqrt{\frac{ML^2 T^{-2} \times L^3}{ML T^{-2} \times L^2}} = L$$

$$3. \sqrt{\frac{Fr^2 (\text{vol})^{2/3}}{(K\epsilon)}} = \sqrt{\frac{ML T^{-2} \times L^2 \times L^2}{ML^2 T^{-2}}} = \sqrt{L^3} = L^{3/2}$$

$$4. \sqrt{\frac{Fr^2 (\text{vol})^{1/3}}{\text{Energy}}} = \sqrt{\frac{ML T^{-2} L^2 \times L}{ML^2 T^{-2}}} = L$$

$$\therefore \text{Dimension } n = \dim\left(\frac{1}{\text{vol}}\right) = L^{-3}$$

5. (1), (2), (4)

S. No.	T	absolute = $ T - T_{\text{mean}} $
(1)	0.52	0.04
(2)	0.56	0.00
(3)	0.57	0.01
(4)	0.54	0.02
(5)	0.59	0.03
	$T_{\text{mean}} = \frac{2.78}{5}$ $T_{\text{mean}} = 0.56$	$(\Delta T)_{\text{mean}} = 0.02$

$$\% \text{ error in } T = \frac{0.02}{0.56} \times 100 = 3.57\%$$

According to the question

$$g \propto \frac{T^2}{R-r}$$

$$\frac{dg}{g} = 2 \frac{dT}{T} + \frac{dR+dr}{R-r}$$

$$\frac{dg}{g} = 2(3.57\%) + \frac{1+1}{60-10} \times 100\%$$

$$\frac{dg}{g} = 11\%$$

Linked Comprehension Type

1. (3) We have $\frac{E}{C} = B$

$$\therefore [B] = \frac{[E]}{[C]} = [E] L^{-1} T^1 \Rightarrow [E] = [B][L][T^{-1}]$$

2. (4) We have $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\therefore [C^2] = \left[\frac{1}{\mu_0 \epsilon_0} \right] \Rightarrow L^2 T^{-2} = \frac{1}{[\mu_0][\epsilon_0]}$$

$$\Rightarrow [\mu_0] = [\epsilon_0]^{-1} [L]^{-2} [T]^2$$

Matrix Match Type

1. (3) p $\rightarrow$ 4; q $\rightarrow$ 2; r $\rightarrow$ 1; s $\rightarrow$ 3

p. $KE = \frac{3}{2} K T \Rightarrow [ML^2 T^{-2}] = K'[K] \Rightarrow K' = [ML^2 T^{-2} K^{-1}]$

q. $F = 6\pi\eta r v \Rightarrow [ML^{-2} T^{-2}] = \eta[L][LT^{-1}] \Rightarrow \eta = [ML^{-1} T^{-1}]$

r. $E = hf \Rightarrow [ML^2 T^{-2}] = \frac{h}{[T]} \Rightarrow h = [ML^2 T^{-1}]$

s. $\frac{dQ}{dt} = \frac{K'A(\Delta T)}{\Delta x} \Rightarrow \frac{[ML^2 T^{-2}]}{[T]} = \frac{k[L^2][K]}{[L]}$
 $K' = [MLT^{-3} K^{-1}]$

Numerical Value Type

1. (3) $d = \rho^a S^b f^c$

$$d = \left(\frac{\text{kg}}{\text{m}^3}\right)^a \times \left(\frac{\text{w}}{\text{m}^2}\right)^b \times \left(\frac{1}{\text{sec}}\right)^c$$

$$d = (ML^{-3})^a \left(\frac{ML^2 T^{-3}}{L^2}\right)^b \left(\frac{1}{\text{sec}}\right)^c$$

$$L^1 = M^{a+b} L^{-3a} T^{-3b-c}$$

$$-3a = 1 \Rightarrow a = -\frac{1}{3}$$

$$a + b = 0$$

$$-3b - c = 0$$

$$\therefore d = \rho^{-\frac{1}{3}} S^{\frac{1}{3}} f^1$$

$$\therefore d \propto S^{1/3}$$

2. (4) $E(t) = A^2 e^{-\alpha t}$

$$\ln E = 2 \ln A - \alpha t$$

$$\therefore \frac{\Delta E}{E} \times 100 = 2 \frac{\Delta A}{A} \times 100 - \alpha \Delta t \times 100$$

$$\text{For time: } \frac{\Delta t}{t} \times 100 = 1.5$$

$$\text{At } t = 5 \text{ s, } \Delta t = \frac{1.5 \times 5}{100} = \frac{7.5}{100}$$

Using Eq(i)

$$\% \text{ error in } E = 2 \times (1.25) + \alpha \times \frac{7.5}{100} \times 100 = 2.5 + 1.5 = 4$$

Chapter 2

Concept Application Exercises

Exercise 2.1

$$1. (1+x)^{-2} = 1 + (-2)x + \frac{(-2)(-2-1)x^2}{2!} + \frac{(-2)(-2-1)(-2-2)x^3}{3!} = 1 - 2x + 3x^2 - 4x^3$$

$$2. \mathcal{Q} \left[\left(1 + \frac{\Delta x}{x} \right)^3 - 1 \right] = \mathcal{Q} \left[1 + \frac{3\Delta x}{x} - 1 \right] = \frac{3\mathcal{Q}\Delta x}{x}$$

$$3. (1+x^2)^{-2} = 1 + (-2)x^2 + \frac{(-2)(-2-1)}{2!}(x^2)^2 + \frac{(-2)(-2-1)(-2-2)}{3!}(x^2)^3$$

$$= 1 - 2x^2 + \frac{(-2)(-3)}{2 \times 1}x^4 + \frac{(-2)(-3)(-4)}{3 \times 2 \times 1}x^6$$

$$= 1 - 2x^2 + 3x^4 - 4x^6$$

$$4. \frac{1+x}{1-x} = (1+x)(1-x)^{-1}$$

$$= (1+x)(1+x)$$

$$\approx 1 + 2x + x^2$$

[by binomial expansion]

$$\approx 1 + 2x$$

[ignore x^2 as x is very small]

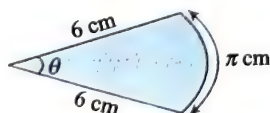
$$5. m = m_0 \left(1 - \frac{v^2}{c^2} \right)^{-1/2} = 10 \left[1 - \left(\frac{3 \times 10^7}{3 \times 10^8} \right)^2 \right]^{-1/2}$$

$$= 10 \left[1 - \frac{1}{100} \right]^{-1/2}$$

$$\approx 10 \left[1 - \left(-\frac{1}{2} \right) \left(\frac{1}{100} \right) \right] = 10 + \frac{10}{200} \approx 10.05 \text{ kg}$$

Exercise 2.2

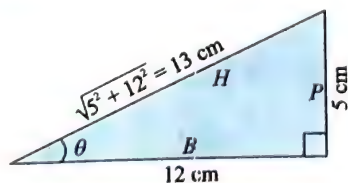
$$1. \theta = \frac{s}{r} = \frac{\pi \text{ cm}}{6 \text{ cm}} = \frac{\pi}{6} \text{ rad} = 30^\circ$$



$$2. \sin \theta = \frac{P}{H} = \frac{5 \text{ cm}}{13 \text{ cm}} = \frac{5}{13}$$

$$\cos \theta = \frac{B}{H} = \frac{12 \text{ cm}}{13 \text{ cm}} = \frac{12}{13} \text{ and}$$

$$\tan \theta = \frac{P}{B} = \frac{5 \text{ cm}}{12 \text{ cm}} = \frac{5}{12}$$



$$3. \cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$4. \text{ Using } \sin(A+B) = \sin A \cos B + \cos A \sin B$$

Clearly $\sin(105^\circ) = \sin(60^\circ + 45^\circ)$

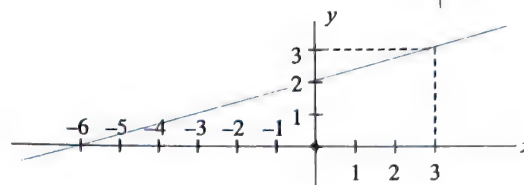
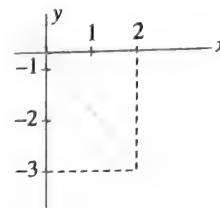
$$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{\sqrt{2}} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{\sqrt{2}} \right) = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{1}{4}(\sqrt{6} + \sqrt{2})$$

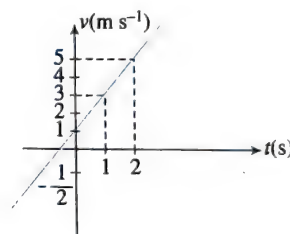
Exercise 2.3

$$1. (a) 3x + 2y = 0$$

$$(b) x - 3y + 6 = 0 \Rightarrow y = \frac{x}{3} + 2$$



$$2. v = 1 + 2t$$



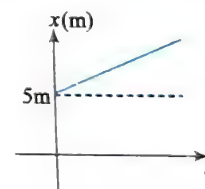
$$3. \text{ Given } F = -10x;$$

The standard equation of straight line is $y = mx + c$. If we compare the given equation with the standard equation of straight line, the intercept is zero and slope is negative. Hence, the graph is passing through origin. Hence, option (3) is correct.

$$4. \text{ We know velocity,}$$

$$v = \frac{\Delta x}{dt} = \frac{x_f - x_i}{t_f - t_i}$$

$$\Rightarrow 2 = \frac{x - 5}{t - 0} \Rightarrow x = 2t + 5$$



If we compare with $y = mx + c$, the graph will be a straight line with positive slope with positive intercept. Hence, option (3) is correct.

$$5. \text{ For side AB: } m = \frac{1-0}{0-(-2)} = \frac{1}{2}, c = 1 \Rightarrow y = \frac{1}{2}x + 1$$

$$\text{For side BC: } m = \frac{1-0}{0-2} = -\frac{1}{2}, c = 1 \Rightarrow y = -\frac{1}{2}x + 1$$

$$\text{For side CD: } m = \frac{0-(-1)}{2-0} = \frac{1}{2}, c = -1 \Rightarrow y = \frac{1}{2}x - 1$$

$$\text{For side DA: } m = \frac{-1-0}{0-(-2)} = -\frac{1}{2}, c = -1 \Rightarrow y = -\frac{1}{2}x - 1$$

Exercise 2.4

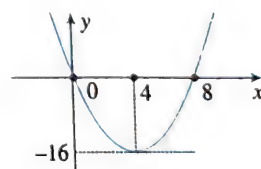
$$1. (a) y = x^2 - 8x, a = 1, b = -8, c = 0$$

$$\text{Vertex, } x = -\frac{b}{2a} = -\left(\frac{-8}{2 \times 1} \right) = 4$$

$$\text{At } x = 4, y = 4^2 - 8 \times 4 = -16,$$

$$\text{At } x = 0, y = 0$$

$$\text{At } x = 8, y = 0$$



(b) $y = -2x^2 + 3$, $a = -2$, $b = 0$, $c = 3$

Vertex, $x = -\frac{b}{2a} = 0$; at $x = 0$, $y = 3$

At $x = 1$, $y = 1$ and at $x = -1$, $y = 1$

(c) $y = x^2 - 6x + 4$, $a = 1$, $b = -6$, $c = 4$

Vertex: $x = -\frac{b}{2a} = -\frac{(-6)}{2 \times 1} = 3$

At $x = 3$, $y = -5$;

at $x = 0$, $y = 4$; at $x = 6$, $y = 4$

2. $x = t + t^2$, $a = 1$, $b = 1$, $c = 0$

Vertex: $t = -\frac{b}{2a} = -\frac{1}{2 \times 1} = -\frac{1}{2}$,

At $t = -\frac{1}{2}$, $x = -\frac{1}{4}$

3. $U = 5x^2$, it is the equation of parabola with concavity is upward
At $x = 0$, $U = 0$, it means graph is passing through origin. Hence, option (1) is correct.

Exercise 2.5

1. (a) $\frac{d}{dx}(9) = 0$, since the differentiation of constant is zero.

(b) $\frac{d}{dx}(\pi^4) = 0$, since the differentiation of constant is zero.

(c) $\frac{d}{dx}(2e^3) = 0$, since the differentiation of constant is zero.

(d) $\frac{d}{dx}(x^2 + 5) = 2x + 0 = 2x$

(e) $\frac{d}{dx}[(x+5)^{-1/2}] = -\frac{1}{2}(x+5)^{-3/2}$

(f) $\frac{d}{dx}(5x^{3/2}) = 5 \frac{d}{dx}(x^{3/2}) = 5 \times \frac{3}{2} x^{1/2} = \frac{15}{2} x^{1/2}$

(g) $\frac{d}{dx}(\sqrt{x+3}) = \frac{d}{dx}[(x+3)^{1/2}] = \frac{1}{2}(x+3)^{-1/2}$

(h) $\frac{d}{dx}[(2x^2 + 9)^3] = 3(2x^2 + 9)^2(4x) = 12(2x^2 + 9)^2 x$

2. (a) Let $y = (x^2 + 3x)(2x + 7)$

$$\frac{dy}{dx} = (x^2 + 3x) \frac{d(2x + 7)}{dx} + \left[\frac{d}{dx}(x^2 + 3x) \right] (2x + 7)$$

$$= (x^2 + 3x) 2 + (2x + 3)(2x + 7) = 6x^2 + 26x + 21$$

(b) Let $y = (3x^2 + 2)(4x - 3x^3)$

$$\frac{dy}{dx} = (3x^2 + 2) \frac{d}{dx}(4x - 3x^3) + \left[\frac{d}{dx}(3x^2 + 2) \right] (4x - 3x^3)$$

$$= (3x^2 + 2)(4 - 9x^2) + 6x(4x - 3x^3) = -45x^4 + 18x^2 + 8$$

(c) Let $y = \sqrt{x}(x^3 + x^2 - 3x) = x^{7/2} + x^{5/2} - 3x^{3/2}$

$$\frac{dy}{dx} = \frac{7}{2} x^{5/2} + \frac{5}{2} x^{3/2} - 3 \times \frac{3}{2} x^{1/2}$$

$$= \frac{1}{2\sqrt{x}}(7x^3 + 5x^2 - 9x)$$

(d) Let $y = \sin x \log x$

$$\frac{dy}{dx} = \sin x \frac{d}{dx}(\log x) + \left[\frac{d}{dx}(\sin x) \right] \log x = \frac{\sin x}{x} + \cos x \log x$$

3. (a) Let $y = \tan^3 x = (\tan x)^3$, $\frac{dy}{dx} = 3(\tan^2 x)(\sec^2 x)$

(b) Let $y = \tan(x^2)$, $\frac{dy}{dx} = \sec^2(x^2) 2x$

(c) Let $y = \sin^2(\sqrt{x}) = (\sin \sqrt{x})^2$

$$\frac{dy}{dx} = 2(\sin \sqrt{x}) \cos(\sqrt{x}) \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} \sin(2\sqrt{x})$$

4. $x = a(\theta + \sin \theta)$

$$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos \theta)$$

$$y = a(1 - \cos \theta) \Rightarrow \frac{dy}{d\theta} = a(0 - (-\sin \theta)) = a \sin \theta$$

Dividing (ii) and (i) $\frac{dy/d\theta}{dx/d\theta} = \frac{dy}{dx} = \frac{\sin \theta}{1 + \cos \theta} = \tan(\theta/2)$

5. $\theta = \frac{t^2}{20} + \frac{t}{5}$, $\omega = \frac{d\theta}{dt} = \frac{2t}{20} + \frac{1}{5} = \frac{t}{10} + \frac{1}{5}$

$$\omega_{t=4s} = \frac{4}{10} + \frac{1}{5} = 0.6 \text{ rad s}^{-1}$$

6. $A = 5t^2 + 4t + 8$

Rate of inc. of area: $\frac{dA}{dt} = 10t + 4$

$$\left(\frac{dA}{dt} \right)_{t=3s} = 10 \times 3 + 4 = 34 \text{ m}^2/\text{s}$$

7. $y = \sin x$; then $\frac{dy}{dx} = \cos x$ and $\frac{d^2y}{dx^2} = -\sin x$

8. Give: $y = x^3$. Then $\frac{dy}{dx} = 3x^2 \Rightarrow \frac{d^2y}{dx^2} = 6x$

9. As $\frac{d}{dx}[\sin(ax + b)] = \cos(ax + b) \times a$;

then $\left(\frac{dy}{dt} \right) = 2 \cos(\omega t + \phi) \times \omega = 2\omega \cos(\omega t + \phi)$

Exercise 2.6

1. (a) $\int (x^2 + 2) dx = \int x^2 dx + \int 2 dx = \frac{x^3}{3} + 2x + c$

(b) $\int (x^3 - \frac{1}{x} + 3x) dx = \int x^3 dx - \int \frac{1}{x} dx + \int 3x dx$

$$= \frac{x^4}{4} - \log x + \frac{3x^2}{2} + c$$

(c) $\int (x^{3/2} - x^{1/2} + 5x) dx$

$$= \int x^{3/2} dx - \int x^{1/2} dx + \int 5x dx$$

$$= \frac{2x^{5/2}}{5} - \frac{2x^{3/2}}{3} + \frac{5}{2} x^2 + c$$

(d) $\int (e^{2x} + 5) dx = \int e^{2x} dx + \int 5 dx = \frac{e^{2x}}{2} + 5x + c$

2. (a) $\int_1^2 x^3 dx = \left(\frac{x^4}{4} \right)_1^2 = \frac{2^4}{4} - \frac{1^4}{4} = \frac{15}{4}$

$$(b) \int_u^v Mv dv = \left(M \frac{v^2}{2} \right)_u^v = \frac{M}{2} (v^2 - u^2)$$

$$(c) \int_3^4 \frac{1}{x} dx = (\log_e x)_3^4 = \log_e 4 - \log_e 3 = \log_e (4/3)$$

$$(d) \int_4^9 \sqrt{x} dx = \left(\frac{x^{3/2}}{3/2} \right)_4^9 = \frac{2}{3} (9^{3/2} - 4^{3/2}) = 38/3$$

$$(e) \int_0^{\pi/4} \cos 2x dx = \left[\frac{\sin 2x}{2} \right]_0^{\pi/4} = \frac{1}{2} [\sin \{2(\pi/4)\} - \sin(2 \times 0)] = \frac{1}{2}$$

Exercise 2.7

$$1. y = 6t^2 + 3t + 4 \Rightarrow v = \frac{dy}{dt} = 12t + 3 \text{ ms}^{-1}$$

$$2. v = 12 + 3(t + 7t^2) = 12 + 3t + 21t^2 \quad a = \frac{dv}{dt} = 3 + 42t$$

$$3. x = 2 + 5t + 7t^2 \Rightarrow \frac{dx}{dt} = v = 5 + 14t$$

$$(a) v_{t=0} = 5 + 14 \times 0 = 5 \text{ ms}^{-1}$$

$$(b) v_{t=4s} = 5 + 14 \times 4 = 61 \text{ ms}^{-1}$$

$$(c) a = \frac{dv}{dt} = 14 \text{ ms}^{-2}$$

$$(d) x_{t=5s} = 2 + 5 \times 5 + 7 \times 5^2 = 202 \text{ m}$$

$$4. a = t^3 - 3t^2 + 5 \quad v = \int a dt + C_1 = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + C_1$$

$$\text{At } t = 1 \text{ s, } v = 6.25 \text{ ms}^{-1}$$

$$6.25 = \frac{1^4}{4} - 1^3 + 5 \times 1 + C_1 \Rightarrow C_1 = 2$$

$$\Rightarrow v = \frac{t^4}{4} - t^3 + 5t + 2$$

$$x = \int v dt + C_2 = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2$$

$$\text{At } t = 1 \text{ s, } x = 8.30 \text{ m}$$

$$8.30 = \frac{1^5}{20} - \frac{1^4}{4} + \frac{5}{2}(1)^2 + 2 \times 1 + C_2 \Rightarrow C_2 = 4$$

$$\Rightarrow x = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5}{2}t^2 + 2t + 4$$

$$\text{Now put } t = 2 \text{ s.}$$

$$v = \frac{2^4}{4} - 2^3 + 5 \times 2 + 2 = 8 \text{ ms}^{-1}$$

$$x = \frac{2^5}{20} - \frac{2^4}{4} + \frac{5}{2}(2)^2 + 2 \times 2 + 4 = 15.6 \text{ m}$$

$$5. x = 2t^3 - 3t^2 + 1 \Rightarrow v = \frac{dx}{dt} = 6t^2 - 6t$$

$$(a) \text{ Put } v = 0 \Rightarrow 0 = 6t^2 - 6t \Rightarrow t = 1 \text{ s, } t = 0 \text{ s}$$

$$(b) x = 0 \Rightarrow 0 = 2t^3 - 3t^2 + 1 \Rightarrow (t-1)^2(2t+1) = 0$$

$$\Rightarrow t = 1 \text{ s, } v_{t=1s} = 6 \times 1^2 - 6 \times 1 = 0 \text{ ms}^{-1}$$

$$6. x = t[t-1][t-2] = (t^2-t)(t-2) = t^3 - 2t^2 - t^2 + 2t = t^3 - 3t^2 + 2t$$

$$v = \frac{dx}{dt} = 3t^2 - 6t + 2$$

$$(a) v_{t=0} = 3 \times 0^2 - 6 \times 0 + 2 = 2 \text{ ms}^{-1}$$

$$(b) a = \frac{dv}{dt} = 6t - 6, a_{t=0} = -6 \text{ ms}^{-2}$$

$$(c) x = 0 \Rightarrow t = 0 \text{ s, } t = 1 \text{ s, } t = 2 \text{ s}$$

$$(d) v = 0$$

$$\Rightarrow 3t^2 - 6t + 2 = 0$$

$$\Rightarrow t = \frac{6 \pm \sqrt{6^2 - 4 \times 3 \times 2}}{6} = \frac{6 \pm \sqrt{12}}{6} = 1 \pm \frac{1}{\sqrt{3}} = \frac{\sqrt{3} \pm 1}{\sqrt{3}} \text{ s}$$

$$\text{At } t = \frac{\sqrt{3}-1}{\sqrt{3}} \text{ s, } x = \left(\frac{\sqrt{3}-1}{\sqrt{3}} \right) \left(\frac{\sqrt{3}-1}{\sqrt{3}} - 1 \right) \left(\frac{\sqrt{3}-1}{\sqrt{3}} - 2 \right) = \frac{2}{3\sqrt{3}} \text{ m}$$

$$\text{and at } t = \frac{\sqrt{3}+1}{\sqrt{3}} \text{ s, } x = \left(\frac{\sqrt{3}+1}{\sqrt{3}} \right) \left(\frac{\sqrt{3}+1}{\sqrt{3}} - 1 \right) \left(\frac{\sqrt{3}+1}{\sqrt{3}} - 2 \right) = \frac{-2}{3\sqrt{3}} \text{ m}$$

$$(e) a = 6t - 6 = 6 \left(\frac{\sqrt{3}-1}{\sqrt{3}} \right) - 6 = -\frac{6}{\sqrt{3}} = -2\sqrt{3} \text{ ms}^{-2}$$

$$\text{and } a = 6 \left(\frac{\sqrt{3}+1}{\sqrt{3}} \right) - 6 = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ ms}^{-2}$$

$$7. (a) \text{ Distance} = \text{Area} = \frac{1}{2} \times 2 \times 10 = 10 \text{ m}$$

$$(b) \text{ Distance} = \text{Area} = 2 \times 10 = 20 \text{ m}$$

$$(c) \text{ Total distance} = 10 + 20 = 30 \text{ m}$$

$$8. (a) \text{ Maximum speed of the car will be at } t = 2 \text{ s, because area is positive till that time.}$$

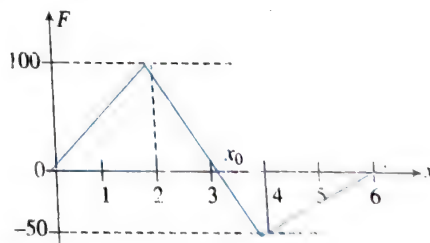
$$\text{Change in velocity} = \text{Area of } a-t \text{ graph}$$

$$\Rightarrow v_{\max} - 0 = 2 \times 2 \Rightarrow v_{\max} = 4 \text{ ms}^{-1}$$

$$(b) \text{ Since finally the car comes to rest, so both area should be same.}$$

$$4 = t_0 \times 4 \Rightarrow t_0 = 1 \text{ s}$$

$$9. (a) \frac{x_0 - 2}{100} = \frac{4 - x_0}{50} \Rightarrow x_0 = \frac{10}{3} \text{ m}$$



$$\text{Total work done, } W = \frac{1}{2} x_0 \times 100 - \frac{1}{2} (6 - x_0) 50 = 100 \text{ J}$$

$$\Rightarrow \frac{1}{2} mv^2 = 100 \Rightarrow \frac{1}{2} \times \frac{15}{10} v^2 = 100$$

$$\Rightarrow v = \sqrt{\frac{2000}{15}} = \frac{20}{\sqrt{3}} \text{ ms}^{-1}$$

$$(b) \text{ Velocity attained will be maximum at } x = x_0, \text{ because after this work done is negative or force is opposite to velocity.}$$

$$W = \frac{1}{2} \times \frac{10}{3} \times 100 = \frac{500}{3} \text{ J}$$

$$\Rightarrow \frac{1}{2} mv^2 = \frac{500}{3} \Rightarrow \frac{1}{2} \times \frac{15}{10} v^2 = \frac{500}{3}$$

$$\Rightarrow v = \sqrt{\frac{5000 \times 2}{45}} = \frac{10\sqrt{20}}{3} \text{ ms}^{-1} = \frac{20}{3} \sqrt{5} \text{ ms}^{-1}$$

$$10. v = \frac{ds}{dt} = 15 - 0.8t$$

$$\Rightarrow 7 = 15 - 0.8t \Rightarrow t = 10 \text{ s}$$

S.18 Mechanics I

$$11. \quad s = t^3 - 6t^2 + 3t + 4 \Rightarrow v = 3t^2 - 12t + 3$$

$$a = dv/dt = 6t - 12 = 0 \Rightarrow t = 2 \text{ s}$$

$$v = 3(2)^2 - 12 \times 2 + 3 = -9 \text{ ms}^{-1}$$

$$12. \quad t = \sqrt{x} + 3 \Rightarrow x = (t - 3)^2$$

$$v = \frac{dx}{dt} = 2(t - 3) = 0$$

$$\Rightarrow t = 3 \text{ s}$$

$$x_{t=3\text{ s}} = (3 - 3)^2 = 0$$

$$13. \text{ (a) When } v_0 = 0 \text{ and } x_0 = 0,$$

$$v_x = \int_0^t (At - Bt^2) dt = \frac{A}{2}t^2 - \frac{B}{3}t^3$$

$$= (0.75 \text{ ms}^{-3})t^3 - (0.04 \text{ ms}^{-4})t^3$$

$$x = \int_0^t \left(\frac{A}{2}t^2 - \frac{B}{3}t^3 \right) dt = \frac{A}{6}t^3 - \frac{B}{12}t^4$$

$$= (0.25 \text{ ms}^{-3})t^3 - (0.010 \text{ ms}^{-4})t^4$$

(b) For the velocity to be a maximum, the acceleration must be zero, this occurs at $t = 0$ and $t = \frac{A}{B} = 12.5 \text{ s}$. At $t = 0$ the velocity is a minimum, and at $t = 12.5 \text{ s}$ the velocity is

$$v_x = (0.75 \text{ ms}^{-3})(12.5 \text{ s})^2 - (0.040 \text{ ms}^{-4})(12.5 \text{ s})^3$$

$$= 39.1 \text{ ms}^{-1}$$

$$14. \text{ We know that } \int dv = \int a dt \Rightarrow \int_2^v dv = \int_0^t (6t + 6) dt$$

$$\Rightarrow v - 2 = \left[\frac{6t^2}{2} + 6t \right]_0^t \Rightarrow v = 3t^2 + 6t + 2$$

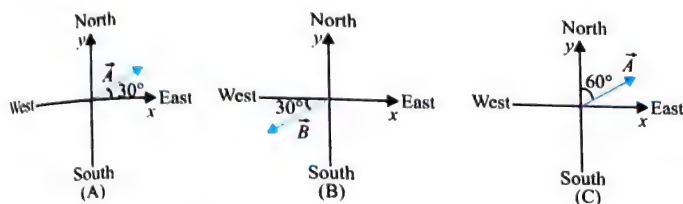
$$\text{Now } \int dx = \int v dt \Rightarrow \int_0^x dx = \int_0^t (3t^2 + 6t + 2) dt$$

$$\Rightarrow x = t^3 + 3t^2 + 2t$$

Concept Application Exercises

Exercise 3.1

- (4) The resultant vector R is drawn from the tail of the first vector to the head of the last vector.
- (2) In this drawing the vector $-C$ is reversed relative to C , while vectors A and B are not reversed.
- (3) In this drawing the vectors $-B$ and $-C$ are reversed relative to B and C , while vector A is not reversed.
- From figure it is clear $\vec{A} = \vec{C}$



- Since the initial force and the resultant force point along the east/west line, the second force must also point along the east/west line. The direction of the second force is not specified; it could point either due east or due west, so there are two answers.

If the second force points due east, both forces point in the same direction and the magnitude of the resultant force is the sum of the two magnitudes: $F_1 + F_2 = F_R$. Therefore,

$$F_2 = F_R - F_1 = 400 \text{ N} - 200 \text{ N} = 200 \text{ N}$$

If the second force points due west, the two forces point in opposite directions, and the magnitude of the resultant force is the difference of the two magnitudes: $F_2 - F_1 = F_R$. Therefore,

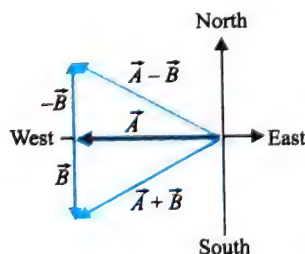
$$F_2 = F_R + F_1 = 400 \text{ N} + 200 \text{ N} = 600 \text{ N}$$

- For convenience, we can assign due east to be the positive direction and due west to be the negative direction. Since all the vectors point along the same east-west line, the vectors can be added just like the usual algebraic addition of positive and negative scalars. We will carry out the addition for all of the possible choices for the two vectors and identify the resultants with the smallest and largest magnitudes.

The resultant vector with the smallest magnitude is $F_1 + F_3 = 10 \text{ N}$, due east.

The resultant vector with the largest magnitude is $F_3 + F_4 = 70 \text{ N}$, due west.

7.



- Arranging the vectors in tail-to-head fashion, we can see that the vector A gives the resultant a westerly direction and vector B gives the resultant a southerly direction. Therefore, the resultant $A + B$ points south of west.

$$\text{Magnitude of } (\vec{A} + \vec{B}) = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ units}$$

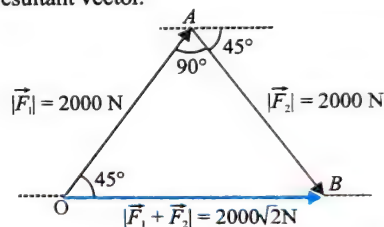
$$\text{Direction, } \theta = \tan^{-1} \left(\frac{10 \text{ units}}{10 \text{ units}} \right) = 45^\circ \text{ south of west}$$

- Arranging the vectors in tail-to-head fashion, we can see that the vector A gives the resultant a westerly direction and vector $-B$ gives the resultant a northerly direction. Therefore, the resultant $A + (-B)$ points north of west.

$$\text{Magnitude of } (\vec{A} - \vec{B}) = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ units}$$

$$\text{Direction } \theta = \tan^{-1} \left(\frac{10 \text{ units}}{10 \text{ units}} \right) = 45^\circ \text{ north of west}$$

- The single force needed to produce the same effect is equal to the resultant of the forces provided by the two ropes. The following figure shows the force vectors drawn to scale and arranged tail to head. The magnitude and direction of the resultant can be found by direct measurement using the scale factor shown in the figure or using the Pythagorean theorem. The drawing shows the two vectors and the resultant vector.



- From the figure, the magnitude of the resultant is $2000\sqrt{2} \text{ N}$.
- The single rope should be directed along the dashed line in the text drawing.

Exercise 3.2

$$1. (2) R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

By substituting, $A = F$, $B = F$ and $R = F$ we get

$$\cos \theta = \frac{1}{2} \therefore \theta = 120^\circ$$

- (3) Resultant of two vectors $\vec{A}$ and $\vec{B}$ can be given by $\vec{R} = \vec{A} + \vec{B}$

$$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\text{If } \theta = 0^\circ \text{ then } |\vec{R}| = A + B = |\vec{A}| + |\vec{B}|$$

- (1) According to problem $P + Q = 3$ and $P - Q = 1$

$$\text{By solving we get } P = 2 \text{ and } Q = 1 \therefore \frac{P}{Q} = 2 \Rightarrow P = 2Q$$

- (4) $R_{\max} = A + B = 17$ when $\theta = 0^\circ$

$$R_{\min} = A - B = 7 \text{ when } \theta = 180^\circ$$

$$\text{by solving we get } A = 12 \text{ and } B = 5$$

$$\text{Now when } \theta = 90^\circ \text{ then } R = \sqrt{A^2 + B^2}$$

$$\Rightarrow R = \sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$$

- (4) If two vectors A and B are given then Range of their resultant can be written as $(A - B) \leq R \leq (A + B)$.
i.e. $R_{\max} = A + B$ and $R_{\min} = A - B$
If $B = 1$ and $A = 4$ then their resultant will lie in between 3 N and 5 N. It can never be 2 N.

6. Let P be the smaller force and Q be the greater force. Then ... (i)

$$P + Q = 18$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} = 12 \quad \dots (ii)$$

$$\tan \phi = \frac{Q \sin \theta}{P + Q \cos \theta} = \tan 90^\circ = \infty$$

$$\therefore P + Q \cos \theta = 0 \quad (iii)$$

By solving (i), (ii), and (iii), we will get $P = 5$ and $Q = 13$.

7. $A = 2F$, $B = \sqrt{2}F$, $R = \sqrt{10}F$

$$R^2 = A^2 + B^2 = 2AB \cos \theta$$

$$\Rightarrow 10F^2 = 4F^2 + 2F^2 + 2(2F)\sqrt{2}F \cos \theta$$

$$\Rightarrow \cos \theta = 1/\sqrt{2} \Rightarrow \theta = 45^\circ$$

8. Let the two forces are A and B , then given

$$A - B = 10, A^2 + B^2 = 50^2$$

Solve to get $A = 40$ N and $B = 30$ N.

9. Resultant of two forces: $\sqrt{(F/2)^2 + (F/2)^2} = F/\sqrt{2}$

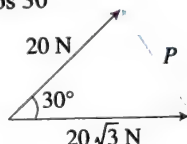
The third force should be opposite to the resultant and of same magnitude. Hence, answer is $F/\sqrt{2}$.

10. Let P be the unknown force.

$$P^2 = (20\sqrt{3})^2 + 20^2 - 2(20)(20\sqrt{3}) \cos 30^\circ$$

$$= 1600 - 1200 = 400$$

$$\text{or } P = 20 \text{ N}$$



Exercise 3.3

1. Net movement along x -direction,

$$S_x = (6 - 4) \cos 45^\circ \hat{i} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \text{ km}$$

Net movement along y -direction,

$$S_y = (6 + 4) \sin 45^\circ \hat{j} = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2} \text{ km}$$

Net movement from starting point,

$$|\vec{S}| = \sqrt{S_x^2 + S_y^2} = \sqrt{(\sqrt{2})^2 + (5\sqrt{2})^2} = \sqrt{52} \text{ km}$$

Angle which makes with the east direction,

$$\tan \theta = \frac{Y\text{-component}}{X\text{-component}} = \frac{5\sqrt{2}}{\sqrt{2}} \therefore \theta = \tan^{-1}(5)$$

2. $\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$

$$|\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$$

$$\tan \theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

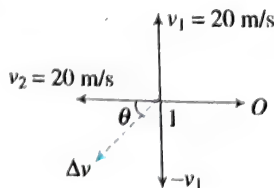
3. From figure, $\vec{v}_1 = 20\hat{j}$ and $\vec{v}_2 = -20\hat{i}$.

$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1 = -20(\hat{i} + \hat{j})$$

$$|\Delta \vec{v}| = 20\sqrt{2}$$

$$\text{and direction } \theta = \tan^{-1}(1) = 45^\circ,$$

i.e., S-W.



4. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors. Then the sum is

$$\vec{n}_s = \hat{n}_1 + \hat{n}_2 \quad \text{or} \quad n_s^2 = n_1^2 + n_2^2 + 2n_1n_2 \cos \theta$$

$$= 1 + 1 + 2 \cos \theta$$

Since it is given that n_s is also a unit vector, therefore

$$1 = 1 + 1 + 2 \cos \theta$$

$$\text{or } \cos \theta = -\frac{1}{2} \quad \text{or } \theta = 120^\circ$$

Now the difference vector is $n_d = n_1 - n_2$

$$\text{or } n_d^2 = n_1^2 + n_2^2 - 2n_1n_2 \cos \theta = 1 + 1 - 2 \cos (120^\circ)$$

$$\therefore n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$$

5. $x = 0$ means y -axis $\Rightarrow \vec{F}_1 = \hat{j}$

$$y = 0 \text{ means } x\text{-axis} \Rightarrow \vec{F}_2 = 2\hat{i}$$

$$\text{So resultant, } \vec{F} = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + \hat{j}$$

6. $\vec{A} - 2\vec{B} + 3\vec{C} = (2\hat{i} + \hat{j}) - 2(3\hat{j} - \hat{k}) + 3(6\hat{i} - 2\hat{k})$

$$= 2\hat{i} + \hat{j} - 6\hat{j} + 2\hat{k} + 18\hat{i} - 6\hat{k} = 20\hat{i} - 5\hat{j} - 4\hat{k}$$

7. $\vec{P}_1 = mv \sin \theta \hat{i} - mv \cos \theta \hat{j}$

$$\text{and } \vec{P}_2 = mv \sin \theta \hat{i} + mv \cos \theta \hat{j}$$

So change in momentum,

$$\Delta \vec{P} = \vec{P}_2 - \vec{P}_1 = 2mv \cos \theta \hat{j}$$

$$|\Delta \vec{P}| = 2mv \cos \theta$$

8. $|\vec{B}| = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25$

$$\text{Unit vector in the direction of } \vec{A} \text{ will be } \hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$$

$$\text{So, required vector} = 25 \left(\frac{3\hat{i} + 4\hat{j}}{5} \right) = 15\hat{i} + 20\hat{j}$$

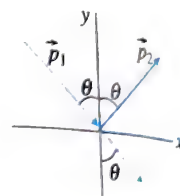
9. Let the components of $\vec{A}$ makes angles α , β , and γ with x , y , and z axis, respectively. Then $\alpha = \beta = \gamma$.

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \Rightarrow 3 \cos^2 \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}} \therefore A_x = A_y = A_z = A \cos \alpha = \frac{A}{\sqrt{3}}$$

10. Unit vector along y -axis = $\hat{j}$. So the required vector,

$$\hat{j} - [(\hat{i} - 3\hat{j} + 2\hat{k}) + (3\hat{i} + 6\hat{j} - 7\hat{k})] = -4\hat{i} - 2\hat{j} + 5\hat{k}$$



Exercise 3.4

$$1. \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$= \frac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{A}| |\vec{B}|} = \frac{2 \times 4 + 4 \times 2 - 4 \times 4}{|\vec{A}| |\vec{B}|} = 0$$

$$\therefore \theta = \cos^{-1}(0) \Rightarrow \theta = 90^\circ$$

2. Let $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

$$\vec{A} \text{ and } \vec{B} \text{ are parallel to each other } \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3},$$

$$\text{i.e., } \frac{2}{-4} = \frac{3}{-6} = \frac{-1}{\lambda} \Rightarrow \lambda = 2$$

3. If $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then

$$\vec{A} \cdot \vec{B} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$$

$$\text{So, } 2(-4) + 3(-6) + (-1)(\lambda) = 0 \Rightarrow \lambda = -26$$

4. $|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

$$|\vec{B}| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1 + 9 + 16} = \sqrt{26}$$

$$\vec{A} \cdot \vec{B} = 2(-1) + 3 \times 3 + (-1)(4) = 3$$

$$\text{The projection of } \vec{A} \text{ on } \vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \frac{3}{\sqrt{26}}$$

$$W = \vec{F} \cdot \vec{S} = FS \cos \theta = 50 \times 10 \times \cos 60^\circ = 50 \times 10 \times \frac{1}{2} = 250 \text{ J.}$$

$$S = \vec{r}_2 - \vec{r}_1; W = \vec{F} \cdot \vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}) \cdot (11\hat{i} + 11\hat{j} + 15\hat{k}) \\ = (4 \times 11 + 1 \times 11 + 3 \times 15) = 100 \text{ J}$$

$(\vec{A} + \vec{B})$ is perpendicular to $(\vec{A} - \vec{B})$.

$$\text{Thus } (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$\text{or } A^2 + B^2 - \vec{A} \cdot \vec{B} - B^2 = 0$$

Because of commutative property of dot product $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

$$\therefore A^2 - B^2 = 0 \text{ or } A = B$$

Thus, the ratio of magnitudes $A/B = 1$.

For motion of the particle from $(0, 0)$ to $(a, 0)$,

$$\vec{F} = -K(0\hat{i} + a\hat{j}) \Rightarrow \vec{F} = -Ka\hat{j}$$

Displacement, $\vec{r} = (a\hat{i} + 0\hat{j}) - (0\hat{i} + 0\hat{j}) = a\hat{i}$

So work done from $(0, 0)$ to $(a, 0)$ is given by

$$W = \vec{F} \cdot \vec{r} = -Ka\hat{j} \cdot a\hat{i} = 0$$

For motion $(a, 0)$ to (a, a)

$$\vec{F} = -K(a\hat{i} + a\hat{j})$$

and displacement, $\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$

So work done from $(a, 0)$ to (a, a) ,

$$W = \vec{F} \cdot \vec{r} = -K(a\hat{i} + a\hat{j}) \cdot a\hat{j} = -Ka^2$$

So, total work done $= -Ka^2$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$$= [1 \times 4 - 2 \times (-2)]\hat{i} + [2 \times 2 - 4 \times 3]\hat{j} + [3 \times (-2) - 1 \times 2]\hat{k} \\ = 8\hat{i} - 8\hat{j} - 8\hat{k}$$

$$\therefore \text{Magnitude of } \vec{A} \times \vec{B} = |\vec{A} \times \vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$$

10. Given $\vec{OA} = \vec{a} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{OB} = \vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

$$\therefore (\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= (12 - 2)\hat{i} + (4 + 6)\hat{j} + (3 + 12)\hat{k}$$

$$= 10\hat{i} + 10\hat{j} + 15\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{10^2 + 10^2 + 15^2} = \sqrt{425} = 5\sqrt{17}$$

$$\text{Area of } \triangle OAB = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{5\sqrt{17}}{2} \text{ sq. unit}$$

11. Let $\vec{A} \cdot (\vec{B} \times \vec{A}) = \vec{A} \cdot \vec{C}$

Here $\vec{C} = \vec{B} \times \vec{A}$ which is perpendicular to both vector $\vec{A}$ and $\vec{B}$

$$\therefore \vec{A} \cdot \vec{C} = 0$$

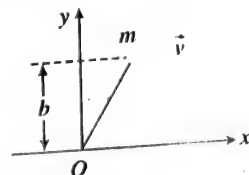
$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

$$= [(2 \times 4) - (3 \times -3)]\hat{i} + [(2 \times 3) - (3 \times 4)]\hat{j} + [(3 \times -3) - (2 \times 2)]\hat{k}$$

$$= 17\hat{i} - 6\hat{j} - 13\hat{k}$$

13. We know that angular momentum, $\vec{L} = \vec{r} \times \vec{p}$ in terms of component becomes

$$\vec{L} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$



As motion is in x - y plane ($z = 0$ and $P_z = 0$),

$$\text{so } \vec{L} = \vec{k} (xp_y - yp_x)$$

Here $x = vt$, $y = b$, $p_x = mv$, and $p_y = 0$.

$$\therefore \vec{L} = \vec{k} [vt \times 0 - bmv] = -mbv\vec{k}$$

Exercises

Single Correct Answer Type

1. (1) Given $|\vec{A}| = |\vec{B}|$ or $A = B$.

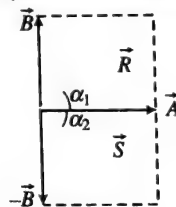
$$\text{Sum } \vec{R} = \vec{A} + \vec{B} \Rightarrow |\vec{R}| = \sqrt{A^2 + B^2} = \sqrt{2}A$$

$$\text{Difference } \vec{S} = \vec{A} - \vec{B} \Rightarrow |\vec{S}| = \sqrt{A^2 + B^2} = \sqrt{2}A$$

$$\alpha_1 = 45^\circ, \alpha_2 = 45^\circ$$

Hence, $\vec{R}$ and $\vec{S}$ will be perpendicular and also of equal lengths.

2. (3) The minimum number of vectors having different planes which can be added to give zero resultant is 4.



3. (4) We see that the dot product of $\hat{i} + \hat{j} + \hat{k}$ with $3\hat{i} + 2\hat{j} - 5\hat{k}$ is zero.

4. (4) In $\triangle MNO$, $\vec{A} + \vec{C} - \vec{D} = 0 \Rightarrow \vec{C} - \vec{D} = -\vec{A}$

Hence, (b) is correct.

In $\triangle MNP$, $\vec{A} + \vec{B} + \vec{E} = 0$

Hence, (a) is correct.

In $\triangle MPNO$, $-\vec{E} - \vec{B} + \vec{C} - \vec{D} = 0$

$$\Rightarrow \vec{B} + \vec{E} - \vec{C} = -\vec{D}$$

Hence, (c) is correct.

5. (4) For the resultant of some vectors to be zero, they should form a closed figure taken in the same order.

$$6. (2) \cos \beta = \frac{R}{B} = \frac{1}{2} \Rightarrow \beta = 60^\circ$$

$$\text{Angle between } \vec{A} \text{ and } \vec{B} = 90^\circ + \beta = 150^\circ$$

$$7. (2) \frac{a+b}{a-b} = \frac{3}{1} \text{ or } 3a - 3b = a + b$$

$$\text{or } 2a = 4b \text{ or } a = 2b$$

8. (2) Note that the angle between two forces is 120° and not 60° .

$$R^2 = F^2 + F^2 + 2F^2 \cos 120^\circ$$

$$= 2F^2 + 2F^2 \left(-\frac{1}{2}\right) F^2$$

$$\text{or } R = F$$

9. (2) Here the angle between two vectors of equal magnitude is 120° . So resultant has the same magnitude as either of the given vectors. Moreover, it is mid-way between the two vectors, i.e., it is along x -axis.

$$10. (1) \left(\frac{1}{3}\right)^2 = 1^2 + 1^2 + 2 \times 1 \times 1 \cos \theta$$

$$\text{or } \frac{1}{9} = 2(1 + \cos \theta) \text{ or } 1 + \cos \theta = \frac{1}{18}$$

$$\text{or } \cos \theta = \frac{1}{18} - 1 = -\frac{17}{18} \text{ or } \theta = \cos^{-1}\left(-\frac{17}{18}\right)$$

11. (3) $\vec{A} + \vec{B}$ will be in the plane containing $\vec{A}$ and $\vec{B}$, whereas $\vec{A} \times \vec{B}$ will be perpendicular to that plane.

12. (4) Consider only x and y components: $\sqrt{3^2 + 1^2} = \sqrt{10}$

13. (1) Given $R = A = B$, it will give $\theta = 120^\circ$.

14. (3) Let the third side be $\vec{C}$, then $|\vec{C}| = |\vec{A} + \vec{B}|$ or $|\vec{C}| = |\vec{A} - \vec{B}|$

$$15. (1) A_1 = 2, A_2 = 3, |\vec{A}_1 + \vec{A}_2| = 3$$

$$\Rightarrow |\vec{A}_1 + \vec{A}_2|^2 = 9$$

$$\Rightarrow A_1^2 + A_2^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 9$$

$$\Rightarrow 2^2 + 3^2 + 2\vec{A}_1 \cdot \vec{A}_2 = 9 \Rightarrow \vec{A}_1 \cdot \vec{A}_2 = -2$$

$$\text{Now, } (\vec{A}_1 + 2\vec{A}_2) \cdot (3\vec{A}_1 - 4\vec{A}_2) = 3A_1^2 - 8A_2^2 + 2\vec{A}_1 \cdot \vec{A}_2 \\ = 3(2)^2 - 8(3)^2 + 2(-2) = -64$$

$$16. (4) \vec{A} \cdot \vec{B} = 0 \text{ (given)} \Rightarrow \vec{A} \perp \vec{B}$$

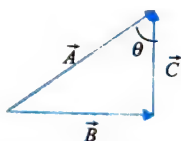
$$\vec{A} \cdot \vec{C} = 0 \text{ (given)} \Rightarrow \vec{A} \perp \vec{C}$$

$\vec{A}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

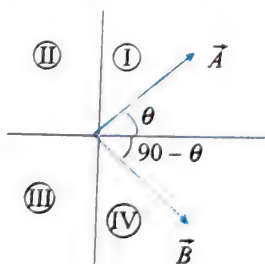
We know from the definition of cross product that $\vec{B} \times \vec{C}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

So $\vec{A}$ is parallel to $\vec{B} \times \vec{C}$.

$$17. (1) \cos \theta = \frac{C}{A} = \frac{3}{5} \text{ or } \theta = \cos^{-1}\left(\frac{3}{5}\right)$$

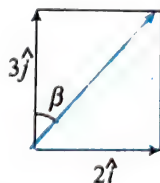


18. (2) Clearly, $\vec{B}$ should be either in the second quadrant or the fourth quadrant. In none of the given options, we have $-\hat{i}$ term. So the second quadrant is ruled out. Also $\vec{B}$ should make an angle of $90^\circ - \theta$ with the x -axis (figure). So, B should be $\vec{B} \cos(90^\circ - \theta) \hat{i} - B \sin(90^\circ - \theta) \hat{j} = B \sin \theta \hat{i} - B \cos \theta \hat{j}$.



19. (3) See figure.

$$\tan \beta = \frac{2}{3} \text{ or } \beta = \tan^{-1}\left(\frac{2}{3}\right)$$



20. (3) $\vec{P}$ is in the fourth quadrant. $4\hat{i} + 3\hat{j}$ is in the first quadrant. Clearly, $4\hat{i} + 3\hat{j}$ can be perpendicular to $\vec{P}$. For confirmation, let us check whether their dot product is zero.

$$(3\hat{i} - 4\hat{j}) \cdot (4\hat{i} + 3\hat{j}) = 12 - 12 = 0$$

This shows that $4\hat{i} + 3\hat{j}$ is perpendicular to $3\hat{i} - 4\hat{j}$.

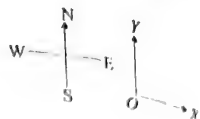
$$21. (3) \vec{S} = 75\hat{j} + [60 \cos 45^\circ \hat{j} - 60 \sin 45^\circ \hat{i}] + 20\hat{i} \\ = (20 - 60 \times 0.7)\hat{i} + (60 \times 0.7 + 75)\hat{j} = -22\hat{i} + 117\hat{j}$$

$$S = \sqrt{22^2 + 117^2} = \sqrt{484 + 13689} = \sqrt{14173} = 119 \text{ km}$$

$$22. (2) \text{Area of parallelogram: } |\vec{A} \times \vec{B}| \\ = AB/2 \text{ (given)}$$

$$\Rightarrow AB \sin \theta = AB/2$$

$$\Rightarrow \sin \theta = 1/2 \Rightarrow \theta = 30^\circ$$



$$23. (2) a^2 + b^2 + 2ab \cos \theta = a^2 + b^2 - 2ab \cos \theta \\ \text{or } 4ab \cos \theta = 0$$

$$\text{But } 4ab \neq 0 \Rightarrow \cos \theta = 0 \text{ or } \theta = 90^\circ$$

Aliter

$(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are the diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$.

Since $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, the two diagonals of a parallelogram are equal. So, think of rectangle. This leads to $\theta = 90^\circ$.

$$24. (1) \vec{A} \times \vec{B} = (4\hat{i} + 6\hat{j}) \times (2\hat{i} + 3\hat{j}) = 12(\hat{i} \times \hat{j}) + 12(\hat{j} \times \hat{i}) \\ = 12(\hat{i} \times \hat{j}) - 12(\hat{i} \times \hat{j}) = 0$$

$$\text{Again, } \vec{A} \cdot \vec{B} = (4\hat{i} + 6\hat{j}) \cdot (2\hat{i} + 3\hat{j}) = 8 + 18 = 26$$

$$\text{Again, } \frac{|\vec{A}|}{|\vec{B}|} = \frac{\sqrt{16+36}}{\sqrt{4+9}} \neq \frac{1}{2}$$

Also, $\vec{B} = \frac{1}{2}\vec{A} \Rightarrow \vec{A}$ and $\vec{B}$ are parallel and not antiparallel.

$$25. (1) \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & p & q \\ 5 & 7 & 3 \end{vmatrix} = 0$$

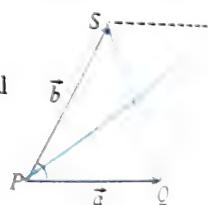
$$\text{or } \hat{i}(3p - 7q) + \hat{j}(5q - 6) + \hat{k}(14 - 5p) = 0$$

$$3p = 7q, 5q - 6 = 0 \text{ or } q = \frac{6}{5}$$

26. (2) If $\vec{A}$ is perpendicular to $\vec{B}$, then $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \times \vec{B} \neq 0$

27. (2) $\vec{PR} = \vec{a} + \vec{b} \Rightarrow$ major diagonal

$$\vec{SQ} = \vec{a} - \vec{b} \Rightarrow \text{minor diagonal}$$



$$28. (2) \vec{A} + \vec{B} = \vec{C}$$

$$(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = \vec{C} \cdot \vec{C}$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = C^2$$

$$4^2 + 5^2 + 2 \times 4 \times 5 \cos \theta = 61$$

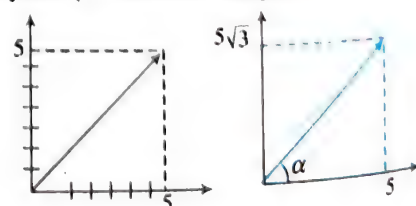
$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

29. (1) Let that vector be $\vec{C}$. Then

$$\vec{C} = C\hat{C} = b\hat{a} \Rightarrow \vec{C} = \frac{b\hat{a}}{a} = \frac{5}{\sqrt{2}}(\hat{i} - \hat{j})$$

30. (2) For the resultant of two vectors to be zero, they should be equal and opposite.

31. (3) In first option (1), vector is along the x -axis (figure).



In (2), angle of vector with the x -axis

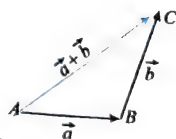
$$\tan \theta = \frac{5}{5} = 1 \Rightarrow \theta = 45^\circ$$

In (c), angle of vector with the x-axis

$$\tan \alpha = \frac{5\sqrt{3}}{5} = \sqrt{3} \Rightarrow \alpha = 60^\circ$$

32. (2) See figure.

$$AC \leq AB + BC \Rightarrow |\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$



33. (4) The resultant of two forces can lie between $A-B$ and $A+B$, i.e., $12-1=11$ N and $12+1=13$ N.

34. (1) Find min $(A-B)$ and max $(A+B)$ value of each case, then check if 4 N lies between them.

$$35. (2) \vec{BA} + \vec{BC} = \vec{BD}$$

$$\vec{BA} + \vec{BC} = 2\vec{BM}$$

Hence, the answer is $2BM$.

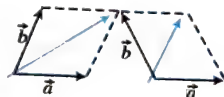
$$36. (2) (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$A^2 - B^2 = 0 \Rightarrow A^2 = B^2$$

$$\Rightarrow A = B$$

37. (3) $\vec{a} + \vec{b} \rightarrow$ major diagonal,

$\vec{a} - \vec{b} \rightarrow$ minor diagonal,



$$38. (2) \tan \theta = \frac{C}{A} = 1$$

$$\Rightarrow \theta = 45^\circ = \frac{\pi}{4}$$

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$



$$39. (1) \vec{F}_2 - \vec{F}_1 = \vec{F}_2 + (-\vec{F}_1) = 250 \text{ N due north} + 500 \text{ N due west}$$

$$\tan \theta = \frac{500}{250} = 2$$

$$|\vec{F}_2 - \vec{F}_1| = \sqrt{(500)^2 + (250)^2} = 250\sqrt{5} \text{ N}$$

40. (3) $\vec{OC}$ and $\vec{OA}$ are equal in magnitude and inclined to each other at an angle of 90° .

So their resultant is $\sqrt{2}r$

. It acts mid-way between $\vec{OC}$ and $\vec{OA}$, i.e., along OB .

Now, both r and $\sqrt{2}r$ are along the same line and in the same direction.

$$\text{Resultant} = r + \sqrt{2}r = r(1 + \sqrt{2})$$

$$41. (2) \tan 45^\circ = \frac{2 \sin 60^\circ}{a + 2 \cos 60^\circ} = \frac{\sqrt{3}}{a + 1}$$

$$\text{or } 1 = \frac{\sqrt{3}}{a + 1} \text{ or } a + 1 = \sqrt{3} \text{ or } a = \sqrt{3} - 1$$

$$42. (4) \tan 90^\circ = \frac{2Q \sin \theta}{P + 2Q \cos \theta} \Rightarrow \infty = \frac{2Q \sin \theta}{P + 2Q \cos \theta}$$

$$\Rightarrow P + 2Q \cos \theta = 0$$

$$\text{Now, } R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$= Q^2 + P[P + 2Q \cos \theta]$$

$$R^2 = Q^2 \Rightarrow R = Q$$

Alternative method:

$$\vec{R} = \vec{P} + \vec{Q} \Rightarrow \vec{P} = \vec{R} - \vec{Q}$$

$$\text{and } \vec{S} = \vec{P} + 2\vec{Q} = \vec{R} - \vec{Q} + 2\vec{Q} = \vec{R} + \vec{Q}$$

Now, $\vec{S}$ and $\vec{P}$ are perpendicular (from figure), so

$$\vec{S} \cdot \vec{P} = 0 \Rightarrow (\vec{R} + \vec{Q}) \cdot (\vec{R} - \vec{Q}) = 0$$

$$\Rightarrow \vec{R} = \vec{Q} \Rightarrow R = Q$$

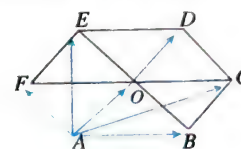
$$43. (1) \text{ Given } \vec{C} = |\vec{B}| \hat{j} \Rightarrow \vec{C} = 5\hat{j}$$

$$\text{Let } \vec{C} = \vec{A} + \vec{B} = \vec{A} + 3\hat{i} + 4\hat{j} \Rightarrow 5\hat{j} = \vec{A} + 3\hat{i} + 4\hat{j}$$

$$\Rightarrow \vec{A} = -3\hat{i} + \hat{j} \Rightarrow |\vec{A}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

$$44. (3) \vec{AB} + \vec{AF} = \vec{AO} \Rightarrow \vec{AB} = \vec{AO} - \vec{AF}$$

$$\vec{AC} = \vec{AB} + \vec{AO}, \vec{AD} = 2\vec{AO}, \vec{AE} = \vec{AO} + \vec{AF}$$



$$\text{Now, } \vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$$

$$= 5\vec{AO} + \vec{AB} + \vec{AF} = 5\vec{AO} + \vec{AO} = 6\vec{AO}$$

$$45. (2) \text{ Displacement} = \vec{AB},$$

angle between $\vec{r}_1$ and $\vec{r}_2$ is $\theta = 75^\circ - 15^\circ = 60^\circ$

$$\text{From figure, } AB^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos \theta$$

$$= 3^2 + 4^2 - 2 \times 3 \times 4 \cos 60^\circ = 13$$

$$\Rightarrow AB = \sqrt{13}$$

$$46. (3) x + y = 16.$$

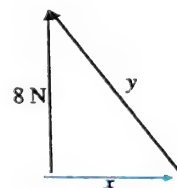
$$\text{Also, } y^2 = 8^2 + x^2$$

$$\text{or } y^2 = 64 + (16 - y)^2 \quad [\because x = 16 - y]$$

$$\text{or } y^2 = 64 + 256 + y^2 - 32y$$

$$\text{or } 32y = 320 \text{ or } y = 10 \text{ N}$$

$$\therefore x + 10 = 16 \text{ or } x = 6 \text{ N}$$



47. (3) Graphically:

$$\angle ROQ = \theta/2, \angle RQO = \theta/2$$

Hence, $\triangle OQR$ is isosceles.

$$\Rightarrow OR = RQ \Rightarrow B = A$$

Analytically:

$$\tan(\theta/2) = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\Rightarrow \frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{2B \sin(\theta/2) \cos(\theta/2)}{A + B[2 \cos^2(\theta/2) - 1]}$$

$$\Rightarrow A + 2B \cos^2(\theta/2) - B = 2B \cos^2(\theta/2) \Rightarrow A = B$$

48. (4) Analytically:

$$R_x = 1 + 2 \cos 120^\circ + 3 \cos 240^\circ = -3/2$$

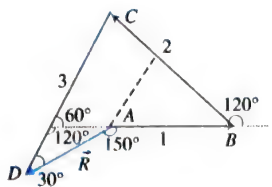
$$R_y = 2 \sin 120^\circ + 3 \sin 240^\circ$$

$$= 2 \times \frac{\sqrt{3}}{2} + 3 \times \left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{R_y}{R_x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{3}{2}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Hence, angle with first vector is $180 - \theta = 150^\circ$.

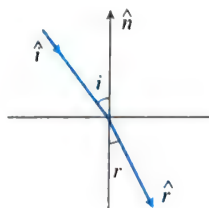
Graphically:



$$AB = 1, BC = 2, CD = 3$$

$\vec{R} = \vec{AD}$ is resultant, clearly angle between $\vec{R}$ and $\vec{AB}$ is 150° .

49. (3) You have to try all the options. Let us discuss the correct option only.



$$\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$$

$$(1)(1) \sin(180^\circ - i) = \mu(1)(1) \sin(180^\circ - r)$$

$$\sin i = \mu \sin r$$

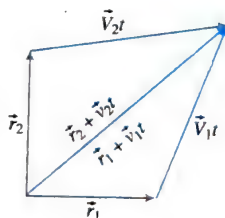
50. (4) The length of the vector is not changed by the rotation of the coordinate axes.

$$\sqrt{(n+1)^2 + 1^2} = \sqrt{n^2 + 3^2}$$

$$\text{or } n^2 + 2n + 2 = n^2 + 9$$

$$\text{or } 2n = 7 \text{ or } n = 3.5$$

51. (1) For collision,



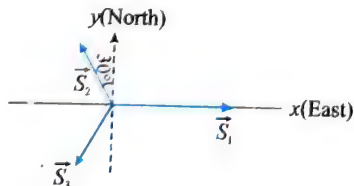
$$\vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$$

$$\text{or } \vec{r}_1 - \vec{r}_2 = (\vec{v}_2 - \vec{v}_1) t$$

$$\text{Equating unit vectors, we get } \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$$

$$52. (3) \cos \theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} - \hat{j} - \hat{k})}{\sqrt{3} \sqrt{3}} = \frac{1 - 1 - 1}{3} = -\frac{1}{3}$$

53. (1)



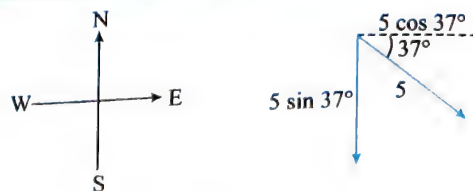
$$\vec{s}_1 + \vec{s}_2 + \vec{s}_3 = 0$$

$$10\hat{i} + [10 \sin 30(-\hat{i}) + 10 \cos 30 \hat{j}] + \vec{s}_3 = 0$$

$$(10 - 5)\hat{i} + 5\sqrt{3}\hat{j} + \vec{s}_3 = 0$$

$$\vec{s}_3 = -5\hat{i} - 5\sqrt{3}\hat{j}$$

54. (1) Given resultant displacement is 6 km due east
Which may be expressed as $6\hat{i}$.



Given displacements are

- (i) 2 km due east or $2\hat{i}$
(ii) 5 km 37° south of east its components

$$\text{along } x\text{-axis} = 5 \cos 37^\circ = 5 \times \frac{4}{5} = 4\hat{i}$$

$$\text{along } y\text{-axis} = 5 \sin 37^\circ = 5 \times \frac{3}{5} = -3\hat{j}$$

- (iii) unknown displacement let it be $a\hat{i} + b\hat{j}$

$$\therefore 2\hat{i} + 4\hat{i} - 3\hat{j} + a\hat{i} + b\hat{j} = 6\hat{i}$$

$$\therefore 6\hat{i} + a\hat{i} - 3\hat{j} + b\hat{j} = 6\hat{i}$$

comparing the two sides

$$a = 0 \text{ and } b = 3$$

$$\therefore \text{Unknown displacement} = 3\hat{j}$$

55. (4) Resultant acts along y-axis

$$\Rightarrow P_x = P \cos 60^\circ = 10 \text{ N or } \frac{P}{2} = 10 \text{ or } P = 20$$

$$\text{Resultant } P_y = P \sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ N}$$

56. (2) $\vec{V} = 50 \cdot \widehat{AB}$

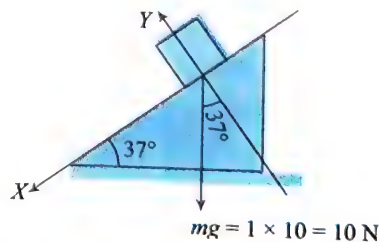
$$\widehat{AB} = (9-2)\hat{i} + (25-1)\hat{j} = (7\hat{i} + 24\hat{j})$$

$$|\widehat{AB}| = \sqrt{(7)^2 + (24)^2}$$

$$\widehat{AB} = \frac{(7\hat{i} + 24\hat{j})}{\sqrt{(7)^2 + (24)^2}} = \frac{(7\hat{i} + 24\hat{j})}{25}$$

$$\vec{V} = 50 \cdot \frac{(7\hat{i} + 24\hat{j})}{25} = 2(7\hat{i} + 24\hat{j}) \text{ m/s}$$

57. (1)



$$mg = 1 \times 10 = 10 \text{ N}$$

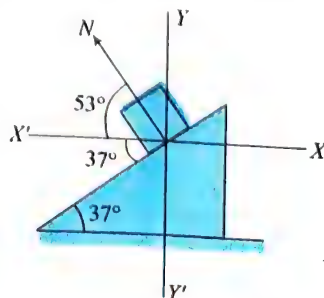
From figure, the x-component is

$$mg \sin 37^\circ = 1 \times 10 \times \frac{3}{5} = 6 \text{ N}$$

and y-component is

$$-mg \cos 37^\circ = -1 \times 10 \times \frac{4}{5} = -8 \text{ N}$$

58. (1) The x-component of normal reaction N is



$$-N \cos 53^\circ = -8 \times \frac{3}{5} = -4.8 \text{ N}$$

The y-component of normal reaction N is $N \sin 53^\circ$
 $= 8 \times \frac{4}{5} = \frac{32}{5} = 6.4 \text{ N}$

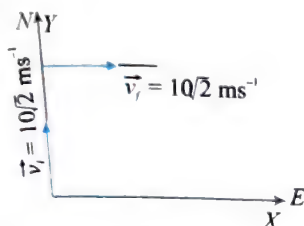
Resultant is along Y-axis. So, the component of resultant force along X-axis should be zero.

$$\therefore \sum F_x = 0$$

$$\text{or } 1 \sin 30^\circ + 2 \sin 30^\circ - 4 \cos 60^\circ + F_0 = 0$$

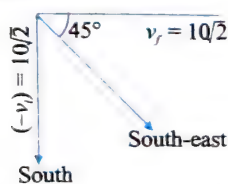
$$\therefore F_0 = \frac{-1}{2} - 1 + 2 = \frac{1}{2} = 0.5 \text{ N}$$

(1) Initial velocity is $\vec{v}_i = 10\sqrt{2} \hat{j}$



Final velocity is $\vec{v}_f = 10\sqrt{2} \hat{j}$

$\therefore$ The change in velocity is



$$\Delta v = v_f - v_i = 10\sqrt{2} \hat{i} - 10\sqrt{2} \hat{j}$$

$$|\Delta v| = \sqrt{(10\sqrt{2})^2 + (10\sqrt{2})^2} = \sqrt{400}$$

$$= 20 \text{ ms}^{-1} \text{ in South-East direction.}$$

Multiple Correct Answers Type

1. (1), (3)

Both x and y components of $\vec{d}_1$ are positive. The x component of $\vec{d}_2$ is negative and y component is positive. Both x and y components of $\vec{d}_1 + \vec{d}_2$ are positive.

2. (1), (2)

Component of $\vec{A}$ along $\vec{B}$ is $|\vec{A}| \cos \theta$ for θ being the angle between the vectors.

Also $\hat{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$. So choice (1) is correct.

The vector $(\hat{i} - \hat{j})$ is perpendicular to the vector $(\hat{i} + \hat{j})$.

So the other resolved component is $|\vec{A}| \sin \theta \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}} \right)$.

3. (1), (2), (3)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(1-1) - \hat{j}(2-1) + \hat{k}(2-1) = -\hat{j} + \hat{k}$$

The unit vector perpendicular to $\vec{A}$ and $\vec{B}$ is $\left(\frac{-\hat{j} + \hat{k}}{\sqrt{2}} \right)$. So choices

(1) and (3) are correct.

Any vector whose magnitude is K (constant) times $(2\hat{i} + \hat{j} + \hat{k})$ is parallel to $\vec{A}$.

So, unit vector $\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{6}}$ is parallel to $\vec{A}$.

So, choice (2) is correct.

4. (2), (4)

If two vectors are normal to each other, then their dot product is zero.

$$(\vec{v}_1 + \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2) = 0 \Rightarrow v_1^2 - v_2^2 = 0$$

$$\Rightarrow v_1^2 - v_2^2 \Rightarrow v_1 = v_2 \text{ or } |\vec{v}_1| = |\vec{v}_2|$$

5. (1), (4)

The resultant of three vectors is zero only if they can form a triangle. But three vectors lying in different planes cannot form a triangle.

6. (1), (2), (3), (4)

$$C^2 = A^2 + B^2 + 2AB \cos \theta$$

then $\theta = 90^\circ$

then $C^2 = A^2 + B^2$

if $\theta > 90^\circ$

then $C^2 = A^2 + B^2 + 2AB \cos \theta < A^2 + B^2$

$\therefore \cos \theta$ will be negative

If $\theta < 90^\circ$

then $C^2 = A^2 + B^2 + 2AB \cos \theta > A^2 + B^2$

$\therefore$ If $C = A - B \Rightarrow \theta = 180^\circ$

7. (1), (2), (3)

$\vec{x} \times \vec{y}$ should point in $\vec{z}$ direction for right-handed coordinate system.

8. (1), (2), (3)

(i) and (ii) give difference of $\vec{a}_x$ and $\vec{a}_y$; in (v), $\vec{a}$ is oppositely directed to what it should have been.

9. (1), (3), (4)

If $R = x\hat{i} + y\hat{j} + z\hat{k}$.

If $R = 0, x = 0, y = 0$ and $z = 0$.

10. (1), (2)

$\therefore$ Particle is in equilibrium.

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

$\therefore$ The sum of x -component is

$$\sum F_x = 10 \cos 30^\circ + F_2 \sin 30^\circ - 15 \sin 53^\circ = 0$$

$$\text{or } 5\sqrt{3} + \frac{F_2}{2} - 12 = 0$$

$$\therefore F_2 = 24 - 10\sqrt{3} = 24 - 17 = 7 \text{ N}$$

The sum of y -component of force is

$$\sum F_y = F_2 \cos 30^\circ + 10 \sin 30^\circ + 15 \cos 53^\circ - F_1 = 0$$

$$\text{or } \frac{\sqrt{3} F_2}{2} + 5 + 15 \times \frac{3}{5} - F_1 = 0$$

$$\text{or } \frac{\sqrt{3}}{2} (24 - 10\sqrt{3}) + 5 + 9 - F_1 = 0$$

$$\text{or } 12\sqrt{3} - 15 + 14 - F_1 = 0$$

$$\therefore F_1 = 12\sqrt{3} - 1 = 20.4 - 1 = 19.4 \text{ N}$$

11. (1), (2), (3)

(1) The x -component of F_1 is

$$F_1 \cos 30^\circ = 6 \cos 30^\circ = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ N}$$

(2) The x -component of F_2 is

$$-F_2 \sin 30^\circ = -4 \times \frac{1}{2} = -2 \text{ N}$$

Here, sum of x-component of forces
 $F_x = 6 \cos 30^\circ - 4 \sin 30^\circ = (3\sqrt{3} - 2) \text{ N}$
 and the sum of y-component of forces is
 $F_y = 6 \sin 30^\circ + 4 \cos 30^\circ = (3 + 2\sqrt{3}) \text{ N}$
 The magnitude of resultant is
 $F = \sqrt{F_x^2 + F_y^2} = \sqrt{52} \text{ N}$

12. (1), (2), (4)

The magnitude of unit vector is 1.

$$\therefore |a| = 1 \text{ or } \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

This equation is satisfied by options (1), (2) and (4). Hence, options (1), (2) and (4) are correct.

13. (1), (3)

$$\therefore \alpha = \beta = \gamma$$

$$\text{But } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\text{or } 3 \cos^2 \alpha = 1 \text{ or } \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

$$\therefore F = F \cos \alpha \hat{i} + F \cos \beta \hat{j} + F \cos \gamma \hat{k}$$

$$\text{or } F = \pm (\hat{i} + \hat{j} + \hat{k}) \text{ N}$$

14. (1), (3), (4)

$$F_z = 50 \cos 45^\circ = 25\sqrt{2} \text{ N}$$

$$F_{xy} = 50 \sin 45^\circ = 25\sqrt{2} \text{ N}$$

$$\therefore F_x = F_{xy} \cos 45^\circ = 25\sqrt{2} \times \frac{1}{\sqrt{2}} = 25 \text{ N}$$

$$\text{and } F_y = F_{xy} \sin 45^\circ = 25\sqrt{2} \times \frac{1}{\sqrt{2}} = 25 \text{ N}$$

Hence, options (1), (3) and (4) are correct.

15. (1), (2)

$$(1) \text{ The resultant } = R = a + b = 7\hat{i} + \hat{j}$$

$$\therefore \text{The magnitude of resultant} \\ = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2} \text{ unit}$$

$$(2) \therefore a \cdot b = |a||b|\cos\theta$$

$$\therefore \cos\theta = \frac{a \cdot b}{|a||b|} = \frac{12 - 12}{\sqrt{9 + 16}\sqrt{16 + 9}} = 0 \Rightarrow \theta = 90^\circ$$

16. (1), (4)

The mirror is in XY-plane. So, normal will be perpendicular to XY-plane (i.e. Z-axis).

If angle of incidence is θ . Then $\cos\theta = (\hat{e}_1 \cdot \hat{n})$

Here, $\hat{e}_1$ = unit vector along incidence ray

$$= \frac{\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + \hat{k}}{\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 + 1^2}} = \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$\hat{n}$ = unit vector along normal on incidence point = $\hat{k}$

$$\therefore \cos\theta = |\hat{e}_1 \cdot \hat{n}| = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

17. (2), (3), (4)

If C and r are perpendicular to each other.

Then, $C \cdot r = 0$

This condition is satisfied by options (2), (3) and (4).

18. (1), (2), (3)

If two vectors are perpendicular to each other, their dot product is zero.

$$(1) (\hat{i} + \hat{j} - \hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k}) = 2 - 1 - 1 = 0$$

So, option (1) is correct.

$$(2) (\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 1 + 1 - 2 = 0$$

So, option (2) is correct.

$$(3) (\hat{i} + \hat{j} - \hat{k}) \cdot (-\hat{i} + 2\hat{j} + \hat{k}) = -1 + 2 - 1 = 0$$

So, option (3) is correct.

19. (1), (2)

$$\vec{L} = \vec{OP} \times m\vec{v} \text{ or } \vec{L}$$

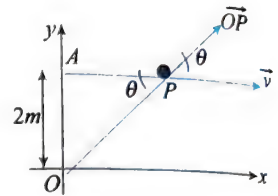
$$= (OP)(mv)\sin\theta(-\hat{k}) \text{ or}$$

$$= mv(OP\sin\theta)(-\hat{k})$$

$$= mv(OA)(-\hat{k})$$

$$= 2 \times 10 \times 2(-\hat{k}) = 40 \text{ kg} \cdot \text{m}^2 \text{s}^{-1}$$

Directed towards negative Z-axis.



20. (1), (2), (3)

$$F = qv \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 4 \end{vmatrix}$$

$$= \hat{i}(4 - 1) - \hat{j}(4 + 2) + \hat{k}(-1 - 2)$$

$$= 3\hat{i} - 6\hat{j} - 3\hat{k}$$

$$\therefore |F| = \sqrt{3^2 + 6^2 + 3^2} = \sqrt{54} \text{ N}$$

Hence, F is perpendicular to v and B .

Linked Comprehension Type

For Problems 1–3

1. (3), 2. (4), 3. (2)

$$\vec{F}_1 + \vec{F}_2 = m \cdot \vec{a}$$

$$(70\hat{j} + \vec{F}_2) = 5(10 \cos 53^\circ)\hat{i} + (10 \sin 53^\circ)\hat{j}$$

$$\Rightarrow (70\hat{j} + \vec{F}_2) = 5\left(10 \times \frac{3}{5}\hat{i} + 10 \times \frac{4}{5}\hat{j}\right)$$

$$(70\hat{j} + \vec{F}_2) = 30\hat{i} + 40\hat{j}$$

$$\therefore \vec{F}_2 = 30\hat{i} - 30\hat{j}$$

at $t = 10$ seconds

$$v = \vec{u} + \vec{at} = 0 + 10(6\hat{i} + 8\hat{j}) = (60\hat{i} + 80\hat{j}) \text{ m/s}$$

Acceleration will be zero if a third force makes the resultant force = 0

$$\therefore \vec{F} + \vec{F}_2 + \vec{F}_3 = 0$$

$$\therefore \vec{F}_3 = -(\vec{F}_1 + \vec{F}_2) = -(70\hat{j} + 30\hat{i} - 30\hat{j})$$

$$\therefore \vec{F}_3 = (-30\hat{i} - 40\hat{j}) \text{ N}$$

For Problems 4 and 5

4. (1), 5. (2)

$$\vec{r} = A(\cos t + t \sin t)\hat{i} + A(\sin t - t \cos t)\hat{j}$$

$$\frac{d\vec{r}}{dt} = A(\sin t + t \cos t + \sin t)\hat{i}$$

$$+ A\{(\cos t - t \times (-\sin t) + \cos t(-1))\}\hat{j}$$

$$\vec{v} = A(t \cos t) \hat{i} + A(t \sin t) \hat{j}$$

$$\frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = A\{t - \sin t + \cos t\} \hat{i} + A\{t \cos t + \sin t\} \hat{j}$$

$$\vec{a} = A\{-t \sin t + \cos t\} \hat{i} + A\{t \cos t + \sin t\} \hat{j}$$

$$\vec{r} \cdot \vec{a} = \cos^2 t + \sin^2 t - t^2 (\sin^2 t + \cos^2 t)$$

$$\Rightarrow \vec{r} \cdot \vec{a} = 1 - t^2$$

If $\vec{r}$ and $\vec{a}$ are perpendicular $\vec{r} \cdot \vec{a} = 0$

$$\text{or } 1 - t^2 = 0 \text{ for } t = 1 \text{ sec}$$

$\therefore$ They are parallel to each other $\vec{r} \cdot \vec{a} = 1$

$$\text{or } 1 - t^2 \text{ for } t = 0.$$

For Problems 6 and 7

6. (2), 7. (4)

The position of the particle, $\vec{r} = (5t^2 \hat{i} + 15t^2 \hat{j}) \text{ m}$

$$\frac{d\vec{r}}{dt} = (10t \hat{i} + 30t \hat{j}) \text{ m/s}$$

$$\text{Acceleration, } \frac{d^2\vec{r}}{dt^2} = (10 \hat{i} + 30 \hat{j}) \text{ m/s}^2$$

$$(\vec{F}_1 + \vec{F}_2 + \vec{F}_3) = m\vec{a}$$

$$\Rightarrow 100 \hat{i} + (F_2 \cos 30^\circ \hat{i} + F_2 \sin 30^\circ \hat{j})$$

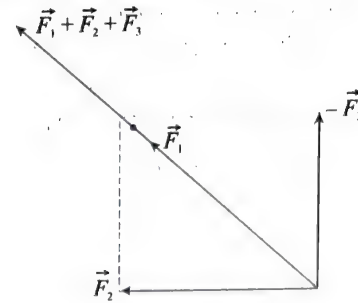
$$+ (F_1 \cos 150^\circ \hat{i} + F_1 \sin 150^\circ \hat{j}) = 10(10 \hat{i} + 30 \hat{j})$$

$$100 \hat{i} + F_2 \left(\frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j} \right) - F_1 \left(\frac{\sqrt{3}}{2} \hat{i} - \frac{1}{2} \hat{j} \right) = 100 \hat{i} + 300 \hat{j}$$

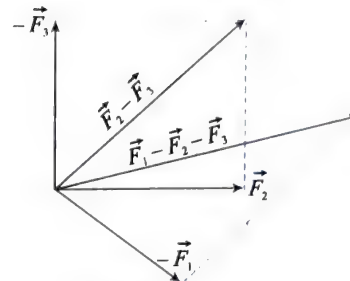
$$\left(100 + \frac{\sqrt{3}}{2} F_2 - \frac{\sqrt{3}}{2} F_1 \right) \hat{i} + \left(\frac{1}{2} F_2 + \frac{1}{2} F_1 \right) \hat{j} = 100 \hat{i} + 300 \hat{j}$$

$$\left(100 + \frac{\sqrt{3}}{2} F_2 - \frac{\sqrt{3}}{2} F_1 \right) = 100 \Rightarrow F_2 = F_1$$

$$\left(\frac{F_1}{2} + \frac{F_2}{2} \right) = 300 \Rightarrow F_1 = F_2 = 300 \text{ N}$$



(iii)



(iv)

3. (4)

$$(i) |\vec{a}| = \sqrt{3^2 + 4^2} = 5$$

$$(ii) \vec{a} + \vec{b} = 3\hat{i} + 4\hat{j} + 2\hat{i} + 2\hat{j} - \hat{k} = 5\hat{i} + 6\hat{j} - \hat{k}$$

$$\therefore |\vec{a} + \vec{b}| = \sqrt{5^2 + 6^2 + (-1)^2} = \sqrt{62} = 7.87$$

$$(iii) \vec{a} - \vec{c} = 3\hat{i} + 4\hat{j} - (3\hat{i} - 4\hat{k}) = 4\hat{j} + 4\hat{k}$$

$$\therefore |\vec{a} - \vec{c}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$$

$$(iv) s\vec{b} = s(2\hat{i} + 2\hat{j} - \hat{k}) = 2s\hat{i} + 2s\hat{j} - s\hat{k}$$

$$\therefore |s\vec{b}| = \sqrt{4s^2 + 4s^2 + s^2} = 1$$

$$\text{or } 3s = 1 \Rightarrow s = \frac{1}{3}$$

Hence, (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (d), (iv) $\rightarrow$ (b)

4. (2)

$$(i) \text{ Let } \vec{a} = 6\hat{i} - 3\hat{j} - 6\hat{k} \text{ and } \vec{b} = \hat{i} + \hat{j} + \hat{k}$$

$$\therefore a_{\parallel} = \frac{(\vec{a} \cdot \vec{b})\vec{b}}{|\vec{b}|^2} = \frac{(6 - 3 - 6)(\hat{i} + \hat{j} + \hat{k})}{(\sqrt{1^2 + 1^2 + 1^2})^2} = -\hat{i} - \hat{j} - \hat{k}$$

$$(ii) \vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$$

Here, $\vec{a}_{\parallel}$ = The component of $\vec{a}$ along $\vec{b}$,

$\vec{a}_{\perp}$ = The component of $\vec{a}$ perpendicular to $\vec{b}$.

$$\therefore \vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = 6\hat{i} - 3\hat{j} - 6\hat{k} - (-\hat{i} - \hat{j} - \hat{k}) = 7\hat{i} - 2\hat{j} - 5\hat{k}$$

$$(iii) \text{ Here, } \vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$$

$$\therefore \vec{a}_{\parallel} = \frac{(\vec{a} \cdot \vec{b})\vec{b}}{|\vec{b}|^2} = \frac{(12 + 3 + 12)(2\hat{i} - \hat{j} - 2\hat{k})}{(\sqrt{4 + 1 + 4})^2} = \frac{27}{9}(2\hat{i} - \hat{j} - 2\hat{k}) = 6\hat{i} - 3\hat{j} - 6\hat{k} = 0$$

$$\therefore \vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = (6\hat{i} - 3\hat{j} + 6\hat{k}) - (6\hat{i} - 3\hat{j} - 6\hat{k}) = 0$$

$$(iv) \text{ Here, } \vec{a} = 2\hat{j} - \hat{k}$$

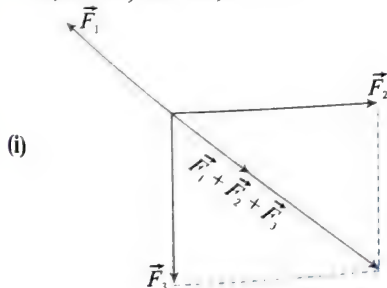
$$\therefore \vec{a}_{\parallel} = \frac{(\vec{a} \cdot \vec{b})\vec{b}}{|\vec{b}|^2} = \frac{(-6 + 6)(2\hat{j} - \hat{k})}{(\sqrt{4 + 1})^2} = 0$$

Hence, (i) $\rightarrow$ (b,d), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (a), (iv) $\rightarrow$ (a)

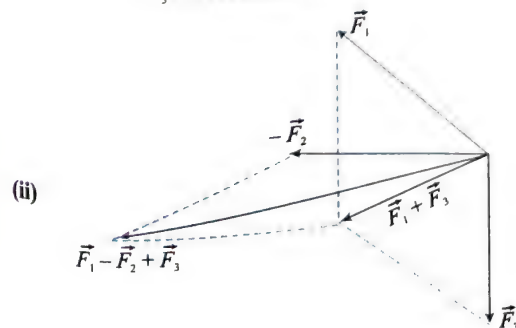
Matrix Match Type

1. i $\rightarrow$ b; ii $\rightarrow$ c; iii $\rightarrow$ a; iv $\rightarrow$ d

2. i $\rightarrow$ b; ii $\rightarrow$ c; iii $\rightarrow$ a; iv $\rightarrow$ d



(i)



(ii)

5. (3)

$$(i) \quad \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}(2+3) - \hat{j}(-1+6) + \hat{k}(1+4) \\ = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$$\therefore (\vec{a} \times \vec{b}) \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -5 & 5 \\ 1 & 3 & -2 \end{vmatrix} \\ = \hat{i}(10-15) - \hat{j}(-10-5) + \hat{k}(15+5) \\ = -5\hat{i} + 15\hat{j} + 20\hat{k} \\ \therefore |(\vec{a} \times \vec{b}) \times \vec{c}| = \sqrt{25+225+400} = \sqrt{650} \\ = \sqrt{25+26} = 5\sqrt{26}$$

$$(ii) \quad \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-4+1) + \hat{k}(6-1) \\ = \hat{i} + 3\hat{j} + 5\hat{k} \\ \therefore \vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 1 & 3 & 5 \end{vmatrix} \\ = \hat{i}(-10+9) - \hat{j}(5+3) + \hat{k}(3+2) \\ = -\hat{i} - 8\hat{j} + 5\hat{k} \\ \therefore |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{(-1)^2 + (-8)^2 + 5^2} \\ = \sqrt{1+64+25} = \sqrt{90} = 3\sqrt{10}$$

$$(iii) \quad \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} = \hat{i}(-2+3) - \hat{j}(-4+1) + \hat{k}(6-1) \\ = \hat{i} + 3\hat{j} + 5\hat{k} \\ \therefore \vec{a} \cdot (\vec{b} \times \vec{c}) = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 3\hat{j} + 5\hat{k}) \\ = 1 - 6 + 15 = 20 \\ \therefore |\vec{a} \cdot (\vec{b} \times \vec{c})| = 20$$

$$(iv) \quad \text{From solution of (i), } \vec{a} \times \vec{b} = 5\hat{i} - 5\hat{j} + 5\hat{k} \\ \vec{b} \cdot \vec{c} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} + 3\hat{j} + 2\hat{k}) = 2 + 3 + 2 = 7 \\ \therefore |(\vec{a} \times \vec{b}) \cdot (\vec{b} \cdot \vec{c})| = 7|a \times b| \\ = 7\sqrt{5^2 + (-5)^2 + 5^2} = 35\sqrt{3}$$

Hence, (i) $\rightarrow$ (d), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (a)

6. (3)

$$(i) \quad 2\hat{i} \times (\hat{i} + \hat{j}) = 2\hat{i} \times \hat{i} + 2\hat{i} \times \hat{j} = 2\hat{k} \\ (ii) \quad 4\hat{i} \times (2\hat{i} - \hat{j}) = 4\hat{i} \times 2\hat{i} - 4\hat{i} \times \hat{j} = 0 - 4\hat{k} = -4\hat{k} \\ (iii) \quad 3\hat{i} \times (3\hat{i} - 4\hat{j}) = 3\hat{i} \times 3\hat{i} - 12\hat{i} \times \hat{j} = 0 - 12\hat{k} = -12\hat{k} \\ (iv) \quad 2(\hat{i} + \hat{j}) \times 2\hat{i} = 4(\hat{i} \times \hat{j}) + 4(\hat{j} \times \hat{i}) = 0 + (-4\hat{k}) = -4\hat{k}$$

Hence, (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (d), (iv) $\rightarrow$ (c)**Numerical Value Type**

1. (6)

Since, acceleration of particle is along X-axis. So, net forces will be along X-axis.

$$\text{Hence, } \sum F_y = 0$$

$$\therefore F_2 \sin 37^\circ - F_1 \sin 30^\circ = 0 \text{ or } \frac{3}{5}F_2 = \frac{F_1}{2}$$

$$\therefore F_1 = \frac{6}{5} \times 5 = 6 \text{ N}$$

2. (1)

$$\vec{A} + \vec{B} = 5\hat{i} - \hat{j} + \hat{k} \\ \text{The unit vector of } \vec{A} + \vec{B} = \frac{5\hat{i} - \hat{j} + \hat{k}}{\sqrt{25+1+1}} = \frac{5\hat{i} - \hat{j} + \hat{k}}{\sqrt{27}}$$

$$\text{Hence, } a = 1$$

3. (2)

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\text{or } \frac{1}{4} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\text{or } \cos^2 \gamma = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \cos \gamma = \pm \frac{1}{\sqrt{2}}$$

Two values of $\cos \gamma$ are possible. Hence, two vectors are possible, one corresponding to direction cosines $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ and

other corresponding to direction cosines $\frac{1}{2}, \frac{1}{2}, -\frac{1}{\sqrt{2}}$.

4. (4)

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\text{or } \cos^2 60^\circ + \cos^2 60^\circ + \cos^2 \gamma = 1$$

$$\text{or } \frac{1}{4} + \frac{1}{4} + \cos^2 \gamma = 1$$

$$\text{or } \cos^2 \gamma = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{or } \cos \gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4} \quad \therefore n = 4$$

5. (5)

$$F_{21} = F_{21}\hat{e} = F_{21} \frac{AB}{|AB|} = F_{21} \frac{(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}} \\ = \frac{3}{5}(\hat{i} + 2\hat{j} + 2\hat{k}) = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\therefore x = 1, y = 2, z = 2$$

$$\therefore x + y + z = 5$$

6. (2)

$$\text{Here, } \vec{F}_1 = 3\hat{k}$$

$$\vec{F}_2 = 5 \cos 37^\circ + 5 \sin 37^\circ \hat{i} = 4\hat{i} + 3\hat{j}$$

$$\text{and } \vec{F}_3 = -4\sqrt{2} \cos 45^\circ \hat{i} - 4\sqrt{2} \sin 45^\circ \hat{j} \\ = -4\hat{i} - 4\hat{j}$$

For equilibrium of the particle

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 0$$

$$\text{or } 3\hat{k} + 4\hat{j} + 3\hat{i} - 4\hat{i} - 4\hat{j} + \vec{F}_4 = 0$$

$$\therefore \vec{F}_4 = \hat{i} - 3\hat{k}$$

$$\therefore \vec{F}_4 = \sqrt{1+9} = \sqrt{10}$$

$$\therefore n = 2$$

7. (1)

If a and b are perpendicular, $a \cdot b = 0$

$$\text{or } 2x - 3 + 1 = 0$$

$$\therefore x = 1$$

(7) The volume of parallelepiped is $|\vec{a} \cdot (\vec{b} \times \vec{c})|$

$$= |(2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot \{(i + 2j - k) \times 3i - j + 2k\}|$$

$$= |(2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot \left\{ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} \right\}|$$

$$= |(2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot \{(4 - 1) - j(2 + 3) + k(-1 - 6)\}|$$

$$= |(2\hat{i} - 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} - 5\hat{j} - 7\hat{k})| = |6 + 15 - 28| = 7 \text{ m}^3$$

(4) If the vectors are coplanar. Then, $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$.

$$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 3 & -y & 5 \end{vmatrix} = 0$$

$$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}) \cdot \{i(10 - 3y) - j(5 + 9) + k(-y - 6)\} = 0$$

$$\Rightarrow (2\hat{i} - \hat{j} + \hat{k}) \cdot \{(10 - 3y)\hat{i} - 14\hat{j} - (6 + y)\hat{k}\} = 0$$

$$\Rightarrow 2(10 - 3y) + 14 - (6 + y) = 0$$

$$\Rightarrow 20 - 6y + 14 - 6 - y = 0$$

$$\Rightarrow 28 - 7y = 0 \quad \therefore y = 4$$

(2) The resultant of F_1 and F_2 is in direction of 1000 N force. So, the sum of component of F_1 and F_2 in perpendicular direction of 1000 N force will be zero.

$$\therefore F_1 \sin 30^\circ = F_2 \sin 45^\circ$$

$$\text{or } \frac{F_1}{F_2} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1}{\sqrt{2} \times \frac{1}{2}} = \sqrt{2}$$

$$\therefore n = 2$$

Archives

Main

Single Correct Answer Type

1. (2) Given $u = 3\hat{i} + 4\hat{j}$, $a = 0.4\hat{i} + 0.3\hat{j}$

$$v = u + at$$

$$= 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j})10$$

$$= 3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$$

So, speed is equal to magnitude of velocity

$$= \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ units}$$

2. (3) $\vec{L} = m(\vec{r} \times \vec{v})$

$$\vec{L} = m \left[v_0 \cos \theta \hat{i} + (v_0 \sin \theta - \frac{1}{2}gt^2) \hat{j} \right]$$

$$\times \left[v_0 \cos \theta \hat{i} + (v_0 \sin \theta - gt) \hat{j} \right]$$

$$= mv_0 \cos \theta \left[-\frac{1}{2} \right] \hat{k} = -\frac{1}{2} mgv_0 t^2 \cos \theta \hat{k}$$

3. (3) For a particle in uniform circular motion, $\vec{a} = \frac{v^2}{R}$ towards centre of circle

$$\vec{a} = \frac{v^2}{R} (-\cos \theta \hat{i} - \sin \theta \hat{j})$$

$$\vec{a} = -\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$$

4. (4) $\vec{v} = Ky\hat{i} + Kx\hat{j}$

$$\frac{dx}{dt} = Ky, \frac{dy}{dt} = Kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{Kx}{Ky}$$

$$y = dy = x dx$$

$$y^2 = x^2 + c$$

5. (1) $x = t$

$$y = 2t - 5t^2$$

Equation of trajectory is $y = 2x - 5x^2$

JEE Advanced

Single Correct Answer Type

1. (3) Let vector from point P to point S be $\vec{C}$

$$\Rightarrow \vec{C} = b|\vec{R}|\hat{R} = b|\vec{R}|\left(\frac{\vec{R}}{|\vec{R}|}\right) = b\vec{R} = b(\vec{Q} - \vec{P})$$

from triangle rule of vector addition

$$\vec{P} + \vec{C} = \vec{S}$$

$$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S}$$

$$\Rightarrow \vec{S} = (1 - b)\vec{P} + b\vec{Q}$$

Numerical Value Type

1. (2) Given $\vec{A} = a\hat{i}$ and $\vec{B} = a(\cos \omega t \hat{i} + \sin \omega t \hat{j})$

$$(\vec{A} + \vec{B}) = (a + a \cos \omega t)\hat{i} + a \sin \omega t \hat{j}$$

and $(\vec{A} - \vec{B}) = (a - a \cos \omega t)\hat{i} - a \sin \omega t \hat{j}$

$$|(\vec{A} + \vec{B})| = \sqrt{(a + a \cos \omega t)^2 + (a \sin \omega t)^2}$$

$$|(\vec{A} - \vec{B})| = \sqrt{(a - a \cos \omega t)^2 + (a \sin \omega t)^2}$$

Which gives $\vec{A} + \vec{B} = 2a \cos \frac{\omega t}{2}$

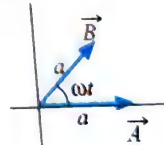
$$\vec{A} - \vec{B} = 2a \sin \frac{\omega t}{2}$$

So $2a \cos \frac{\omega t}{2} = \sqrt{3} \left(2a \sin \frac{\omega t}{2} \right)$

$$\tan \frac{\omega t}{2} = \frac{1}{\sqrt{3}}$$

$$\frac{\omega t}{2} = \frac{\pi}{6} \Rightarrow \omega t = \frac{\pi}{3}$$

or $\frac{\pi}{6} t = \frac{\pi}{3} \Rightarrow t = 2.00 \text{ sec}$



Chapter 4

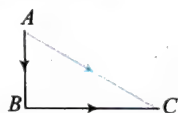
Concept Application Exercises

Exercise 4.1

- No; Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations.
- Yes, if its velocity slows down towards north.
 - Yes, if velocity and acceleration are in opposite directions.
 - Yes, if an accelerating object starts decelerating.
 - No, because average speed can also be greater than average velocity.
 - No, change in velocity takes place in the direction of acceleration.
- Yes, for example, a freely falling particle at its highest point has zero velocity but accelerating downward.
 - No, if distance is zero, then displacement is also zero.
 - Yes, if velocity is also in the negative direction.
 - Yes, if motion takes place continuously in one direction.

$$4. \quad v_{av} = \frac{s}{t} = \frac{100 \times 10 + 200 \times 20}{10 + 20} = \frac{500}{3} \text{ ms}^{-1}$$

$$5. \quad AB = 10 \times 10 = 100 \text{ m}, BC = 20 \times 10 = 200 \text{ m}$$



$$\text{Average velocity} = \frac{AC}{t} = \frac{100\sqrt{5}}{20} = 5\sqrt{5} \text{ ms}^{-1}$$

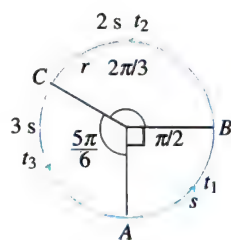
$$\text{Average speed} = \frac{AB + BC}{t} = \frac{100 + 200}{20} = 15 \text{ ms}^{-1}$$

$$\text{Displacement, } AC = \sqrt{AB^2 + BC^2} = 100\sqrt{5} \text{ m}$$

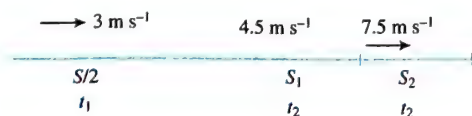
$$6. \quad t_1 = \frac{AB}{s} = \frac{\frac{\pi}{2}r}{s}, \quad t_2 = \frac{BC}{2s} = \frac{\frac{2\pi}{3}r}{2s},$$

$$t_3 = \frac{CA}{3s} = \frac{\frac{5\pi}{6}r}{3s}$$

$$\text{Average speed} = \frac{2\pi r}{t_1 + t_2 + t_3} = 1.8 \text{ s}$$



$$7. \quad t_1 = \frac{S/2}{3} = \frac{S}{6}, \quad t_2 = \frac{S_1}{4.5} = \frac{S_2}{7.5}$$



$$S_1 + S_2 = S/2$$

$$\Rightarrow 4.5 t_2 + 7.5 t_2 = S/2$$

$$\Rightarrow t_2 = \frac{S}{24}$$

$$\text{Average speed} = \frac{S}{t_1 + 2t_2} = 4 \text{ ms}^{-1}$$

- Average velocity on each lap will be zero, because displacement is zero.

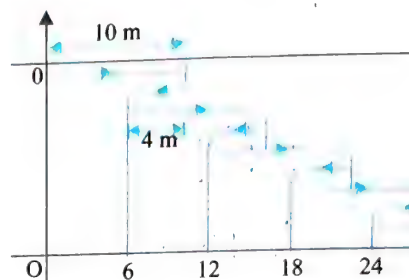
$$\text{Average speed} = \frac{2\pi r}{t} = \frac{2\pi \times 200}{62.8} = 20 \text{ ms}^{-1}$$

$$9. \quad (a) \quad v_{av1} = \frac{50}{20} = 2.5 \text{ ms}^{-1},$$

$$(b) \quad v_{av2} = \frac{50}{22} = 2.27 \text{ ms}^{-1}$$

(c) v_{av} is zero for round trip because displacement is zero.

10.



$$(a) \quad (i) \quad D = 4(|\vec{s}_1| + |\vec{s}_2|) = 4(10 + 4) = 56 \text{ m}$$

$$(ii) \quad \vec{s} = 4 \times (\vec{s}_1 + \vec{s}_2) = 4(10 + (-4)) = +24 \text{ m}$$

$$(b) \quad (i) \quad u_{av} = \frac{D}{\Delta t} = \frac{56}{10} = 5.6 \text{ m/minute}$$

$$(ii) \quad \vec{v}_{av} = \frac{\vec{s}}{\Delta t} = \frac{(+24)}{10} = +2.4 \text{ m/minute}$$

- Let the distance between his house and school is s .

$$\therefore s = \frac{s}{\frac{t}{3} + \frac{1}{60}} \quad \text{(for 1st event)} \quad \dots(i)$$

$$\text{And } 2 + 1 = \frac{s}{\left(t - \frac{3}{60}\right)} \quad \text{(for 2nd event)} \quad \dots(ii)$$

On solving equations (i) and (ii), we get, $s = 0.6 \text{ km}$

Exercise 4.2

$$1. \quad \text{Average velocity } \langle \vec{v} \rangle = \frac{\text{displacement}}{\text{time}} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{1}{2}at$$

$$2. \quad s = 4t \frac{1}{2}(1)t^2 = 2t + \frac{1}{2}(2)t^2$$

$$\text{or } 4t + 0.5t^2 = 2t + t^2$$

$$\text{or } \frac{t^2}{2} = 2t$$

$$\text{or } t = 0 \text{ and } t = 4 \text{ s}$$

$$s = (4)(4) + \frac{1}{2}(1)(4)^2 = 16 + 8 = 24 \text{ m}$$

$$3. \quad \text{Here, } u = 54 \times \frac{5}{18} = 15 \text{ ms}^{-1}$$

$$a = -0.3 \text{ ms}^{-2}$$

$$\therefore v = u + at$$

$$\text{or } 0 = 15 - 0.3t_0$$

$$\therefore t_0 = 50 \text{ s}$$

After 50 s, car permanently comes in rest.

$$\therefore s = \left(\frac{u+v}{2}\right)t = \left(\frac{15+0}{2}\right) \times 50 = 375 \text{ m}$$

Thus, the distance of car from check post is $(400 - 375) \text{ m} = 25 \text{ m}$

$$s = \left(\frac{u+v}{2} \right) t \quad \text{or} \quad 250 = \left(\frac{0+50}{2} \right) t$$

$$\therefore t = 10 \text{ s}$$

$$\therefore a = \frac{u-v}{t}$$

$$\therefore a = \frac{50-0}{10} = 5 \text{ ms}^{-2}$$

(b) The final velocity after 250 m will be initial velocity of the 2nd part of the motion of car.

$$\therefore v = u + at$$

$$\text{or } 60 = 50 + a \times 10$$

$$\therefore a = 1 \text{ ms}^{-2}$$

$$\text{Also } s = \left(\frac{u+v}{2} \right) t = \left(\frac{50+60}{2} \right) \times 10 = 550 \text{ m}$$

$$(c) s = \left(\frac{u+v}{2} \right) t = \left(\frac{60+0}{2} \right) \times 6 = 180 \text{ m}$$

5. The distance travelled by the head light of engine in 60 s is

$$s_1 = ut + \frac{1}{2} at^2 = 0 \times 60 + \frac{1}{2} \times 0.04 \times (60)^2 = 72 \text{ m}$$

The distance travelled by tail light of the train in 120 s is

$$s_2 = ut + \frac{1}{2} at^2 = 0 \times 120 + \frac{1}{2} \times 0.04 \times (120)^2 = 288 \text{ m}$$

Thus, the distance between these two events

$$= |(200 + 72 - 288) \text{ m}| = 16 \text{ m}$$

$$6. u = 108 \text{ km h}^{-1} = 30 \text{ ms}^{-1}, v = 36 \text{ km h}^{-1} = 10 \text{ ms}^{-1}, v^2 = u^2 + 2as$$

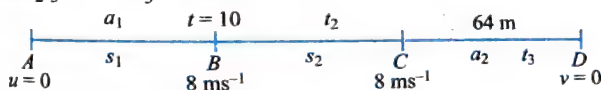
$$\Rightarrow 10^2 = 30^2 + 2a \times 200 \Rightarrow a = -2 \text{ ms}^{-2}$$

$$v = u + at \Rightarrow 10 = 30 - 2t \Rightarrow t = 10 \text{ s}$$

$$7. 8 = 0 + a_1 \times 10 \Rightarrow a_1 = 0.8 \text{ ms}^{-2}$$

$$0^2 = 8^2 + 2a_2 \times 64 \Rightarrow a_2 = -0.5 \text{ ms}^{-2}$$

$$0 = 8 + a_2 t_3 \Rightarrow t_3 = 16 \text{ s}$$



$$s_1 = \frac{1}{2} a_1 t^2 = \frac{1}{2} (0.8) 10^2 = 40 \text{ m}$$

$$s_2 = 584 - 40 - 64 = 480 \text{ m}, t_2 = \frac{s_2}{v} = \frac{480}{8} = 60 \text{ s}$$

$$\text{Total time taken} = 10 + 60 + 16 = 86 \text{ s}$$

$$8. 10 = u + \frac{a}{2} (2 \times 2 - 1) \text{ and } 25 = u + \frac{a}{2} (2 \times 5 - 1)$$

$$\text{Solving, we get } u = \frac{5}{2} \text{ ms}^{-1}, a = 5 \text{ ms}^{-2}$$

$$D_7 = \frac{5}{2} + \frac{5}{2} [2 \times 7 - 1] = 35 \text{ m}$$

$$9. 25 = u + \frac{a}{2} (2 \times 5 - 1) \text{ and } 33 = u + \frac{a}{2} (2 \times 7 - 1)$$

$$\text{Solving, we get } u = 7 \text{ ms}^{-1}, a = 4 \text{ ms}^{-2}$$

$$10. v_{\text{rel}} = 50 + 40 = 90 \text{ km h}^{-1} = 25 \text{ ms}^{-1}$$

$$a_{\text{rel}} = 30 + 20 = 50 \text{ cm s}^{-1} = 0.5 \text{ ms}^{-2}$$

$$s_{\text{rel}} = u_{\text{rel}} t + \frac{1}{2} a_{\text{rel}} t^2$$

$$\Rightarrow 100 + 100 = 25 t + \frac{1}{2} 0.5 t^2$$

$$\text{Solving, we get } t = 10\sqrt{33} - 50 \text{ s}$$

$$11. v_0 = 0 + \alpha t_1 \Rightarrow t_1 = v_0 / \alpha,$$

$$0 = v_0 - \beta t_2 \Rightarrow t_2 = v_0 / \beta$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{\beta}{\alpha}$$

Exercise 4.3

1. (a) i. False, time of ascent = time of descent.

$$\text{ii. True, } D_n = u + \frac{a}{2} (2n-1) = 0 + \frac{10}{2} (2 \times 3 - 1) = 25 \text{ m}$$

iii. True, because at the time of dropping the velocity of the packet is upward, and same as that of balloon.

iv. True, acceleration due to gravity is same for all bodies.

(b) i. Only velocity is zero

ii. Time of descent

iii. Second half

$$2. (a) H = 20 = \frac{u^2}{2g} \Rightarrow u = \sqrt{2g \times 20} = 20 \text{ ms}^{-1}$$

$$(b) t_a = \frac{u}{g} = \frac{20}{10} = 2 \text{ s}$$

(c) On hitting the ground, the velocity will be equal to the initial velocity but in the opposite direction. Hence, answer is 20 ms^{-1} .

$$(d) S_1 = 20 \times 0.5 - \frac{1}{2} 10 (0.5)^2; S_2 = 20 \times 2.5 - \frac{1}{2} 10 (2.5)^2$$

$$\text{Required displacement} = S_2 - S_1 = 10 \text{ m}$$

$$(e) 15 = 20t - \frac{1}{2} 10t^2 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1 \text{ s}, 3 \text{ s}$$

$$3. v = u + at$$

$$\Rightarrow 0 = 20 - g \sin \theta \times 4$$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

$$v^2 = u^2 + 2as$$

$$0^2 = (20)^2 + 2(-g \sin \theta) (h/\sin \theta)$$

$$\Rightarrow h = 20 \text{ m}$$

$$4. (a) s = ut + \frac{1}{2} at^2 \text{ fi } -200 = 0 \times t + \frac{1}{2} (-10) t^2 \text{ fi } t = \sqrt{40} \text{ s}$$

$$(b) -200 = 10t - \frac{1}{2} 10t^2 \text{ fi } t^2 - 2t - 40 = 0 \text{ fi } t = 1 + \sqrt{41} \text{ s}$$

$$(c) -200 = -10t - \frac{1}{2} 10t^2 \text{ fi } t^2 + 2t - 40 = 0 \text{ fi } t = -1 + \sqrt{41} \text{ s}$$

$$5. u = \sqrt{2 \times \frac{g}{8} H} \Rightarrow u = \frac{\sqrt{gH}}{2}$$

$$-H = ut - \frac{1}{2} gt^2 \Rightarrow gt^2 - 2ut - 2H = 0$$

$$gt^2 - \sqrt{gH} t - 2H = 0$$

$$\Rightarrow gt^2 - 2\sqrt{gH} t + \sqrt{gH} t - 2H = 0$$

$$\sqrt{g} t [\sqrt{g} t - 2\sqrt{H}] + \sqrt{H} [\sqrt{g} t - 2\sqrt{H}] = 0$$

$$\Rightarrow (\sqrt{g} t + \sqrt{H})(\sqrt{g} t - 2\sqrt{H}) = 0 \Rightarrow t = 2 \sqrt{\frac{H}{g}}$$

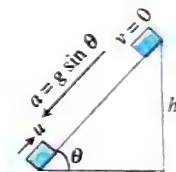
$$6. \text{ Velocity after 50 m fall, } u_1 = \sqrt{2gh} = \sqrt{100} \text{ g}$$

$$3^2 = u_1^2 + 2 \times (-2)h \Rightarrow 9 = 100g - 4h$$

$$h = \frac{100 \times 9.8 - 9}{4} = 242.75 = 243 \text{ m}$$

$$\text{Net height} = 243 + 50 = 293 \text{ m}$$

$$7. h = \frac{1}{2} gn^2, \frac{h}{2} = \frac{1}{2} g(n-1)^2, \text{ where } n \text{ is the total time taken.}$$



Solving, we get

$$n = \frac{\sqrt{2}}{\sqrt{2}-1} \Rightarrow n = \left(\frac{\sqrt{2}}{\sqrt{2}-1} \right) \left(\frac{\sqrt{2}+1}{\sqrt{2}+1} \right) = 2 + \sqrt{2} \text{ s}$$

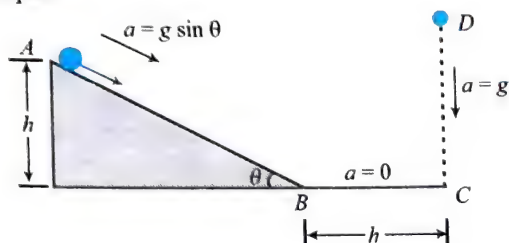
8. The time needed for the flower to fall is

$$t = \sqrt{\frac{2\Delta h}{g}} = \sqrt{\frac{2(46.0 \text{ m} - 1.80 \text{ m})}{(9.80 \text{ m s}^{-2})}} = 3.00 \text{ s}$$

and so the professor should be at a distance

$$v_y t = (1.20 \text{ m s}^{-1})(3.00 \text{ s}) = 3.60 \text{ m}$$

9. If both particles collide, the time of travel for both particles should be equal.



Path A-B-C: $T_1 = t_{AB} + t_{BC}$

For Path AB: $S_{AB} = t_{AB}^2 \Rightarrow \frac{h}{\sin \theta} = \frac{1}{2} g \sin \theta \cdot t_{AB}^2$

$$t_{AB} = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}}$$

Velocity of the particle when it reaches B,

$$v_B = 0 + (g \sin \theta) \cdot \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}}$$

Path BC: As acceleration of the particle is zero, time to cover distance BC,

$$t_{BC} = \frac{BC}{v_B} = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}}$$

$$\text{Hence, } T_1 = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}} + \sqrt{\frac{h}{2g}} \quad \dots(i)$$

Path DC: The particle will free fall,

$$H = \frac{1}{2} g t_2^2 \Rightarrow T_2 = \sqrt{\frac{2H}{g}} \quad \dots(ii)$$

For particle collide $T_1 = T_2$, which gives

$$H = \frac{h}{4 \sin^2 \theta} (2 + \sin \theta)^2$$

10. Maximum height released by particle $H = \frac{u^2}{2g}$

$$\text{Given } v_2 = 2v_1 \Rightarrow \frac{v_1}{v_2} = \frac{1}{2} \quad \dots(i)$$

Motion from A to B:

$$v_1^2 = u^2 - 2gh \quad \dots(ii)$$

Motion from A to C:

$$v_2^2 = u^2 - 2g(-h) \quad \dots(iii)$$

(ii)/(iii)

$$\left(\frac{v_1}{v_2} \right)^2 = \frac{u^2 - 2gh}{u^2 + 2gh} = \frac{\left(\frac{u^2}{2g} - h \right)}{\left(\frac{u^2}{2g} + h \right)}$$

$$\frac{1}{4} = \frac{(H-h)}{(H+h)} \Rightarrow 4(H-h) = H+h \left(\text{from (i)} H = \frac{u^2}{2g} \right)$$

$$\Rightarrow 3H = 5h \Rightarrow H = \frac{5}{3}h$$

11. The falling ball moves a distance of $(15\text{m} - h)$ before they meet, where h is the height above the ground where they meet. We apply

$$y_f = y_i + v_i t - \frac{1}{2} g t^2$$

to the falling ball to obtain

$$-(15.0 \text{ m} - h) = -\frac{1}{2} g t^2$$

$$\text{or } h = 15.0 \text{ m} - \frac{1}{2} g t^2$$

Applying $y_f = y_i + v_i t - \frac{1}{2} g t^2$ to the rising ball gives

$$h = (25 \text{ m/s})t - \frac{1}{2} g t^2$$

Combining equations [1] and [2] gives

$$(25 \text{ m/s})t - \frac{1}{2} g t^2 = 15.0 \text{ m} - \frac{1}{2} g t^2$$

$$\text{or } t = \frac{15 \text{ m}}{25 \text{ m/s}} = 0.60 \text{ s}$$

12. (a) The keys, moving freely under the influence of gravity ($a = -g$) undergo a vertical displacement of $\Delta y = +h$ in time t . We use $\Delta y = v_i t + \frac{1}{2} a t^2$ to find the initial velocity as

$$\Delta y = v_i t + \frac{1}{2} a t^2 = h$$

$$\Rightarrow h = v_i t - \frac{1}{2} g t^2$$

$$\text{or } v_i = \frac{h + \frac{1}{2} g t^2}{t} = \frac{h}{t} + \frac{g t}{2}$$

- (b) We find the velocity of the keys just before they were caught (at time t) using $v = v_i + at$:

$$v = v_i + at$$

$$v = \left(\frac{h}{t} + \frac{g t}{2} \right) - g t$$

$$v = \frac{h}{t} - \frac{g t}{2}$$

Exercise 4.4

1. Distance to be covered = $1000 + 200 = 1200 \text{ m}$

$$\text{Velocity} = 72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}$$

$$t = \frac{\text{Distance}}{\text{Velocity}} = \frac{1200}{20} = 60 \text{ s}$$

2. Required separation = $(v_2 - v_1)t = (72 - 36) \frac{20}{60} = 12 \text{ km}$

3. $t = \frac{\text{Total length}}{\text{Relative velocity}} = \frac{110 + 90}{(36 + 54) \times (5/18)} = 8 \text{ s}$

4. (a) When she walks in the same direction, relative velocity of woman w.r.t. ground:

$$v_1 = 1 + 1.5 = 2.5 \text{ m s}^{-1}$$

$$\text{Time taken} = \frac{s}{v_1} = \frac{35}{2.5} = 14 \text{ s}$$

- (b) When she walks in the opposite direction, relative velocity of woman w.r.t. ground:

$$v_2 = 1.5 - 1 = 0.5 \text{ m s}^{-1}$$

$$\text{Time taken} = \frac{s}{v_2} = \frac{35}{0.5} = 70 \text{ s}$$

5. (a) $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = g + a$

$$\vec{s} = \vec{u} t + \frac{1}{2} \vec{a}_{BL} t^2; 0 = uT - \frac{1}{2} (g + a)T^2$$

$$t = \frac{2u}{(g + a)}$$

$$(b) \quad v^2 - u^2 = 2as$$

$$0 - u^2 = -2(g+a)H \Rightarrow H = \frac{u^2}{2(g+a)}$$

Acceleration of car 1 w.r.t. car 2

$$\vec{a}_{12} = \vec{a}_1 - \vec{a}_2 = \vec{a}_{C_1} - \vec{a}_{C_2} = 0 - a = (-a)$$

$$\vec{u}_{12} = \vec{u}_1 - \vec{u}_2 = 12 - 10 = 2 \text{ ms}^{-1}$$

The collision is just avoided if the relative velocity becomes zero just at the moment the two cars meet each other, i.e.,

$$v_{12} = 0 \text{ when } s_{12} = 200$$

Now $v_{12} = 0$, $\vec{u}_{12} = 2$, $\vec{a}_{12} = -a$, and $s_{12} = 200$.

$$v_{12}^2 - u_{12}^2 = 2a_{12}s_{12} \quad \therefore 0 - 2^2 = -2 \times a \times 200$$

$$\Rightarrow a = \frac{1}{100} \text{ ms}^{-2} = 0.1 \text{ ms}^{-2} = 1 \text{ cms}^{-2}$$

Therefore, minimum acceleration needed by car $C_2 = 1 \text{ cms}^{-2}$

Let the length of each step is x . Time taken by the first boy to complete $p_1 = 1$ step up and $q_1 = 2$ steps down is

$$t = \frac{3x}{v_r} = \frac{3x}{50}$$

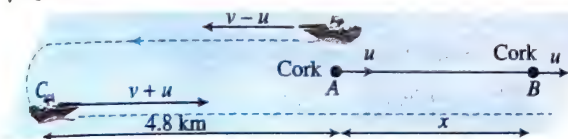
Average speed w.r.t. escalator, $v_{bl/e} = \frac{-x \times 50}{3x} = \frac{-50}{3} \text{ cms}^{-1}$

Velocity of boy 1 w.r.t. ground $= v_e - \frac{50}{3}$

$$\text{So finally } \left(v_e - \frac{50}{3}\right)250 = \left(v_e + \frac{50}{3}\right)50$$

$$\Rightarrow v_e = 25 \text{ cms}^{-1}$$

$$g. \quad v = 8 \text{ ms}^{-1}, u = 3 \text{ ms}^{-1}$$



$$t_{AB} = t_{AC} + t_{CB}$$

$$\frac{x}{u} = \frac{4.8}{v-u} + \frac{4.8+x}{v+u}$$

$$\frac{x}{3} = \frac{4.8}{8-3} + \frac{4.8+x}{8+3} \Rightarrow x = 28.8/5 \text{ km}$$

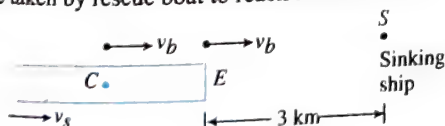
$$\text{Time taken: } \frac{x}{u} = \frac{28.8 \times 1000}{5 \times 3} = 1920 \text{ s} = 32 \text{ min}$$

$$9. \quad t_A = \frac{s}{nu+u} + \frac{s}{nu-u} = \frac{2snu}{(n^2-1)u^2} = \frac{2ns}{(n^2-1)u}$$

$$t_B = \frac{2s}{\sqrt{(nu)^2 - u^2}} = \frac{2s}{u\sqrt{n^2-1}}$$

$$\frac{t_A}{t_B} = \frac{n}{\sqrt{n^2-1}}$$

10. Time taken by rescue boat to reach from C to E:



$$t_1 = \frac{75}{v_b - v_s} = \frac{75}{20-10} = 7.5 \text{ s}$$

Time taken by rescue boat to go from E to S:

$$t_2 = \frac{300}{v_b} = \frac{3000}{20} = 150 \text{ s}$$

Time spent $t_0 = 1 \text{ min} = 60 \text{ s}$

Distance travelled by ship in time,

$$t_2 + t_0 = v_s(150 + 60) = 10 \times 210 = 2100 \text{ m}$$

Time taken by rescue boat to reach from S to C,

$$t_3 = \frac{(3000 - 2100) + 75}{v_b + v_s} = \frac{975}{30} = 32.5 \text{ s}$$

$$\text{Total time: } T = t_1 + t_2 + t_0 + t_3$$

$$= 7.5 + 150 + 60 + 32.5 = 250 \text{ s}$$

$$11. \quad 2 = (v - v_w)t$$

$$8 = (v + v_w)t$$



$$\Rightarrow 10 = 2vt \Rightarrow t = \frac{10}{2v} = \frac{10}{2 \times 20} = \frac{1}{4} \text{ h}$$

$$\text{and } v_w = 12 \text{ km h}^{-1}$$

Now when they again meet at C: [first cyclist takes rest at B for time t_0]

$$\frac{10}{v - v_w} + \frac{4}{v + v_w} = \frac{10}{v + v_w} + t_0 + \frac{6}{v - v_w}$$

Solve to get $t_0 = 18.75 \text{ min}$

Exercise 4.5

1. (a) We know that the slope of the position-time graph is equal to velocity. So

- as the slope is zero, it is zero velocity.
- as the slope is positive and constant, it is constant positive velocity.
- as the slope is infinite, it is infinite velocity.
- as the slope is negative and constant, it is constant negative velocity.
- as the slope is positive and increasing, it is positive increasing velocity.
- as the slope is positive and decreasing, it is positive decreasing velocity.
- as the slope is positive and increasing, it is positive increasing velocity.
- as the slope is positive and decreasing, it is positive decreasing velocity.

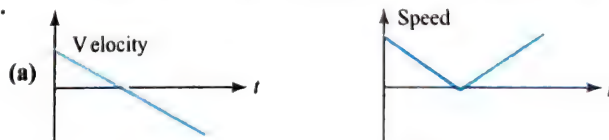
(b) True, as $dv/dt = \text{acceleration}$

(c) Change in velocity

(d) No, because it will indicate infinite acceleration, which is not possible

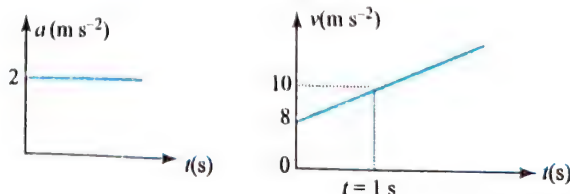
(e) Zero, because acceleration will be zero in uniform motion.

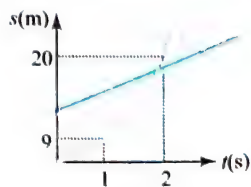
2.



3. (a) $a = 2 \text{ ms}^{-2}$, $u = 8 \text{ ms}^{-1}$

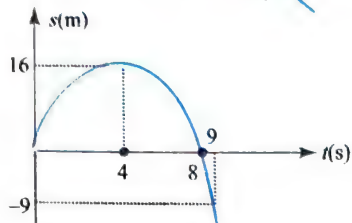
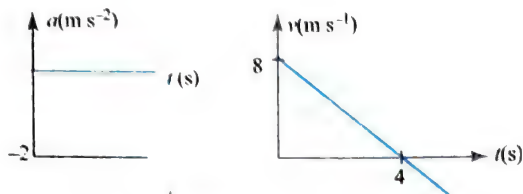
$$v = u + at = 8 + 2t, S = ut + \frac{1}{2}at^2 = 8t + t^2$$





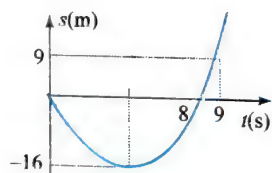
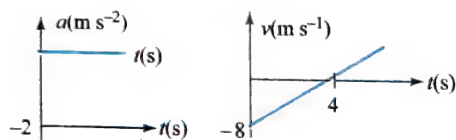
(b) $a = -2 \text{ ms}^{-2}$, $u = 8 \text{ ms}^{-1}$

$$v = u + at = 8 - 2t, S = ut + \frac{1}{2}at^2 = 8t - t^2$$



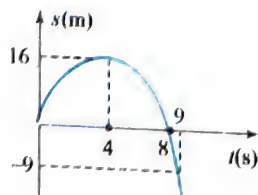
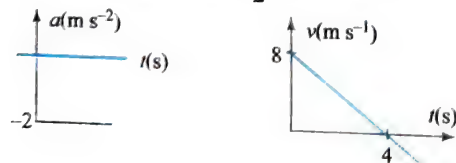
(c) $a = 2 \text{ ms}^{-2}$, $u = -8 \text{ ms}^{-1}$

$$v = u + at = -8 + 2t, S = ut + \frac{1}{2}at^2 = -8t + t^2$$



(d) $a = -2 \text{ ms}^{-2}$, $u = -8 \text{ ms}^{-1}$

$$v = u + at = -8 - 2t, S = ut + \frac{1}{2}at^2 = -8t - t^2$$



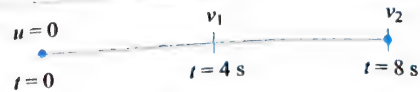
4. Let us first find the change in velocity. Change in velocity is the area under the acceleration-time graph. For first 20 s,

$$\text{Area} = \Delta v = \frac{1}{2} \times 10 \times 20 + 10 \times 20 = 300 \text{ ms}^{-1}$$

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t} = \frac{300}{20} = 15 \text{ ms}^{-2}$$

5. From 0 to 4 s, at $t = 0$, $a = 5 \text{ ms}^{-2}$

$$v_1 = u + at = 0 + 5 \times 4 = 20 \text{ ms}^{-1}$$

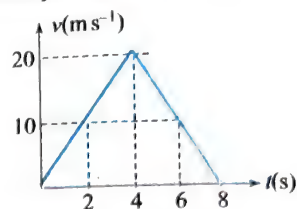


From 4 to 8 s, $a = -5 \text{ ms}^{-2}$

$$v_2 = v_1 + at$$

$$\Rightarrow v_2 = 20 - 5 \times 4 = 0 \text{ ms}^{-1}$$

Maximum velocity is 20 ms^{-1} at $t = 4 \text{ s}$.



Displacement from 2 to 6 s is equal to area from 2 to 6 s, i.e.,
 $10 \times 4 + (1/2) \times 4 \times 10 = 60 \text{ m}$

Since the motion takes place along the same direction only,
 distance = displacement = 60 m

6. (a) The slope of position time graph gives the velocity

$$\text{Slope of line A} = \text{velocity of car A} = \frac{14 - 6}{1.6 - 0} = \frac{8}{1.6} = 5 \text{ km h}^{-1}$$

$$\text{Slope of line B} = \text{velocity of car B} = \frac{14 - 6}{1.4 - 0} = 10 \text{ km h}^{-1}$$

$$\text{Slope of line C} = \text{velocity of car C} = \frac{14 - 2}{1 - 0} = 12 \text{ km h}^{-1}$$

Here car C has highest speed and car A has lowest speed.

- (b) As clear from graph, the position of all three cars are not same at any time. Hence, the three cars never at the same point on the road.

- (c) From graph, it is clear car B is approximately 6 km from the origin.

- (d) From graph, car A travels between the time it passed cars B and C = $1.2 - 0.6 = 0.6 \text{ h}$

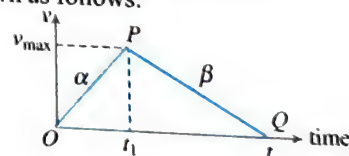
- (e) Relative velocity of car C with respect to car A,

$$\vec{v}_{C,A} = \vec{v}_C - \vec{v}_A = 12 - 5 = 7 \text{ km h}^{-1}$$

- (f) Relative velocity of car B with respect to car C

$$\vec{v}_{B,C} = \vec{v}_B - \vec{v}_C = 10 - 12 = -2 \text{ km h}^{-1}$$

7. Let $v_{\max}$ is the maximum velocity attained and t_1 is the time at which maximum velocity will occur, the velocity vs time graph can be drawn as follows:



The slope of line OP,

$$\alpha = \frac{v_{\max}}{t_1}$$

$$v_{\max} = \alpha t_1$$

...(i)

The slope of line PQ,

$$\beta = \frac{v_{\max}}{t - t_1}$$

$$v_{\max} = \beta(t - t_1)$$

...(ii)

From Eqs. (i) and (ii), we get

$$\alpha t_1 = \beta(t - t_1)$$

$$\text{which gives } t_1 = \frac{\beta t}{\alpha + \beta}$$

...(iii)

Substituting value of t_1 in Eq. (i), we get

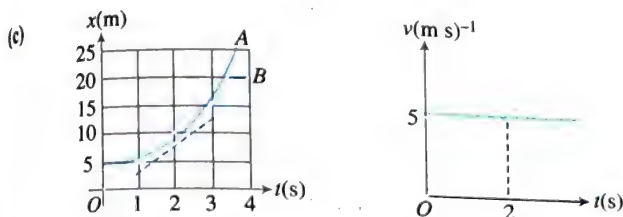
$$(a) v_{\max} = \frac{\alpha\beta t}{\alpha + \beta}$$

$$(b) \text{ Total displacement, } \bar{s} = \text{Area of } \bar{v}-t \text{ graph}$$

$$= \frac{1}{2} \times v_{\max} \times t$$

$$= \frac{1}{2} \times \frac{\alpha\beta t}{\alpha + \beta} \times t = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$

8. (a) The two cars have the same position at times when their $x-t$ graphs cross. The figure shows this occurs at $t = 1$ s and $t = 3$ s.
- (b) A and B have the same velocity when slope of the tangent on parabola is equal to the slope of the straight line. From position-time graph, it is clear it occurs at $t = 2$ s.
- Car B travels with constant velocity.
- $$v_B = \frac{20 - 0}{4 - 0} = 5 \text{ m s}^{-1}$$



- (d) Car A passes car B when x_A moves above x_B in the $x-t$ graph. This happens at $t = 3$ s.
- (e) Car B passes car A when x_B moves above x_A in the $x-t$ graph. This happens at $t = 1$ s.

9. For $t = 0$ to 5 s, v_x is positive and the ball moves in the $+x$ direction. For $t = 5$ s to 20 s, v_x is negative ball moves in the $-x$ direction. The acceleration a_x is the slope of the v_x versus t graph.

- (a) For $t = 0$ to 5 s, Displacement = Change in position = Area under v_x-t graph

$$x_5 - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t = \left(\frac{0 + 30}{2} \right) (5) = 75 \text{ m}$$

The ball travels a distance of 75 m.

For $t = 5$ s to 20 s, displacement = change in position

$$x_{20} - x_5 = \left(\frac{-20 + 0}{2} \right) (15) = -150 \text{ m}$$

The total displacement is $x_{20} - x_0 = 55 + (-150 \text{ m}) = -75 \text{ m}$. The ball ends up 75 m in the negative x direction from where it started

- (b) The total distance traveled is $75 \text{ m} + 150 \text{ m} = 225 \text{ m}$.
- (c) Instantaneous acceleration is the slope of velocity-time graph.

For

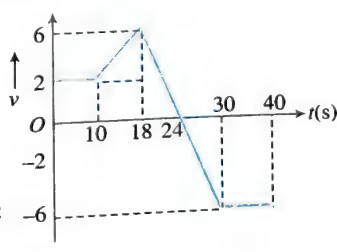
$t = 0$ to 5 s,

$$a_x = \frac{30 - 0}{5} = 6 \text{ m s}^{-2}$$

For

$t = 5$ s to 20 s,

$$a_x = \frac{0 - (-20)}{15} = \frac{4}{3} \text{ m s}^{-2}$$



The graph of a_x versus t is given in figure

- (d) The ball is in contact with the floor for a small but non-zero period of time and the direction of the velocity does not change instantaneously. So, no, the actual graph of $v_x(t)$ is not really vertical at 5 s.

$$10. (a) a = v \frac{dv}{ds} = \frac{v_i}{2} \frac{dv}{ds} = \frac{20}{2} \left[\frac{-20}{50} \right] = -4 \text{ m s}^{-2}$$

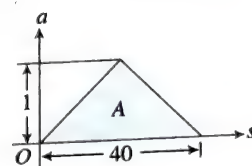
- (b) From graph, total distance covered is 50 m.

$$11. v = \sqrt{v_0^2 + 2(A)}$$

where, $A = \frac{1}{2} \times 40 \times 1 = 20$

and $v_0 = 0$

This gives $v = \sqrt{40} \text{ m s}^{-1}$



$$12. a = -\frac{s}{20} + 20$$

$$\int_0^v v dv = \int_0^{250} \left(-\frac{s}{20} + 20 \right) ds \Rightarrow \frac{v^2}{2} = \left[-\frac{s^2}{40} + 20s \right]_0^{250}$$

$$\frac{v^2}{2} = -\frac{(250)^2}{40} + 20 \times 250$$

$$\Rightarrow v^2 = 6875 \Rightarrow v = 25\sqrt{11} \text{ m s}^{-1}$$

Exercises

Single Correct Answer Type

$$1. (3) S_1 = \frac{1}{2} at^2 = \frac{1}{2} (at)t = \frac{60t}{2} = 30t$$

$$S_2 = 60 \times 8t = 480t, S_3 = S_1 = 30t$$

$$u = 0 \quad \begin{array}{c} t \\ \xrightarrow{S_1} \quad \xrightarrow{8t} \quad \xrightarrow{S_2} \quad \xrightarrow{S_3} \quad \xrightarrow{-a} \end{array} \quad v = 0$$

$$v_{av} = \frac{S_1 + S_2 + S_3}{t + 8t + t} = 54 \text{ km h}^{-1}$$

$$2. (1) 30 = u + a \times 2, 60 = u + a \times 4$$

Solve to get $u = 0$.

$$3. (4) 2ax = (50)^2 - (10)^2 \text{ and } 2(-a)(-x) = v^2 - (50)^2$$

This gives $v^2 - (50)^2 = (50)^2 - (10)^2$ i.e. $v = 70 \text{ m s}^{-1}$

$$4. (1) v^2 = 108 - 9x^2 \quad \dots(i)$$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{d(\sqrt{108 - 9x^2})}{dx} \cdot \frac{dx}{dt}$$

$$a = \frac{1(-18x)}{2\sqrt{108 - 9x^2}} \cdot \sqrt{108 - 9x^2} = -9x \text{ m s}^{-2}$$

Alternative: Differentiating w.r.t. x , we get

$$2v \frac{dv}{dx} = -18x$$

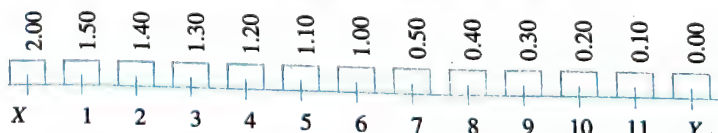
$$\frac{v dv}{dx} = -9x \Rightarrow a = -9x \quad \left(\because v \frac{dv}{dx} = a \right)$$

$$5. (3) \text{ We have } h = \frac{1}{2} g T^2$$

$$\text{In } T/3 \text{ second, distance fallen} = \frac{1}{2} g \left(\frac{T}{3} \right)^2 = \frac{h}{9}$$

$$\text{So position of the ball from the ground is } h - \frac{h}{9} = \frac{8h}{9} \text{ m}$$

6. (2) Because one taxi leaves every 10 min, at any instant there will be 11 taxis on the way towards each station, one will be arriving and another leaving the other station. Figure shows the location of taxis going from X to Y at the instant 2.00 PM. The taxi which leaves the station X at 0.00 PM has just arrived at the station Y. Consider the taxi leaving the station Y at 2.00 PM.



It will meet all the 11 taxis marked 1 to 11 as well as 12 other

taxies which would leave the station X from 2.00 PM to 3.50 PM. When it arrives at the station X at 4.00 PM, there will be one more taxi leaving that station. However, it will not be counted among the taxis crossed by taxi under consideration. That is, it will cross 23 taxis leaving the station X from 0.10 PM to 3.50 PM.

7. (4) $v^2 - u^2 = 2as$, $v = 0$
 $s \propto u^2$

When the initial velocity is made n times, the distance over which it can be stopped becomes n^2 times.

8. (4) Relative velocity of policeman w.r.t. the thief is $10 - 9 = 1 \text{ ms}^{-1}$. Since the relative separation between them is 100 m, the time taken will be = relative separation/relative velocity = $100/1 = 100 \text{ s}$.

9. (4) Distance covered by the object in first 2 s

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

Similarly, distance covered by the object in next 2 s will also be 20 m. hence the required height = $H - 20 - 20 = H - 40 \text{ m}$

10. (3) $h = \frac{1}{2}gt^2$ and $h - 20 = \frac{1}{2}g(t-1)^2$

Solving them, we get $t = 2.5 \text{ s}$ and $h = 31.25 \text{ m}$.

11. (1) Relative velocity of overtaking = $40 - 30 = 10 \text{ ms}^{-1}$. Total relative distance covered with this relative velocity during overtaking = $100 + 200 = 300 \text{ m}$

So time taken = $300/10 = 30 \text{ s}$

12. (1) Time of ascent = $1 \text{ s} \Rightarrow \frac{u}{g} = 1 \Rightarrow u = 10 \text{ ms}^{-1}$

$$\text{Maximum height attained} = \frac{u^2}{2g} = \frac{10^2}{2 \times 10} = 5 \text{ m}$$

13. (3) Maximum height attained $\propto u^2$

14. (2) Given $7x = \frac{g}{2}(2n-1)$ and $x = \frac{1}{2}g(1)^2$
 Solving these two equations, we get $n = 4 \text{ s}$.

15. (1) $t = \alpha x^2 + \beta x$

$$\text{Differentiating: } 1 = 2\alpha \frac{dx}{dt} \cdot x + \beta \frac{dx}{dt}$$

$$v = \frac{dx}{dt} = \frac{1}{\beta + 2\alpha x}; \frac{dv}{dt} = \frac{-2\alpha v}{(\beta + 2\alpha x)^2} = -2\alpha v^3$$

16. (1) $t = \sqrt{x} + 3$, Differentiating with respect to t , we get

$$1 = \frac{1}{2\sqrt{x}} \frac{dx}{dt} + 0 \text{ or } \frac{dx}{dt} = 2\sqrt{x}$$

When velocity is zero, $2\sqrt{x} = 0$ or $x = 0$.

17. (2) The required ratio is $1 : 3 : 5 : \dots$ so on

18. (1) Since the last five steps covering 5 m land the drunkard fell into the pit, the displacement prior to this is $(11 - 5) \text{ m} = 6 \text{ m}$.

Time taken for first eight steps (displacement in first eight steps = $5 - 3 = 2 \text{ m}$) = 8 s . Then time taken to cover first 6 m of

$$\text{journey} = \frac{6}{2} \times 8 = 24 \text{ s}$$

Time taken to cover last 5 m = 5 s

Total time = $24 + 5 = 29 \text{ s}$

19. (3) Here $h = \frac{1}{2} \times 10 \times (5)^2 = 125 \text{ m}$

$$\text{In } 3 \text{ s it falls through: } h_1 = \frac{1}{2} \times 10 \times (3)^2 = 45 \text{ m}$$

Rest 80 m is covered in 4 s. Hence, total time taken is $3 \text{ s} + 4 \text{ s} = 7 \text{ s}$

20. (2) $200 = u \times 2 - (1/2)at(2)^2$

$$\text{or } u - a = 100$$

...(i)

$$200 + 220 = u(2+4) - (1/2)(2+4)^2a$$

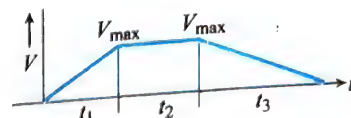
$$\text{or } u - 3a = 70$$

Solving Eqs (i) and (ii), we get $a = 15 \text{ cms}^{-2}$ and $u = 115 \text{ cms}^{-1}$.

Further, $v = u - at = 115 - 15 \times 7 = 10 \text{ cms}^{-1}$.

21. (3) Graphically, the area of $v-t$ curve represents displacement:

$$S = \frac{1}{2}v_{\max}t_1 \text{ or } t_1 = \frac{2S}{v_{\max}}$$



$$2S = v_{\max}t_2 \text{ or } t_2 = \frac{2S}{v_{\max}}$$

$$5S = \frac{1}{2}v_{\max}t_3 \text{ or } t_3 = \frac{10S}{v_{\max}}$$

$$v_{\text{av}} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{S + 2S + 5S}{\frac{2S}{v_{\max}} + \frac{2S}{v_{\max}} + \frac{10S}{v_{\max}}}$$

$$\frac{v_{\text{av}}}{v_{\max}} = \frac{8S}{14S} = \frac{4}{7}$$

Alternative:

$$\frac{v_{\text{av}}}{v_{\max}} = \frac{\text{Total displacement}}{2 \left(\frac{\text{Total displacement during acceleration and retardation}}{2} \right) + \left(\frac{\text{Displacement during uniform velocity}}{v_{\max}} \right)}$$

$$\frac{v_{\text{av}}}{v_{\max}} = \frac{8S}{2(S + 5S) + 2S} = \frac{8}{14} = \frac{4}{7}$$

22. (2) When a body slides on an inclined plane, component of weight along the plane produces an acceleration.

$$a = \frac{mg \sin \theta}{m} = g \sin \theta = \text{Constant}$$

If s is the length of the inclined plane, then

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}g \sin \theta \times t^2$$

$$\frac{s'}{s} = \frac{t'^2}{t^2} \text{ or } \frac{s}{s'} = \frac{t^2}{t'^2}$$

$$\text{Given } t = 4 \text{ s and } s' = \frac{s}{4}$$

$$t' = t \sqrt{\frac{s'}{s}} = 4 \sqrt{\frac{s}{4s}} = \frac{4}{2} = 2 \text{ s}$$

23. (1) Bomb B_1 will have less velocity upwards on dropping, so it will reach ground first.

24. (3) Time taken to cover first n meter is given by

$$n = \frac{1}{2}gt_n^2 \text{ or } t_n = \sqrt{\frac{2n}{g}}$$

Time taken to cover first $(n+1)$ m is given by

$$t_{n+1} = \sqrt{\frac{2(n+1)}{g}}$$

So time taken to cover $(n+1)$ th m is given by

$$t_{n+1} - t_n = \sqrt{\frac{2(n+1)}{g}} - \sqrt{\frac{2n}{g}} = \sqrt{\frac{2}{g}} [\sqrt{n+1} - \sqrt{n}]$$

This gives the required ratio as

$$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), \dots, \text{etc., (starting from } n = 0)$$

25. (1) Time of fall = $\sqrt{\frac{2h}{g}}$

$$\text{Time taken by the sound to come out} = \frac{h}{c}$$

$$\text{Total time} = \sqrt{\frac{2h}{g}} + \frac{h}{c} = h \left[\sqrt{\frac{2}{gh}} + \frac{1}{c} \right]$$

$$\text{3. (2)} \quad t_1 = \frac{s}{v_1}, t_2 = \frac{s}{v_2}, \dots, t_n = \frac{s}{v_n}$$

Average speed = (Total distance)/(Total time)

$$\Rightarrow \bar{v} = \frac{ns}{t_1 + t_2 + \dots + t_n}$$

$$= \frac{ns}{\frac{s}{v_1} + \frac{s}{v_2} + \dots + \frac{s}{v_n}} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n}}$$

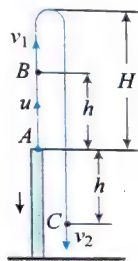
Taking reciprocal, we get $\frac{1}{\bar{v}} = \frac{1}{n} \left(\frac{1}{v_1} + \frac{1}{v_2} + \dots + \frac{1}{v_n} \right)$

3. (3) $H = \frac{u^2}{2g}$; given $v_2 = 2v_1$... (i)

A to B: $v_1^2 = u^2 - 2gh$... (ii)

A to C: $v_2^2 = u^2 - 2g(-h)$... (iii)

Solving (i), (ii) and (iii), we get the value of u^2 as $10gh/3$ and then we get the value of H by using $H = \frac{u^2}{2g}$.



2. (2) Time taken by the same ball to return to the hands of the juggler

is $\frac{2u}{g} = \frac{2 \times 20}{10} = 4$ s. So he is throwing the balls after 1 s each.

Let at some instant he throws ball number 4. Before 1 s of throwing it, he throws ball 3. So the height of ball 3 is

$$h_3 = 20 \times 1 - \frac{1}{2} 10(1)^2 = 15 \text{ m}$$

Before 2 s, he throws ball 2. So the height of ball 2 is

$$h_2 = 20 \times 2 - \frac{1}{2} 10(2)^2 = 20 \text{ m}$$

Before 3 s, he throws ball 1. So the height of ball 1 is

$$h_1 = 20 \times 3 - \frac{1}{2} 10(3)^2 = 15 \text{ m}$$

2. (4) $a = g \sin \alpha = g \sin (90^\circ - \theta) = g \cos \theta$
 $l = 2R \cos \theta$

Now using $s = ut + \frac{1}{2} at^2$, we get

$$l = 0t + \frac{1}{2} g \cos \theta t^2$$

$$\Rightarrow 2R \cos \theta = \frac{1}{2} g \cos \theta t^2 \Rightarrow t = 2 \sqrt{\frac{R}{g}}$$

This is independent of θ .

3. (3) Suppose the body be projected vertically upwards from A with a speed u_0 .

Using equation $s = ut + \left(\frac{1}{2}\right) at^2$, we get

For first case: $-h = u_0 t_1 - \left(\frac{1}{2}\right) g t_1^2$... (i)

For second case: $-h = -u_0 t_2 - \left(\frac{1}{2}\right) g t_2^2$... (ii)

(i) - (ii) $\Rightarrow 0 = u_0(t_2 + t_1) + \left(\frac{1}{2}\right) g(t_2^2 - t_1^2)$
 $\Rightarrow u_0 = \left(\frac{1}{2}\right) g(t_1 - t_2)$... (iii)

Putting the value of u_0 in (ii), we get

$$-h = -\left(\frac{1}{2}\right) g(t_1 - t_2)t_2 - \left(\frac{1}{2}\right) g t_2^2$$

$$\Rightarrow h = \frac{1}{2} g t_1 t_2$$
 ... (iv)

For third case: $u = 0, t = ?$

$$-h = 0 \times t - \left(\frac{1}{2}\right) g t^2 \quad \text{or} \quad h = \left(\frac{1}{2}\right) g t^2$$
 ... (v)

Combining Eq. (iv) and Eq. (v), we get

$$\frac{1}{2} g t^2 = \frac{1}{2} g t_1 t_2 \quad \text{or} \quad t = \sqrt{t_1 t_2}$$

31. (4) Here $\frac{dv}{dt} = -kv^3$ or $\frac{dv}{v^3} = -k dt$ or $\int_{v_0}^v \frac{dv}{v^3} = \int_0^t -k dt$

or $\left[-\frac{1}{2v^2} \right]_{v_0}^v = -kt$ or $-\frac{1}{2v^2} + \frac{1}{2v_0^2} = -kt$

or $v^2 = \frac{v_0^2}{1 + 2v_0^2 kt}$ or $v = \frac{v_0}{\sqrt{1 + 2v_0^2 kt}}$

32. (2) Given $v = 3x^2 - 2x$; differentiating v , we get

$$\frac{dv}{dt} = (6x - 2) \frac{dx}{dt} = (6x - 2)v$$

$$\Rightarrow a = (6x - 2)(3x^2 - 2x)$$

Now put $x = 2$ m

$$\Rightarrow a = (6 \times 2 - 2)(3(2)^2 - 2 \times 2) = 80 \text{ ms}^{-2}$$

33. (1) Suppose h be the height of each storey. Then

$$25h = 0 + \frac{1}{2} \times 10 \times t^2 = \frac{1}{2} \times 10 \times 5^2 \quad \text{or} \quad h = 5 \text{ m}$$

In first second, let the stone passes through n storey. So

$$n \times 5 = \frac{1}{2} \times 10 \times (1)^2 \quad \text{or} \quad n = 1$$

34. (2) Suppose v be the velocity attained by the body after time t_1 . Then $v = u - gt_1$... (i)

Let the body reach the same point at time t_2 . Now velocity will be downwards with same magnitude v . Then

$$-v = u - gt_2$$
 ... (ii)

(i) - (ii) $\Rightarrow 2v = g(t_2 - t_1)$

or $t_2 - t_1 = \frac{2v}{g} = \frac{2}{g}(u - gt_1) = 2 \left(\frac{u}{g} - t_1 \right)$

35. (1) The velocity v acquired by the parachutist after 10 s:

$$v = u + gt = 0 + 10 \times 10 = 100 \text{ ms}^{-1}$$

Then, $s_1 = ut + \frac{1}{2} g t^2 = 0 + \frac{1}{2} \times 10 \times 10^2 = 500 \text{ m}$

The distance travelled by the parachutist under retardation is

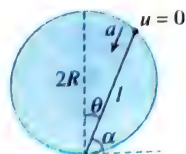
$$s_2 = 2495 - 500 = 1995 \text{ m}$$

Let v_g be the velocity on reaching the ground. Then $v_g^2 - v^2 = 2as_2$

or $v_g^2 - (100)^2 = 2 \times (-2.5) \times 1995$ or $v_g = 5 \text{ ms}^{-1}$

36. (3) If police is able to catch the dacoit after time t , then

$$vt = x + \frac{1}{2} \alpha t^2$$



This gives $\frac{\alpha}{2}t^2 - vt + x = 0$

$$\text{or } t = \frac{v \pm \sqrt{v^2 - 2\alpha x}}{\alpha}$$

For t to be real, $v^2 \geq 2\alpha x$

37. (4) Here relative velocity of the train w.r.t. other train is $V - v$.

Hence, $0 - (V - v)^2 = 2\alpha x$

$$\text{or } a = -\frac{(V - v)^2}{2x} \quad \text{Minimum retardation} = \frac{(V - v)^2}{2x}$$

38. (2) Distance covered = $S = v_{av} \times \text{time}$

For first second: $S_1 = 5 \times 1 = 5 \text{ m}$

For second second: $S_2 = 10 \times 1 = 10 \text{ m}$

For third second: $S_3 = 15 \times 1 = 15 \text{ m}$

Total distance travelled

$$S = S_1 + S_2 + S_3 = 5 + 10 + 15 = 30 \text{ m}$$

39. (2) Given $v_{av} = \frac{v+u}{2} = 0.34$ and $v - u = 0.18$

Solving these two equations, we get

$$u = 0.25 \text{ ms}^{-1}, v = 0.43 \text{ ms}^{-1}. \text{ Given } s = 3.06 \text{ m}$$

Now use $v^2 - u^2 = 2as$ to find $a = 0.02 \text{ ms}^{-2}$.

40. (4) By the time fifth water drop starts falling, the first water drop reaches the ground.

$$u = 0, h = \frac{1}{2}gt^2 \Rightarrow 5 = \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 1 \text{ s}$$

Hence, the interval of falling of each water drop is $\frac{1 \text{ s}}{4} = 0.25 \text{ s}$

When the fifth drop starts its journey towards ground, the third drop travels in air for $0.25 + 0.25 = 0.5$

Therefore, height (distance) covered by third drop in air is

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.5)^2 = 5 \times 0.25 = 1.25 \text{ m}$$

The third water drop will be at a height of

$$5 - 1.25 = 3.75 \text{ m}$$

41. (2) $S_1 + S_2 + S_3 + S_4 = 16 \text{ m}$,

$$S_1 : S_2 : S_3 : S_4 = 1 : 3 : 5 : 7$$

Solve to get $S_1 = 1 \text{ m}$,

$$S_2 = 3 \text{ m}, S_3 = 5 \text{ m}, S_4 = 7 \text{ m}$$

42. (3) $x^2 = 1 + t^2$ or $x = (1 + t^2)^{1/2}$

$$\frac{dx}{dt} = \frac{1}{2}(1 + t^2)^{-1/2} 2t = t(1 + t^2)^{-1/2}$$

$$a = \frac{d^2x}{dt^2} = (1 + t^2)^{-1/2} + t \left(-\frac{1}{2} \right) (1 + t^2)^{-3/2} 2t$$

$$= \frac{1}{x} - \frac{t^2}{x^3}$$

43. (2) Suppose u be the initial velocity.

Velocity after time t_1 : $v_{11} = u + at_1$

Velocity after time $t_1 + t_2$: $v_{22} = u + a(t_1 + t_2)$

Velocity after time $t_1 + t_2 + t_3$:

$$v_{33} = u + a(t_1 + t_2 + t_3)$$

$$\text{Now } v_1 = \frac{u + v_{11}}{2} = \frac{u + u + at_1}{2} = u + \frac{1}{2}at_1$$

$$v_2 = \frac{v_{11} + v_{22}}{2} = u + at_1 + \frac{1}{2}at_2$$

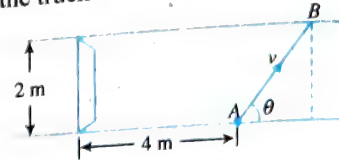
$$v_3 = \frac{v_{22} + v_{33}}{2} = u + at_1 + at_2 + \frac{1}{2}at_3$$

$$\text{So } v_1 - v_2 = -\frac{1}{2}a(t_1 + t_2)$$

$$v_2 - v_3 = -\frac{1}{2}a(t_2 + t_3)$$

$$(v_1 - v_2) : (v_2 - v_3) = (t_1 + t_2) : (t_2 + t_3)$$

44. (3) Let the man start crossing the road at an angle θ with the roadside. For safe crossing, the condition is that the man must cross the road by the time the truck describes the distance $(4 + 2 \cot \theta)$



$$\text{So, } \frac{4 + 2 \cot \theta}{8} = \frac{2 / \sin \theta}{v} \quad \text{or } v = \frac{8}{2 \sin \theta + \cos \theta}$$

For minimum v , $\frac{dv}{d\theta} = 0$

$$\text{or } \frac{-8(2 \cos \theta - \sin \theta)}{(2 \sin \theta + \cos \theta)^2} = 0 \quad \text{or } 2 \cos \theta - \sin \theta = 0$$

$$\text{or } \tan \theta = 2, \text{ so } \sin \theta = \frac{2}{\sqrt{5}}, \cos \theta = \frac{1}{\sqrt{5}}$$

$$v_{\min} = \frac{8}{2 \left(\frac{2}{\sqrt{5}} \right) + \frac{1}{\sqrt{5}}} = \frac{8}{\sqrt{5}} = 3.57 \text{ ms}^{-1}$$

45. (3) Displacement = Area under graph

$$= 2 \times 2 + \frac{1}{2}(2 + 6) \times 1$$

$$+ \frac{1}{2} \times 1 \times 6 - \frac{1}{2} \times 1 \times 6 - 1 \times 6 + 2 \times 4 = 10 \text{ m}$$

46. (1) Area from 0 to 10 s = $\frac{1}{2}[10 + 4]5 = 35 \text{ m}$

$$\text{Area from 10 to 12 s} = \frac{1}{2} \times 2 \times (-2.5) = -2.5 \text{ m}$$

$$\text{Distance travelled} = 35 + 2.5 = 37.5 \text{ m}$$

47. (3) Maximum height will be attained at 110 s. Because after 110 s, velocity becomes negative and rocket will start coming down.

Area from 0 to 110 s is

$$\frac{1}{2} \times 110 \times 1000 = 55,000 \text{ m} = 55 \text{ km}$$

48. (4) Displacement in 12 s = Area under $v-t$ graph

$$= \frac{1}{2} \times (12 + 5)4 = 34 \text{ m}$$

$$V_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{34}{12} = \frac{17}{6} \text{ ms}^{-1}$$

Hence, (1) is incorrect; (2) is incorrect because during first 3 s, velocity increases from 0 to 4 ms^{-1} option; (3) is incorrect, because in part AB velocity is constant.

49. (3) Acceleration between 8 and 10 s (or at $t = 9 \text{ s}$):

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5 - 15}{10 - 8} = -5 \text{ ms}^{-2}$$

50. (1) Maximum acceleration will be from 30 to 40 s, because slope in this interval is maximum.

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{60 - 20}{40 - 30} = 4 \text{ ms}^{-2}$$

51. (4) From 0 to t_1 , velocity is positive and constant as indicated by positive and constant slope.

From t_1 to t_3 , slope is zero, hence velocity is zero.
 From t_3 to t_4 , velocity is negative and constant as indicated by negative and constant slope.
 Option (4) satisfies all these observations.

62. (1) At $t = 0$, velocity is positive and maximum. As the particle goes up, velocity decreases and becomes zero at the highest point. When the particle starts coming down, velocity increases in the negative direction.

63. (1) At time t , let the displacement of first stone be $S_1 = \frac{1}{2}gt^2$ and that of the second stone

$$S_2 = ut - \frac{1}{2}gt^2$$

Distance between two stones at time t :

$$S = S_1 + S_2 = u \Rightarrow S = ut$$

So the graph should be a straight line passing through origin as shown in option (1).

64. (1) Let the particle be thrown up with initial velocity u .

$$\text{Displacement (s) at any time } t \text{ is } S = ut - \frac{1}{2}gt^2.$$

The graph should be parabolic downwards as shown in option (b).

65. (2) In (I), slope is negative and its magnitude is decreasing with time. It means slope is increasing numerically. So velocity is increasing towards right, and so acceleration is positive.

In (IV), slope is positive and its magnitude is increasing with time. So velocity is increasing towards right, and so acceleration is positive.

66. (1) We know that for a body thrown up, its displacement is given as

$$S = ut - \frac{1}{2}gt^2. \text{ So the } s-t \text{ graph is parabolic downwards.}$$

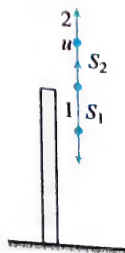
Also the ball collides inelastically, so it will rebound to less height every time as shown in the graph. t_1, t_2, t_3 are the instants when the ball collides with ground. Here slope of the $s-t$ graph is suddenly changing from negative to positive. It means velocity before collision is negative (downwards) and after collision is positive (upwards).

67. (1) From 0 to t_1 , acceleration is increasing linearly with time; hence, $v-t$ graph should be parabolic upwards.

From t_1 to t_2 , acceleration is decreasing linearly with time; hence, the $v-t$ graph should be parabolic downwards.

68. (1) At $t = 0$, slope of the $x-t$ graph is zero; hence, velocity is zero at $t = 0$. As time increases, slope increases in negative direction; hence, velocity increases in negative direction. At point (1), slope changes suddenly from negative to positive value; hence, velocity changes suddenly from negative to positive and then velocity starts decreasing and becomes zero at (2). Option (1) represents all these clearly.

69. (4) Before the second ball is dropped, the first ball would have travelled some distance say $S_0 = \frac{1}{2}gt_0^2$. After dropping the second ball, the relative acceleration of both balls becomes zero. So distances between them increase linearly. After some time, the first ball will collide with the ground and the distance between them will start decreasing and the magnitude of relative velocity will be increasing for this time. Option (4) represents all these clearly.

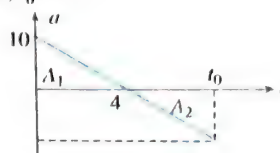


$$60. (2) a = -\frac{4}{2}t + 4 \Rightarrow a = -2t + 4$$

$$v = \int a dt + C = \int (-2t + 4) dt + C = -t^2 + 4t + C$$

Hence, graph will be parabolic.

61. (3) Particle will acquire the initial velocity when areas A_1 and A_2 are equal. For this, $t_0 = 8$ s.



62. (1) For 0 to 5 s, acceleration is positive, for 5 to 15 s acceleration is negative, for 15 to 20 s acceleration is positive.

63. (1) At $t = 0$, $v = 0$, $x = 4/3$ m

$$a = \frac{4 \times 3}{3} - 4 = 0$$

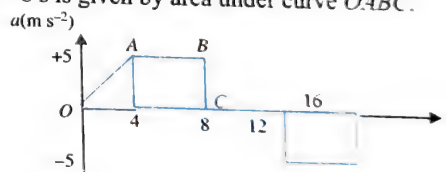
As both the particle velocity and acceleration are zero at $t = 0$, it will always remain at rest and hence distance travelled at any time interval would be zero.

$$64. (2) x^3 = t^3 + 1 \Rightarrow 3x^2 \frac{dx}{dt} = 3t^2 \Rightarrow \frac{dx}{dt} = \frac{t^2}{x^2} \quad \dots(i)$$

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{2tx^2 - 2x \frac{dx}{dt} t^2}{x^4} \\ &= \frac{1}{x^4} \left[2tx^2 - 2xt^2 \frac{t^2}{x^2} \right] \\ &= \frac{2tx^2}{x^4} \left[1 - \frac{t^3}{x^3} \right] = \frac{2tx^2}{x^4} \left[1 - \left\{ \frac{x^3 - 1}{x^3} \right\} \right] = \frac{2t}{x^5} \end{aligned}$$

65. (2) Maximum speed is at $t = 8$ s

v at $t = 8$ s is given by area under curve $OACB$.



Maximum speed

$$= \frac{1}{2}(AB + OC) \times BC = \frac{1}{2} \times (12) \times 5 = 30 \text{ ms}^{-1}$$

$$66. (1) a = v \frac{dv}{ds} = 4 \times (-\tan 60^\circ) = -4\sqrt{3} \text{ ms}^{-2}$$

67. (1) From $a = v dv/ds$, we can find the sign of acceleration at various points. v is positive for all three points 1, 2 and 3. dv/ds is positive for point 1, zero for point 2, negative for point 3. So, only for point 1, velocity and acceleration have same sign, so the object is speeding up at point 1 only.

68. (2) From the graph, velocity-displacement equation can be written as $v = v_0 + \alpha x$ $\dots(ii)$

Here v_0 and α are positive constants.

Differentiating Eq. (i) with respect to x , we get

$$\frac{dv}{dx} = \alpha = \text{constant}$$

Acceleration of the particle can be written as

$$a = v \frac{dv}{dx} = (v_0 + \alpha x)\alpha$$

The $a-x$ equation is a linear equation. Thus acceleration increases linearly with x .

69. (1) From graph $v = s$

$$\Rightarrow \frac{dv}{dt} = \frac{ds}{dt} \Rightarrow a = v$$

Acceleration versus velocity graph will be a straight line passing through origin with slope 1 s^{-1} .

70. (3) Acceleration-velocity equation from the graph is

$a = kv$, where k is the slope of given line.

This can also be written as

$$v \frac{dv}{ds} = kv \Rightarrow \frac{dv}{ds} = k$$

i.e. slope of velocity-displacement graph is same as slope of acceleration-velocity graph which is constant.

71. (2) $v = t^2 - t \Rightarrow a = \frac{dv}{dt} = 2t - 1$

For retardation, $av < 0$

$$\Rightarrow (t^2 - t)(2t - 1) < 0$$

$$\Rightarrow t(t - 1)(2t - 1) < 0$$

This is possible for $\frac{1}{2} < t < 1$

72. (3) $\vec{r} = x\hat{i} + y\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

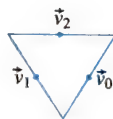
If $\vec{r}$ and $\vec{v}$ are in opposite directions, then definitely the particle will be moving towards origin O , for this $\vec{r} \cdot \vec{v} < 0$

$$\Rightarrow xv_x + yv_y < 0$$

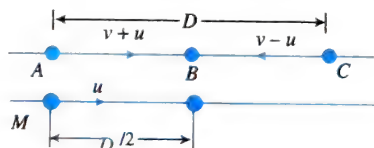
73. (2) $\vec{v}_2 = \vec{v}_1 - \vec{v}_0$, $\vec{v}_2 + \vec{v}_0 = \vec{v}_1$

From the law of triangle, we can draw the velocity vector diagram (figure)

From triangle property, sum of two sides $\geq$ third side. So $v_1 \leq v_0 + v_2$

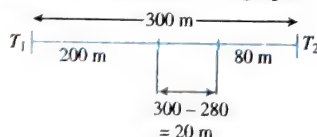


74. (4) Time taken for swimmer for $(AC + CB) =$ Time taken to float for AB



$$\Rightarrow \frac{D}{v+u} + \frac{D/2}{v-u} = \frac{D/2}{u} \Rightarrow \frac{v}{u} = 3$$

75. (4) Initial distance between trains is 300 m. Displacements of trains can be calculated by area under $V-t$ graph.



$$\text{Displacement of train 1} = \frac{1}{2} \times 10 \times 40 = 200 \text{ m}$$

$$\text{Displacement of train 2} = \frac{1}{2} \times 8 \times (-20) = -80 \text{ m}$$

which means it moves towards left.

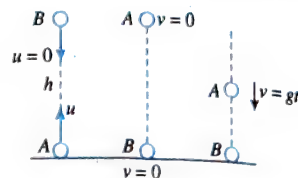
Hence, distance between the two is $300 - 200 - 80 = 20 \text{ m}$.

76. (2) $a = v \frac{dv}{dx} \Rightarrow \int_u^v v dv = \int_0^{1.4} a dx$

$$\Rightarrow \frac{v^2 - u^2}{2} = \text{Area of } a-x \text{ graph}$$

$$\Rightarrow v^2 - (0.8)^2 = 2(0.4) \Rightarrow v = 1.2 \text{ ms}^{-1}$$

77. (3) Initial relative velocity of A w.r.t. $B = u - 0 = u$. It will remain constant till B reaches ground. B stops on colliding with ground, and A comes down with increasing velocity.



Multiple Correct Answers Type

1. (2), (3), (4)

A body having a constant speed can have a varying velocity due to change in the direction of velocity. Thus a body having constant speed can have an acceleration.

If velocity and acceleration are in the same direction, then distance is equal to displacement, because then there is no change in direction of motion. The body will continuously travel in one direction only.

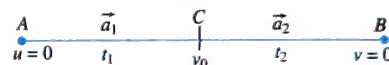
2. (2), (4)

Freely falling block will reach the ground first, because it has to travel less distance and with greater acceleration in comparison to the other block.

Both the blocks will reach the ground with the same speed, because the potential energies of both decrease by the same amount, which gets converted into KE.

3. (1), (3)

$$a_1 = 2 \text{ ms}^{-2}, a_2 = -4 \text{ ms}^{-2}$$



$$v_0 = a_1 t_1 = 2t_1, v_0 = a_2 t_2 = 4t_2$$

$$t_1 + t_2 = 6 \Rightarrow \frac{v_0}{2} + \frac{v_0}{4} = 6 \Rightarrow v_0 = 8 \text{ ms}^{-1}$$

Total distance travelled

$$S = AC + CB$$

$$= \frac{1}{2} v_0 t_1 + \frac{1}{2} v_0 t_2 = \frac{1}{2} \times 8 \times 6 = 24 \text{ m}$$

4. (1), (2), (3)

Let they meet at height h after time t .

$$h = 100t - \frac{1}{2}gt^2 \rightarrow \text{for first arrow}$$

$$= 100(t - 5) - \frac{1}{2}g(t - 5)^2 \rightarrow \text{for second arrow}$$

$$\Rightarrow t = -12.5 \text{ s (after solving). So (1) is correct.}$$

$$\text{Time of flight of first arrow: } T = \frac{2u}{g} = \frac{2 \times 100}{10} = 20 \text{ s}$$

Second arrow will reach after 5 s of reaching first. So (2) is correct.

$$v_1 = 100 - 10 \times 20 = -100 \text{ ms}^{-1}$$

$$v_2 = 100 - 10 \times 15 = -50 \text{ ms}^{-1}$$

$$\text{Ratio: } \frac{v_1}{v_2} = 2 : 1. \text{ So (3) is correct.}$$

Maximum height attained

$$H = \frac{u^2}{2g} = \frac{(100)^2}{2 \times 10} = 500 \text{ m. Hence, (4) is incorrect.}$$

5. (1), (3)

$$t = \sqrt{\frac{2h}{g}}, v = \sqrt{2gh}$$

6. (1), (3)

Ball A will return to the top of tower after

$$T = \frac{2u}{g} = \frac{2 \times 10}{10} = 2 \text{ s}$$

with speed of 10 m s^{-1} downward.

And this time, B is also projected downwards with 10 m s^{-1} . So both reach ground simultaneously. Also they will hit the ground with the same speed.

7. (1), (4)

$$\text{From } s = \frac{1}{2}at^2, u = 0$$

$s \propto t^2$, since the particle starts from rest and acceleration is constant, so there is no change in the direction of velocity and the particle moves in a straight line always.

8. (1), (2), (4)

$a = dv/dt$, if velocity changes, definitely there will be acceleration. If speed changes, then velocity also changes, so definitely there will be acceleration.

Acceleration may be due to change in the direction of velocity only and not magnitude.

If body has acceleration, its speed may change if acceleration is due to change in the magnitude of velocity.

9. (1), (4)

The body will speed up if the angle between velocity and acceleration is acute.

10. (3), (4)

Since average acceleration = Change in velocity/time, so average acceleration is in the direction of change in velocity. Also if the initial velocity is zero, then the final velocity and change in velocity will be in the same direction.

11. (1), (2), (3)

For vertically projected body, if it returns to the starting point, displacement and average velocity become zero. As acceleration is constant, average speed during upward or downward motion is $(u+0)/2 = u/2$. The same will be the average speed for the whole motion.

12. (3), (4)

Average velocity,

$$\begin{aligned} \vec{v} &= \frac{\int_0^5 v dt}{\int_0^5 dt} = \frac{\int_0^5 (4t - t^2) dt}{\int_0^5 dt} \\ &= \frac{\left[2t^2 - \frac{t^3}{3} \right]_0^5}{5} = \frac{50 - \frac{125}{3}}{5} = \frac{25}{3 \times 5} = \frac{5}{3} \end{aligned}$$

For average speed, let us put $v = 0$, which gives $t = 0$ and $t = 4 \text{ s}$

$$\begin{aligned} \therefore \text{Average speed} &= \frac{\left| \int_0^4 v dt \right| + \left| \int_4^5 v dt \right|}{\int_0^5 dt} = \frac{\left| \int_0^4 (4t - t^2) dt \right| + \left| \int_4^5 v dt \right|}{5} \\ &= \frac{\left[2t^2 - \frac{t^3}{3} \right]_0^4 + \left[2t^2 - \frac{t^3}{3} \right]_4^5}{5} \\ &= \frac{\left[2t^2 - \frac{t^3}{3} \right]_0^4 + \left[2t^2 - \frac{t^3}{3} \right]_4^5}{5} = \frac{13}{5} \text{ m s}^{-1} \end{aligned}$$

$$\text{For acceleration: } a = \frac{dv}{dt} = \frac{d}{dt}(4t - t^2) = 4 - 2t$$

$$\text{At } t = 0, a = 4 \text{ m s}^{-2}$$

Therefore, options (3) and (4) are correct, and options (1) and (2) are wrong.

13. (1), (4)

Since the graph is a straight line, its slope is constant. It means acceleration of the particle is constant.

Velocity of the particle changes from positive to negative at $t = 10 \text{ s}$, so particle changes direction at this time.

The particle has zero displacement up to 20 s , but not for the entire motion.

The average speed in the interval of 0 to 10 s is the same as the average speed in the interval of 10 s to 20 s because distance covered in both time intervals is same.

14. (1), (2), (3), (4)

Particle changes direction of motion at $t = T$. Acceleration remains constant, because the velocity-time graph is a straight line. Displacement is zero, because net area is zero. Initial and final speeds are equal.

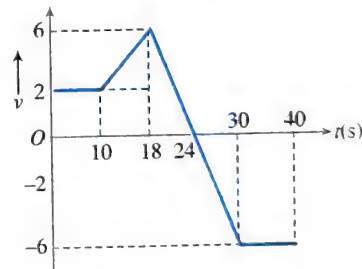
15. (1), (2), (3)

Initially at origin, slope is not zero, so the particle has some initial velocity but with time we see that slope is decreasing and finally the slope becomes zero, so the particle stops finally.

As the magnitude of velocity is decreasing, velocity and acceleration will be in opposite directions.

16. (1), (3), (4)

Maximum value of position coordinate = Initial coordinate + Area under the graph up to $t = 24 \text{ s}$ (As up to $t = 24 \text{ s}$, the displacement of the particle will be positive (figure).



Maximum value of position coordinate

$$\begin{aligned} &= -16 + \left[(2 \times 10) + \left(\frac{2+6}{2} \right) \times (18-10) + \frac{1 \times 6}{2} \times (24-18) \right] \\ &= -16 + [20 + 32 + 18] = 54 \text{ m} \end{aligned}$$

At time $t = 18 \text{ s}$

$$\begin{aligned} \text{Position} &= -16 + \text{Area of graph up to } t = 18 \text{ s} \\ &= -16 + [20 + 32 + 18] = 54 \text{ m} \end{aligned}$$

At time $t = 30 \text{ s}$

$$\begin{aligned} \text{Position} &= -16 + \text{Area of graph up to } t = 30 \text{ s} \\ &= -16 + \left[70 - \frac{1}{2} \times 6 \times 6 \right] = 36 \text{ m} \end{aligned}$$

17. (1), (3), (4)

Self-explanatory.

18. (1), (3), (4)

Velocity is given by the slope of the $x-t$ graph. Here the slope is zero at A and at peak of region CD and bottom most point of EF . For AB and EF , acceleration is positive. For these regions, graph is concave up.

For BC and DE , acceleration is zero as here velocity is constant. For CD , acceleration is negative as graph is concave down for this region.

19. (2), (3), (4)

Point of steepest slope corresponds to the maximum speed. Particle will speed up when the directions of acceleration and velocity are same. For region AB , both the acceleration and velocity are positive while for CD both are negative, so particle is speeding up in these regions.

20. (2), (3), (4)

Based on thought.

21. (1), (2), (3)

This is the example of non-uniform acceleration.

$$a = \frac{dv}{dt} = -0.5t$$

$$\int_{16}^v dv = -\int_0^t 0.5t dt \Rightarrow v = 16 - \frac{0.5t^2}{2}$$

Direction of velocity changes at the moment when it becomes zero momentarily.

$$0 = 16 - \frac{0.5t^2}{2} \Rightarrow t = 8 \text{ s}$$

$$\frac{dx}{dt} = v = 16 - \frac{0.5t}{2}$$

Let us consider that at $t = 0$, particle is at $x = 0$

$$\int_0^x dx = \int_0^t \left(16 - \frac{0.5t}{2} \right) dt; \quad x = 16t - \frac{0.5t^2}{6};$$

Distance travelled = |displacement| for $t \leq 8 \text{ s}$. So, distance travelled in 4 s

$$x = 16 \times 4 - \frac{0.5 \times 4^2}{6} = 59 \text{ m}$$

Distance travelled in 10 s

$$\begin{aligned} &= |\text{Displacement in 8 s}| \times 2 - \text{Displacement in 10 s} \\ &= 85.33 \times 2 - 76.55 = 94 \text{ m} \\ v(t = 10 \text{ s}) &= -9 \text{ ms}^{-1} \end{aligned}$$

22. (2), (3), (4)

Initially, the FBD of ball would be as shown in figure.

Here, directions of velocity and acceleration are opposite, so particle is slowing down and moving in upward direction. At one instant its velocity becomes zero, which would be the highest point of trajectory. Then it starts falling down, for its descent FBD would be as shown in figure.



During its descent, the ball will acquire the terminal speed (assuming ball is not striking ground before acquiring the terminal speed).

As work has been done against the resistive force, so speed of throwing is greater than speed with which the particle lands.

Hence, force of air friction is greatest (when speed is greatest) just after it is thrown.



23. (2), (3)

$$v = \frac{dx}{dt} = -9 + t^2 = 0 \Rightarrow t = 3 \text{ s}$$

The particle's velocity is getting zero at $t = 3 \text{ s}$, where it changes its direction of motion.

For $0 < t < 3 \text{ s}$, v is negative, a is positive, so particle is slowing down.

For $t > 3$, both v and a are positive, so the particle is speeding up.

24. (1), (2), (4)

As the particle is going up, it is slowing down, i.e., speed is decreasing and hence we can say that time taken by the particle to cover equal distances is increasing as the particle is going up. Hence, $t_1 < t_2 < t_3$.

As $v_{av} = \frac{\text{Distance}}{\text{Time}}$, we have $v_{av} \propto \frac{1}{\text{Time}}$

$$\text{So, } v_{av_1} > v_{av_2} > v_{av_3}$$

Acceleration throughout the motion remains same from equation,

$$\vec{v} = \vec{u} + \vec{at}, \Delta v \propto t$$

$$\text{So } \Delta v_1 < \Delta v_2 < \Delta v_3$$

25. (2)

$$\text{Let } \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}$$

As $\vec{a}$ is constant, so its magnitude as well as direction is not changing, but v_x , v_y and v_z all can be varying (any combination of these if a_x , a_y , or $a_z = 0$, then corresponding velocity component may be constant).

$$\begin{aligned} |\vec{v}| &= v = \sqrt{v_x^2 + v_y^2 + v_z^2} \\ \frac{d|\vec{v}|}{dt} &= \frac{1}{2\sqrt{v_x^2 + v_y^2 + v_z^2}} \\ &\times \left[2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt} + 2v_z \frac{dv_z}{dt} \right] = \frac{\vec{v} \cdot \vec{a}}{v} \end{aligned}$$

which is a variable quantity.

$$\left| \frac{d\vec{v}}{dt} \right| = |\vec{a}|, \text{ which would be constant.}$$

$$\frac{dv^2}{dt} = \frac{d(v_x^2 + v_y^2 + v_z^2)}{dt} = 2\vec{v} \cdot \vec{a} \quad (\text{a variable quantity})$$

$$\frac{d(\vec{v}/v)}{dt} = \frac{d}{dt} \left[\frac{v_x \hat{i} + v_y \hat{j} + v_z \hat{k}}{\sqrt{v_x^2 + v_y^2 + v_z^2}} \right] \quad (\text{a variable quantity})$$

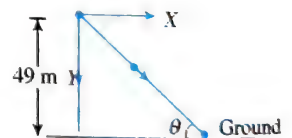
26. (1), (3), (4)

$$u_x = u_y = 0, a_x = 4.9 \text{ ms}^{-2}, a_y = 9.8 \text{ ms}^{-2}$$

$$\text{So } x = \frac{1}{2} a_x t^2$$

$$\text{and } y = \frac{1}{2} a_y t^2$$

$$\frac{x}{y} = \frac{a_x}{a_y} = \frac{1}{2}$$



So path of the ball is a straight line.

Let t be the time taken by the ball to reach ground, then

$$49 = \frac{1}{2} a_y t^2 \Rightarrow t^2 = 10 \Rightarrow t = 3.16 \text{ s}$$

$$\tan \theta = \frac{a_y}{a_x} = 2 \Rightarrow \theta = \tan^{-1}(2)$$

27. (2), (4)

A body moving on a circular path with uniform speed must have varying velocity as well as acceleration.

28. (1), (3), (4)

Total displacement ball A in 2 s,

$$S = ut - \frac{1}{2} g t^2 = 10 \times 2 - \frac{1}{2} \times 10 \times (2)^2 = 0$$

Hence, both A and B will reach the ground simultaneously and strike with the same velocity

29. (2), (4)
Slope of A is greater than slope of B.

$$\therefore a_A > a_B$$

$$S = \frac{1}{2}at^2 \quad (\because u = 0)$$

$$\therefore S_A > S_B$$

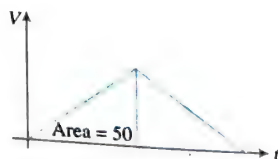
30. (2), (3), (4)

$$u = 10 \text{ m s}^{-1}, t = 5 \text{ s}, S = 50 \text{ m}$$

$$V_{\text{av}} = \frac{50}{5} = 10 \text{ m s}^{-1}$$

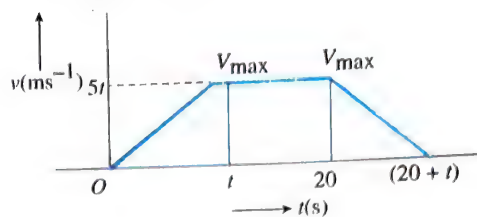
The motion may be with constant velocity.

The body may be first accelerated and then retarded.



$$\frac{\text{Total distance}}{\text{Total time}} = 20$$

$$\Rightarrow \frac{\text{Area of graph}}{20 + t} = 20$$



$$\Rightarrow \frac{1}{2}(20 + t + 20 - t) \times 5t = 20(20 + t) \Rightarrow t = 5 \text{ s}$$

$$10. (2) \text{ Maximum velocity} = 5t = 5 \times 5 = 25 \text{ m s}^{-1}$$

$$11. (1) 25(20 - t) = 25 \times 15 = 375 \text{ m}$$

For Problems 12 and 13

12. (2) In graphs (i) and (iii), magnitude of slope is greater at t_1 than that at t_2 .

13. (3) In graph (iii), magnitude of slope of graph is decreasing. Hence, magnitude of velocity is decreasing but in negative direction, hence acceleration is positive.

For Problems 14 and 15

14. (2) For the graphs (i) and (iv), slope is constant, hence the velocity is constant.

15. (3) For the graph (iii), the particle's velocity first decreases and then increases in negative direction. It means negative acceleration is involved in this motion.

For Problems 16–20

$$16. (1) h = \frac{1}{2}gt^2$$

$$180 = \frac{1}{2}g \times (6)^2$$

$$\Rightarrow g = 10 \text{ m s}^{-2}$$

17. (3) Let time taken by rocketeer in catching the boy at the top of 100th

floor be t , then time for which boy is in free fall motion is $(t + 5)$ s. So

$$t + 5 = 6 \Rightarrow t = 1 \text{ s}$$

$$\text{Now } 180 = v_0 t + \frac{gt^2}{2},$$

where v_0 is the initial downward speed of racketeer, which gives $v_0 = 175 \text{ m s}^{-1}$

18. (2) Initially, the rocketeer is under free fall and acceleration is along vertical downward, direction, i.e., along positive Y-axis, so $y-t$ graph is concave up but when he uses his jet pack acceleration starts acting in upward direction and hence $y-t$ graph would be concave down.

19. (1) From solution 2, velocity of rocketeer at the time of catching the boy is

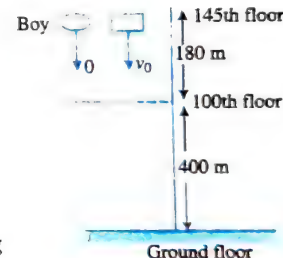
$$v = v_0 + gt = 185 \text{ m s}^{-1}$$

For safe landing, final velocity = 0

$$\Rightarrow 0 = v^2 - 2 \times a \times s \quad [s = 400 \text{ m}] \Rightarrow a \gg 4g$$

So, the acceleration produced by jet pack along would be $(4g + g) \text{ m s}^{-2}$ in upward direction, as due to gravity, acceleration g is acting acting in downward direction.

20. (2) Initially, the rocketeer's velocity increases with time linearly having acceleration g and then decrease due to use of jet pack



$$1. (2) s = \frac{1}{2}gt_1^2 \quad \text{or} \quad t_1^2 = \frac{50 \times 2}{g} = \frac{100}{g} \quad \text{or} \quad t_1 = \frac{10}{\sqrt{g}}$$

$$\text{and } 100 = \frac{1}{2}gt^2 \quad \text{or} \quad t = \frac{10\sqrt{2}}{\sqrt{g}}$$

$$t_2 = t - t_1 = \frac{10}{\sqrt{g}}(\sqrt{2} - 1) = 0.4t_1 \quad t_1 > t_2$$

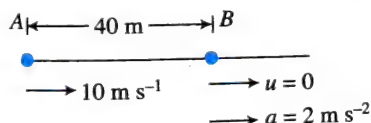
$$2. (1) t_1 : t_2 = \frac{10}{\sqrt{g}} \times \frac{\sqrt{g}}{10(\sqrt{2} - 1)} = \frac{1}{0.4} = \frac{10}{4} = \frac{5}{2}$$

$$3. (2) t : t_1 = \frac{10\sqrt{2}}{\sqrt{g}} \times \frac{\sqrt{g}}{10} = \sqrt{2}$$

For Problems 4 and 5

4. (3) Distance between the particles will be minimum when velocity of B becomes equal to that of A, i.e., 10 m s^{-1} .

$$\text{Apply } v = u + at \Rightarrow 10 = 0 + 2t \Rightarrow t = 5 \text{ s}$$



5. (2) Distance travelled by A in time 5 s, $S_1 = 10 \times 5 = 50 \text{ m}$
Distance travelled by B in time 5 s

$$S_2 = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 5^2 = 25 \text{ m}$$

$$\text{Minimum distance} = 40 + S_2 - S_1 = 15 \text{ m}$$

For Problems 6–8

6. (1) Distance covered = area of speed–time graph
$$= \frac{1}{2} \times (4 + 2) \times 4 + \frac{1}{2} \times (4 + 2) \times 2 = 18 \text{ m}$$

7. (4) Distance from the origin will be equal to the final displacement of the particle which is equal to the area of the velocity–time graph, i.e.,

$$\frac{1}{2} \times (4 + 2) \times 4 - \frac{1}{2} \times (4 + 2) \times 2 = 6 \text{ m}$$

$$8. (2) a_{\text{av}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{-2 - 4}{6 - 2} = -\frac{3}{2} \text{ m s}^{-2}$$

For Problems 9–11

$$9. (1) \text{ Average speed} = 20 \text{ m s}^{-1}$$

with an acceleration (acting upwards) of $\leq 5g$.

So, the most suitable option is (2).

For Problems 21–25

21. (1) Initial velocity of ball w.r.t ground.

$$\vec{v}_{BG} = \vec{v}_{BE} + \vec{v}_{EG} = 15 + 10 = 25 \text{ ms}^{-1} \uparrow$$

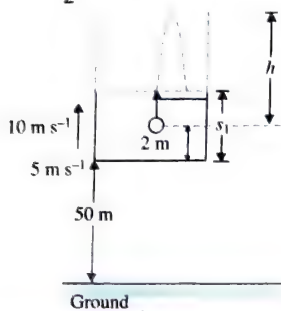
$$\vec{v}_{BE} = 15 \text{ ms}^{-1} \uparrow, \vec{a}_{BG} = 10 \text{ ms}^{-2} \downarrow$$

$$\vec{a}_{BE} = \vec{a}_{BG} - \vec{a}_{EG} = 10 - (-5) = 15 \text{ ms}^{-2} \downarrow$$

$$\vec{s}_{BE} = 2 \text{ m} \downarrow. \text{ Using } s = ut + \frac{1}{2}at^2$$

w.r.t. elevator frame,

$$-2 = 15t - \frac{1}{2} \times 15t^2 \Rightarrow t = 2.13 \text{ s}$$



22. (3) The height of the ball from ground at the time of projection is 52 m.

Maximum height from point of projection is

$$h = \frac{|\vec{v}_{BG}|^2}{2a_{BG}} = \frac{(25)^2}{2 \times 10} = 31.25 \text{ m}$$

$$\text{Required height} = 52 + 31.25 = 83.25 \text{ m}$$

23. (4) Displacement of floor of elevator in 2.13 s is

$$s_1 = 10t + \frac{1}{2} \times 5 \times t^2 = 32.64 \text{ m}$$

So displacement of ball w.r.t. ground,

$$\vec{s}_{BG} = \vec{s}_{BE} + \vec{s}_{EG} = -2 + 32.64 = 30.64 \text{ m}$$

24. (2) From the Figure, it is very clear that distance travelled by the ball is

$$H = h + [h - (s_1 - 2)] = 2h - s_1 + 2 = 31.86 \text{ m}$$

25. (4) Displacement of ball w.r.t. elevator is given by

$$s = 15t - \frac{1}{2} \times 15t^2$$

Here $s + 2$ will be separation between the floor of elevator and ball.

For s to be maximum, $\frac{ds}{dt} = 0$

which gives $t = 1 \text{ s}$

So maximum value of separation is

$$\left(15 \times 1 - \frac{1}{2} \times 15 \times (1)^2\right) + 2 = 9.5 \text{ m}$$

For Problems 26–28

26. (2) Since the x - t graph is parabolic, its slope (velocity) should change linearly. At $t = 0$, slope is positive, so velocity should be positive.

At $t = T$, slope is zero, so velocity should be zero. At $t = 0$, slope is negative, so velocity should be negative.

27. (4) Since velocity decreases linearly, acceleration should be negative and constant.

28. (3) Speed is always positive. So we can obtain speed–time graph from velocity–time graph by rotating the part of velocity–time graph below time axis by 180° .

Matrix Match Type

1. i $\rightarrow$ b; ii $\rightarrow$ a; iii $\rightarrow$ d; iv $\rightarrow$ c

$$v = \frac{dS}{dt} = \beta + 2\gamma t \quad a = \frac{dv}{dt} = 2\gamma \quad \text{So, i} \rightarrow b$$

$$(v) = \frac{\int_2^3 v dt}{\int_2^3 dt} = \frac{\beta(3-2) + \gamma(9-4)}{1} = \beta + 5\gamma \quad \text{So, ii} \rightarrow a.$$

Velocity at $t = 1 \text{ s}$: $v = \beta + 2\gamma \times 1 = \beta + 2\gamma$, So iii $\rightarrow$ d.

Initial displacement i.e., $t = 0$, $S = a$ So, iv $\rightarrow$ c

2. i $\rightarrow$ a; b; ii $\rightarrow$ c; iii $\rightarrow$ d; iv $\rightarrow$ d

For a round trip, displacement is zero; hence $\vec{v}_{av} = 0$. Also

$$\vec{v}_{av} = \frac{\vec{v}_1 + \vec{v}_2}{2}, \text{ when } \vec{v}_1 \text{ is initial, } \vec{v}_2 \text{ is final. Hence i.} \rightarrow a, b$$

$$\text{Average speed } (v_{av}) = \frac{\text{Total distance}}{\text{Time of flight}} = \frac{2(v_0^2 / 2g)}{2v_0 / g} = \frac{v_0}{2}$$

Hence ii $\rightarrow$ c.

$$T_{\text{ascent}} = T_{\text{descent}} = \frac{v_0}{g}$$

Hence, iii. $\rightarrow$ d., iv $\rightarrow$ d.

3. i $\rightarrow$ a; ii $\rightarrow$ c, d; iii $\rightarrow$ b; iv $\rightarrow$ b

When the ball is above the point of projection, its displacement is always positive, but its velocity may be positive (when moving up), zero (at top point), or negative (when coming down).

Acceleration is always directed downwards, so it is always negative.

4. i $\rightarrow$ a; ii $\rightarrow$ c; iii $\rightarrow$ b; iv $\rightarrow$ d

$$\text{In OA, } S \propto t^2, \quad v = \frac{dS}{dt} \propto 2t$$

$$\text{i.e., } v \propto t$$

i.e., velocity increases with time.

$$\text{In AB, } S \propto t, \quad v = \frac{dS}{dt} \propto 1$$

i.e., velocity is uniform, i.e., constant or independent of time. In

BC, body is retarded, i.e., velocity decreases with time. In CD,

$S \propto t^0$, i.e., $v = \text{zero}$ i.e., body is at rest.

5. i $\rightarrow$ b; d; ii $\rightarrow$ a, d; iii $\rightarrow$ c; iv $\rightarrow$ a

(a) Area of v - t graph lies below the time axis, so displacement is negative, but slope is positive, so acceleration is positive.

(b) Area of v - t graph lies above the time axis, so displacement is positive, and slope is positive, so acceleration is also positive.

(c) Displacement is zero, because half area is above time axis and half below. Slope is negative, so acceleration is negative.

(d) Area of v - t graph lies above the time axis, so displacement is positive, and slope is negative, so acceleration is also negative.

6. i $\rightarrow$ b; ii $\rightarrow$ c; iii $\rightarrow$ b, c; iv $\rightarrow$ b, c

If initial velocity and acceleration are in opposite directions, velocity reaches zero and then increases in the opposite direction. In (ii), initial velocity and acceleration are in the same direction, so velocity increases continuously and particle moves along the direction of acceleration.

In (iii), $x > 0$ but velocity can be either along positive or negative x -axis. Similarly in (iv).

7. i $\rightarrow$ a; ii $\rightarrow$ a; iii $\rightarrow$ b; iv $\rightarrow$ c

Slope of velocity–time graph gives acceleration. If the directions of acceleration and velocity are the same, then particle is speeding up, otherwise slowing down.

Particle moves in the direction of velocity.

8. i → b, d; ii → a, c; iii → a, c; iv → b, d

Particle is accelerating when both velocity and acceleration are having the same direction (sign), and decelerating when velocity and acceleration have opposite directions (sign).

9. i → b, d; ii → a, d; iii → c; iv → a

- (a) Area of $v-t$ graph lies below time axis, so displacement is negative, but slope is positive, so acceleration is positive.
 (b) Area of $v-t$ graph lies above time axis, so displacement is positive, and slope is positive, so acceleration is also positive.
 (c) Displacement is zero, because half area is above time axis and half below. Slope is negative, so acceleration is negative.
 (d) Area of $v-t$ graph lies above time axis, so displacement is positive, and slope is negative, so acceleration is also negative.

10. (1)

$$x = \frac{t^3}{3} - 2t^2 + 3 + 5$$

$$v = t^2 - 4t + 3 = t^2 - 3t - t + 3 = (t-1)(t-3)$$

$$a = 2t - 4 = 2(t-2)$$

$t=1$	$t=2$	$t=3$
$V > 0$	$V < 0$	$V < 0$
$a < 0$	$a < 0$	$a > 0$

11. (2)

In case A and B, acceleration is constant but speed first decreases and then increases.

In case C and D, the velocity does not change sign. Hence, direction of acceleration is constant. Speed and magnitude of acceleration decrease with time.

Numerical Value Type

1. (2)

Taking upward direction as positive, let us work in the frame of lift. Acceleration of ball relative to lift = $(g+a)$ downwards, so $a_{\text{rel}} = -(g+a)$, initial velocity: $u_{\text{rel}} = v$, final velocity: $v_{\text{rel}} = -v$ as the ball will reach the man with same speed w.r.t lift

$$\text{Apply } v_{\text{rel}} = u_{\text{rel}} + a_{\text{rel}}t \Rightarrow -v = v + (-g-a)t \Rightarrow t = 2 \text{ s}$$

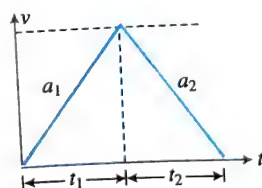
2. (2)

$$t_1 + t_2 = 4 \text{ min}, v = a_1 t_1 = a_2 t_2$$

$$S = \frac{1}{2} \times 4v \Rightarrow 4 = 2v \Rightarrow v = 2$$

$$t_1 + t_2 = v \left[\frac{1}{a_1} + \frac{1}{a_2} \right]$$

$$\Rightarrow 4 = 2 \left[\frac{1}{a_1} + \frac{1}{a_2} \right] \Rightarrow \frac{1}{a_1} + \frac{1}{a_2} = 2$$



3. (8)

$$t_1 = t_2 - t, v_1 = v_2 = v, S = \frac{1}{2} a_1 t_1^2, S = \frac{1}{2} a_2 t_2^2$$

$$v_1 = a_1 t_1, v_2 = a_2 t_2 \Rightarrow v_2 + v = a_1 t_1$$

$$\Rightarrow a_2 t_2 + v = a_1 t_1 = a_1 t_2 \Rightarrow t_2 = \frac{v + a_1 t_1}{a_1 - a_2}$$

$$\sqrt{\frac{a_2}{a_1}} = \frac{t_1}{t_2} = 1 - \frac{t}{t_2} \Rightarrow \sqrt{\frac{a_2}{a_1}} = 1 - \frac{t(a_1 - a_2)}{(v + a_1 t)}$$

$$\Rightarrow \frac{\sqrt{a_2}}{\sqrt{a_1}} = \frac{v + a_2 t}{v + a_1 t} \Rightarrow \sqrt{a_2} (v + a_1 t) = \sqrt{a_1} (v + a_2 t)$$

$$v = (\sqrt{a_1 a_2}) t = 8 \text{ ms}^{-1}$$

4. (5)

$$\text{For rat } S = \frac{1}{2} \beta t^2 \quad \dots(i)$$

$$\text{For cat } S = d = ut + \frac{1}{2} \alpha t^2 \quad \dots(ii)$$

Putting the value of S from Eq. (i) in Eq. (ii),

$$(a-b)t^2 + 2ut - 2d = 0$$

$$t = \frac{2u \pm \sqrt{4u^2 - 8d(\beta - \alpha)}}{2(\beta - \alpha)}$$

$$\text{For } t \text{ to be real, } \frac{u^2}{2d} \geq (\beta - \alpha) \therefore \beta = \alpha + \frac{u^2}{2d}$$

Substituting a, d and u , we get

$$\beta = 2.5 + \frac{5^2}{2 \times 5} = 2.5 + 2.5 = 5 \text{ ms}^{-2}$$

5. (5)

$$s = u + \frac{a}{2}(2n-1)$$

$$u = 100 \text{ ms}^{-1}, a = -10 \text{ ms}^{-2} \text{ and } s = 5 \text{ m}$$

$$5 = 100 - 5(2n-1) \text{ gives } n = 10 \text{ s}$$

Body when thrown up with velocity 200 ms^{-1} will take 20 s to reach the highest point. Distance travelled in 20th second is $200 - 5(20 - 1) = 5 \text{ m}$

In the last second of upward journey, the bodies will travel same distance.

6. (1)

$$v^2 = u^2 - 2gs$$

$$0 = u^2 - (2)(10) \text{ will give } u = 10 \text{ ms}^{-1}$$

$$\text{Further, } v = u - gt$$

$$0 = 10 - (10)t \text{ gives } t = 1 \text{ s}$$

7. (1)

$$V_p = 90 \text{ km h}^{-1} = 25 \text{ ms}^{-1}$$

$$V_c = 72 \text{ km h}^{-1} = 20 \text{ ms}^{-1}$$

In 10 s culprit reaches point B from A. Distance covered by culprit, $S = vt = 20 \times 10 = 200 \text{ m}$.

At time $t = 10 \text{ s}$, the police jeep is 200 m behind the culprit.

Relative velocity between jeep and culprit is $25 - 20 = 5 \text{ ms}^{-1}$.

$$\text{Time} = \frac{S}{V} = \frac{200}{5} = 40 \text{ s (Relative velocity is considered)}$$

In 40 s, the police jeep will move from A to a distance S . Where $S = vt = 25 \times 40 = 1000 \text{ m} = 1.0 \text{ km}$ away.

The jeep will catch up with the bike 1 km far from the turning.

8. (5)

As the velocity of the ball changes from $\vec{v}_1$ to $\vec{v}_2$, the change in velocity $\Delta \vec{v}$ is given by

$$|\Delta \vec{v}| = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos \theta}$$

where $v_1 = 30 \text{ ms}^{-1}$, $v_2 = 40 \text{ ms}^{-1}$ and $\theta = 90^\circ$.

$$\text{Then, } |\Delta \vec{v}| = 5 \text{ ms}^{-1}$$

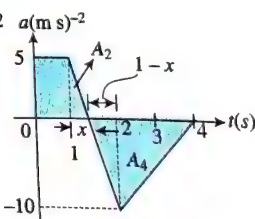
$$a_{\text{av}} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{5}{0.01} = 5 \times 10^2 \text{ ms}^{-2}$$

9. (3)

The velocity of the particle at $t = 4 \text{ s}$ can be given as

$$\vec{v}_4 = \vec{v}_0 + \Delta \vec{v}$$

... (i)



where $\Delta \bar{v} \equiv A$ (= area under $a-t$ graph during first 4 s)

Referring to $a-t$ graph (shown in figure), we have

$$A = A_1 + A_2 - A_3 - A_4 \quad \dots(ii)$$

where $A_1 = 5 \times 1 = 5$, $A_2 = \frac{1}{2} \times x \times 5$

$$A_3 = \frac{1}{2} \times (1-x) \times 10 \text{ and } A_4 = \frac{1}{2} \times 2 \times 10 = 10$$

We can find the value of x as follows:

Using properties of similar triangles, we have $\frac{x}{5} = \frac{1-x}{10}$.

This yields $x = \frac{1}{3}$.

Substituting $x = \frac{1}{3}$ in $A_2 = \frac{1}{2} \times x \times 5$ and $A_3 = \frac{1}{2} (1-x) \times 10$,

we have $A_2 = 5/6$ and $A_3 = 10/3$.

Then substituting A_1, A_2, A_3 and A_4 in Eq. (ii), we have

$$A = -7.5 \Rightarrow \Delta v = -7.5 \text{ and } \bar{v}_0 = 10.5 \text{ ms}^{-1}.$$

Hence, we have $v_4 = v_0 + \Delta v = 10.5 - 7.5 = 3 \text{ ms}^{-1}$.

10. (5)

Let the particle meet after a time t . First of all, we choose the point of collision above the top of the cliff.

For the first particle, $s = s_1$, $v_0 = v_1$, $a = -g$

$$\text{Then, } s_1 = v_1 t - \frac{1}{2} g t^2 \quad \dots(i)$$

For the second particle, $s = s_2$, $v_0 = v_2$, $a = -g$

$$\text{Then, } s_2 = v_2 t - \frac{1}{2} g t^2 \quad \dots(ii)$$

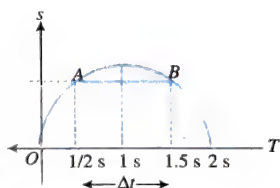
Referring to the figure, $s_2 - s_1 = h$... (iii)

Substituting s_1 from Eq. (i), s_2 from Eq. (ii) in Eq. (iii), we have

$$t = \frac{h}{v_2 - v_1} = \frac{10}{4 - 2} = 5 \text{ s}.$$

11. (0)

As $s-t$ graph is a parabola, it can be given as $s = k_1 t - k_2 t^2$, where k_1 and k_2 are constants. Since it is symmetrical $t = 1 \text{ s}$, we have equal displacements at $t = 1/2 \text{ s}$ and $t = 3/2 \text{ s}$ that tells us that line AB joining two coordinates is parallel to the t -axis.



Hence, slope of AB is zero. This implies that the average velocity during the time interval $\Delta t = 1 \text{ s}$ is zero.

12. (8)

$$V_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{(1 \times 5^2 + 1) - (1 \times 3^2 + 1)}{5 - 3} = \frac{16}{2} = 8 \text{ ms}^{-1}$$

13. (14)

$$s_1 = 5(-3) = 8 \text{ m}$$

$$s_2 = -7 - (-3) = -4 \text{ m}$$

$$s_3 = -3 - 7 = -10 \text{ m}$$

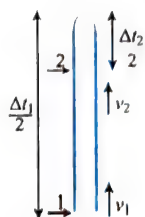
Required value $= (s_2 + s_3) = (-14) = 14$

14. (8)

$$1.5 = \frac{v_1}{a}$$

$$1 = \frac{v_2}{a}$$

$$5 = \frac{v_1^2 - v_2^2}{2a}, a = 8$$



15. (4)

$$\frac{v dv}{dx} = 4 - 2x$$

$$\int_0^v v dv = \int_0^x (4 - 2x) dx \Rightarrow \frac{v^2}{2} = 4x - x^2$$

when $v = 0$, $4x - x^2 = 0$

$$x = 0, 4$$

$\Rightarrow$ Thus, at $x = 4$, the particle will again come to rest.

16. (2)

$$0 = vt - \frac{1}{2} g t^2 \quad \dots(i)$$

$$160 = vt + \frac{1}{2} g t^2 \quad \dots(ii)$$

On solving, $v = 20 \text{ ms}^{-1}$

17. (21)

From figure, $h = u \times 1 - \frac{1}{2} g (1)^2$

$$h + 1 = u \times 1.1 - \frac{1}{2} g (1.1)^2$$

$$\Rightarrow u = 20.5 \text{ ms}^{-1}$$

$$H = \frac{u^2}{2g} = \frac{(20.5)^2}{2 \times 10} = 21 \text{ m}$$



18. (3.5)

$$(n-2)l = \frac{1}{2} a t^2$$

$$(n-1)l = \frac{1}{2} a (t+3)^2$$

$$nl = \frac{1}{2} a (t+5)^2$$

$$l = \frac{1}{2} a [(1+3)^2 - t^2] = \frac{1}{2} a [(t+5)^2 - (t+3)^2]$$

$$t^2 + 9 + 6t - t^2 = t^2 + 25 + 10t - t^2 - 9 - 6t$$

$$9 + 6t = 16 + 4t$$

$$2t = 7$$

$$t = 3.5 \text{ sec}$$

19. (60)

Time taken by the particle to reach the highest point

$$= (t_1 + t_2 + t_3) / 2 = 4 \text{ s}$$

At highest point, $0 = u - g \left(\frac{t_1 + t_2 + t_3}{2} \right)$

$$0 = u - 10 \times 4 \Rightarrow u = 40 \text{ m/s}$$

Time taken to reach upto midpoint of A and B ,

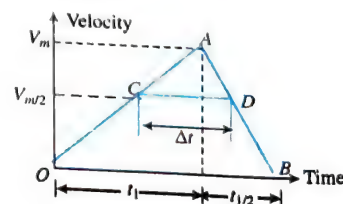
$$t_0 = t_1 + \frac{(t_2 - t_1)}{2} = 2 \text{ s}$$

Height of midpoint

$$h_{\text{middle}} = ut_0 - \frac{1}{2} g t_0^2 = 40 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 80 - 20 = 60 \text{ m}$$

20. (5)

Velocity time curve of the process is shown. Acceleration before application of brake,



$$a_1 = \frac{v_m}{t_1} \Rightarrow t_1 = \frac{v_m}{a_1}$$

$$\text{Displacement} = (1/2) v_m (t_1 + t_2/2)$$

But velocity ($v_m/2$) is observed at C and D.

$$CD = \Delta t = 1 \text{ s (given)}$$

C and D are mid-points of OA and OB.

From similar triangles, $OB = (t_1 + t_2/2) = 2CD = 2\Delta t$

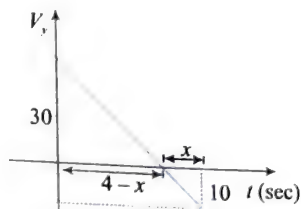
$$\text{Displacement} = \left(\frac{1}{2}\right) v_m (2\Delta t) = v_m \Delta t = 5 \times 1 = 5 \text{ m}$$

21. (8) Using similar triangle properties

$$\frac{30}{10} = \frac{4-x}{x}$$

$$x = 1$$

Area of curve given maximum height when V_y becomes zero.



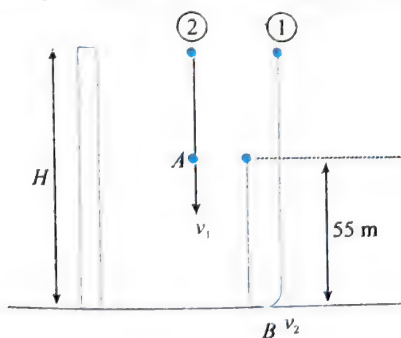
$$h = \frac{1}{2}(30)(3) - \frac{1}{2}(1)(10) = 45 - 5 = 40 \text{ m}$$

22. (180)

Since both the balls move identically, the first ball must move down from the place of collision to the ground and again return back there in time that equals to delay in release of the second ball.

$$v_1 = \sqrt{2g(H-55)}, v_2 = \sqrt{2gH}$$

Time taken by ball 2 from A to B should be $\Delta t/2$.



$$\text{So, } v_2 = v_1 + g\left(\frac{\Delta t}{2}\right)$$

$$\Rightarrow \sqrt{2gH} = \sqrt{2g(H-55)} + g\left(\frac{\Delta t}{2}\right)$$

$$\text{Solve to get } H = \frac{[8h + g(\Delta t)^2]^2}{32g(\Delta t)^2} = 180 \text{ m}$$

23. (0.64)

Let length of each coach is x . For third coach, $x = \frac{1}{2}a(5)^2$

Total number of coach = n , then $(n-2)x = \frac{1}{2}a(20)^2$.

From these equations: $n = 18$.

Let t is time taken to cross 15 coaches (from 3rd to 17th) then

$$15x = \frac{1}{2}at^2 \Rightarrow 15\left[\frac{1}{2}a(5)^2\right] = \frac{1}{2}at^2 \Rightarrow t = 5\sqrt{15}$$

So time taken to cross last coach = $20 - 5\sqrt{15} = 0.64 \text{ s}$

Archives

JEE Main

Single Correct Answer Type

1. (2) $v = u + at$

$$\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$$

$$\Rightarrow \vec{v} = (3+4)\hat{i} + (4+3)\hat{j}$$

$$\Rightarrow |\vec{v}| = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2} \text{ unit}$$

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

2. (1) Here, $\vec{v} = K(y\hat{i} + x\hat{j})$

$$\vec{v} = Ky\hat{i} + Kx\hat{j}$$

$$\text{Also, } \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

Equating equations (i) and (ii), we get

$$\frac{dx}{dt} = Ky; \frac{dy}{dt} = Kx$$

$$\text{Now, } \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \therefore \frac{dy}{dx} = \frac{Kx}{Ky} = \frac{x}{y}$$

Integrating both sides of the above equation, we get

$$\int y dy = \int x dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + \text{const.}$$

$$\text{or } y^2 = x^2 + \text{constant}$$

3. (2) We are given $\frac{dv}{dt} = -0.25\sqrt{v}$ or $\frac{1}{\sqrt{v}} dv = 2.5 dt$

On integrating, within limit (at $t = 0$, $v_1 = 6.25 \text{ m s}^{-1}$ and at time, $v_2 = 0$), we get

$$\int_{6.25 \text{ ms}^{-1}}^0 v^{-1/2} dv = -2.5 \int_0^t dt$$

$$2 \times [v^{1/2}]_{6.25}^0 = -(2.5)t \Rightarrow t = \frac{-2 \times (6.25)^{1/2}}{-2.5} = 2 \text{ s}$$

4. (4) Let u be the velocity of projection of the stone.

The maximum height a boy can throw a stone,

$$H_{\max} = \frac{u^2}{2g} = 10 \text{ m}$$

The maximum horizontal distance the boy can throw the stone is equal to maximum range

$$R_{\max} = \frac{u^2 \sin(2 \times 45^\circ)}{g} = \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g}$$

$$H_{\max} = \frac{u^2}{g} = 20 \text{ m} \quad (\text{Using (i)})$$

5. (3) For 1st stone: $y_1 = u_1 t - \frac{1}{2}gt^2$

For 2nd stone: $y_2 = u_2 t - \frac{1}{2}at^2$

Relative position of 2nd, w.r.t. 1st

$$\Delta y = y_2 - y_1 = (u_2 - u_1)t = (40 - 10)t = 30t$$

or $\Delta y \propto t$

It means up to the time both the stones are in the air (From $t = 0$ to $t = 8 \text{ sec}$) the graph should be a straight line.

When second stone hits the ground, first stone is still moving under gravitational acceleration, hence the graph should be parabolic. The speed of first stone will keep on increasing. Hence parabola should open downwards.

Hence, option (3) is correct.

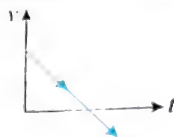
6. (1) Velocity at any time t is given by, $v = u + at$

If we take upward direction as positive and downward direction as negative, we can write

$$v = v_0 + (-g)t$$

$$v = v_0 - gt$$

⇒ Straight line with negative slope



7. (1) Graph in option (1) represents



- Negative acceleration
- Velocity is decreasing
- The situation is same as the object thrown upward and moving under gravity.

First moving up than moving down.

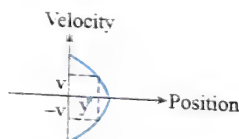
Mathematically the object is moving according to equation

$$v = u - at$$

For velocity position relation it should follow the equation

$$v = \sqrt{u^2 - 2ay} \quad \dots(i)$$

The graph in option (2)



The particle can have same magnitude of velocity at same position. Positive when moving away from initial position and negative when it is returning to initial position.

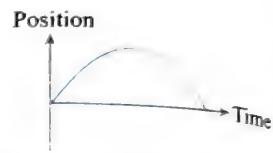
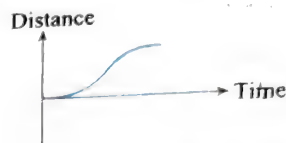
Hence this graph (2) represent same motion as (1)

Now come to analyse graph of option (3).

We can represent the motion of the particle given in option (1) and (2) with the equation

$$y = ut - \frac{1}{2}at^2$$

It means position time graph should be parabola open down. The option (4) satisfy this condition.



But option (3) does not represent same motion as at $t = 0$, its slope is zero and slope is increasing with time, which represents speed is increasing.

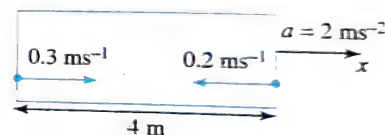
While in first part of motion, speed should decrease.

JEE Advanced

Numerical Value Type

1. (2)

Consider motion of two balls with respect to rocket



Maximum distance of ball A from left wall

$$= \frac{u^2}{2a} = \frac{0.3 \times 0.3}{2 \times 2} = \frac{0.09}{4} \approx 0.02 \text{ m}$$

So collision of two balls will take place very near to left wall

$$\text{For } BS = ut + \frac{1}{2}at^2$$

$$-4 = -0.2t - \left(\frac{1}{2}\right)2t^2 \Rightarrow t^2 + 0.2t - 4 = 0$$

$$\Rightarrow t = \frac{-0.2 \pm \sqrt{0.04 + 16}}{2} = 1.9$$

Nearest integer = 2 s

Concept Application Exercises

Exercise 5.1

The acceleration of the particle,

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = (-1.0 \text{ m/s}^2) \hat{i} + (-0.50 \text{ m/s}^2) \hat{j}$$

We can analyse the motion as two independent motions individually (for x and y)

Along the x -direction, we have

$$x(t) = v_{0x}t + \frac{1}{2}a_x t^2, \quad v_x(t) = v_{0x} + a_x t$$

Similarly, along the y -direction, we get

$$y(t) = v_{0y}t + \frac{1}{2}a_y t^2, \quad v_y(t) = v_{0y} + a_y t$$

We are given $a_x = -1.0 \text{ m/s}^2$, $a_y = -0.5 \text{ m/s}^2$.

Substituting the values given, the components of the velocity are,

$$v_x(t) = 3.0 - 1.0t \quad \text{and} \quad v_y(t) = 0 - 0.50t$$

When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.0 - 1.0t_m = 0$ or $t_m = 3.0 \text{ s}$. The y component of the velocity at this time is $v_y(t) = 0 - 0.50 \times 3 = -1.50 \text{ m/s}$.

Thus, $\vec{v}_m = (-1.5 \text{ m/s}) \hat{j}$.

At $t = 3.0 \text{ s}$, the components of the position are

$$\begin{aligned} x(t = 3.0 \text{ s}) &= v_{0x}t + \frac{1}{2}a_x t^2 \\ &= (3.0 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-1.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 4.5 \text{ m} \end{aligned}$$

$$y(t = 3.0 \text{ s}) = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5 \text{ m/s}^2)(3.0 \text{ s})^2 = 2.25 \text{ m}$$

Using unit-vector notation, the results can be written as

$$\vec{r}_m = (4.50 \text{ m}) \hat{i} - (2.25 \text{ m}) \hat{j}$$

$$\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 : 16 = 0 + 4t + \frac{1}{2}4.0t^2 \Rightarrow t = 2 \text{ s}$$

Therefore, its x component of velocity (when $\Delta x = 16.0 \text{ m}$) is

$$v_x(t) = v_{0x} + a_x t = 4 + 4 \times 2 = 12 \text{ m/s}$$

Its y component of velocity at this time is

$$v_y(t) = v_{0y} + a_y t = 0 + 8 \times 2 = 16 \text{ m/s}$$

Thus, the magnitude of

$$\vec{v} \text{ is } |\vec{v}| = \sqrt{(12.0 \text{ m/s})^2 + (16.0 \text{ m/s})^2} = 20.0 \text{ m/s}$$

The angle of $\vec{v}$ measured from $+x$

$$\theta = \tan^{-1} \left(\frac{16.0 \text{ m/s}}{12.0 \text{ m/s}} \right) = \tan^{-1} \left(\frac{4}{3} \right)$$

Collision between particles A and B requires two things.

First, the y motion of B must satisfy,

$$y = \frac{1}{2}a_y t^2 \Rightarrow 30 \text{ m} = \frac{1}{2}[(0.40 \text{ m/s}^2) \cos \theta] t^2 \quad \dots (i)$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \Rightarrow (3.0 \text{ m/s})t = \frac{1}{2}[(0.40 \text{ m/s}^2) \sin \theta] t^2$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2) \sin \theta}$$

This is then plugged into the equation (i) to produce,

$$30 \text{ m} = \frac{1}{2}[(0.40 \text{ m/s}^2) \cos \theta] \left[\frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2) \sin \theta} \right]^2$$

which, with the use of $\sin^2 \theta = 1 - \cos^2 \theta$ simplifies to

$$30 = \frac{9.0}{0.20} \left(\frac{\cos \theta}{1 - \cos^2 \theta} \right) \Rightarrow 1 - \cos^2 \theta = \frac{9.0}{(0.20)(30)} \cos \theta$$

We use the quadratic formula (choosing the positive root) to solve for $\cos \theta$:

$$\cos \theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields $\theta = \cos^{-1} \left[\frac{1}{2} \right] = 60^\circ$.

4.

- (a) The initial position is $\vec{r}_0 = [a \sin \omega t \hat{i} + a \cos \omega t \hat{j} + bt^2 \hat{k}]_{t=0} = a \hat{j}$
 (b) The speed of the particle at $t = 0$ is

$$\begin{aligned} \vec{v}_0 &= \left[\frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} \right]_{t=0} \\ &= [a\omega \cos \omega t \hat{i} - a\omega \sin \omega t \hat{j} + 2bt \hat{k}]_{t=0} = a\omega \end{aligned}$$

- (c) The initial acceleration is $\vec{a}_0 = \left[\frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k} \right]_{t=0}$
 $= [-a\omega^2 \sin \omega t \hat{i} - a\omega^2 \cos \omega t \hat{j} + 2b \hat{k}]_{t=0} = -a\omega^2 \hat{j} + 2b \hat{k}$

5. Here $a_x = a$, $a_y = -g$ and $v_{0x} = v_0$ and $v_{0y} = 0$

The velocity components are $v_x = v_{0x} + a_x t = v_0 + at$

$$v_y = v_{0y} + a_y t = 0 + (-g)t = -gt$$

Then, $\vec{v} = v_x \hat{i} + v_y \hat{j} = (v_0 + at) \hat{i} - gt \hat{j}$

The displacement components are

$$\Delta x = x = v_{0x}t + \frac{1}{2}a_x t^2 = v_0 t + \frac{1}{2}at^2$$

$$\Delta y = y = v_{0y}t + \frac{1}{2}a_y t^2 = -\frac{1}{2}gt^2$$

Then, the net displacement is $\vec{s} = \vec{r} = \left(v_0 t + \frac{1}{2}at^2 \right) \hat{i} - \frac{1}{2}gt^2 \hat{j}$

6. Displacement along X -axis:

$$x = u_x t + \frac{1}{2}a_x t^2 = \frac{1}{2} \times 6 \times (4)^2 = 48 \text{ m}$$

Displacement along Y -axis:

$$y = u_y t + \frac{1}{2}a_y t^2 = \frac{1}{2} \times 8 \times (4)^2 = 64 \text{ m}$$

Total distance from the origin

$$= \sqrt{x^2 + y^2} = \sqrt{(48)^2 + (64)^2} = 80 \text{ m}$$

7. Body moves horizontally with constant initial velocity 3 ms^{-1} upto 4 s .

$$\therefore x = ut = 3 \times 4 = 12 \text{ m}$$

And in perpendicular direction it moves under the effect of constant force with zero initial velocity upto 4 s .

$$\therefore y = ut + \frac{1}{2}(a)t^2 = 0 + \frac{1}{2}\left(\frac{F}{m}\right)t^2 = \frac{1}{2}\left(\frac{4}{2}\right)4^2 = 16\text{ m}$$

So its distance from O is given by

$$d = \sqrt{x^2 + y^2} = \sqrt{(12)^2 + (16)^2} = 20\text{ m}$$

Exercise 5.2

1. Standard equation of projectile motion

$$y = x \tan \theta \left[1 - \frac{x}{R} \right]$$

Given equation: $y = 16x - \frac{5x^2}{4} = 16x \left[1 - \frac{x}{64/5} \right]$

By comparing above equations, $R = \frac{64}{5} = 12.8\text{ m}$

2. For first particle angle of projection from the horizontal is α .

So $T_1 = \frac{2u \sin \alpha}{g}$

For second particle, angle of projection from the vertical is α it mean from the horizontal is $(\theta - \alpha)$

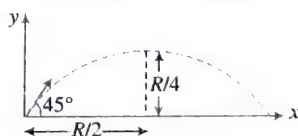
$\therefore T_2 = \frac{2u \sin(90^\circ - \alpha)}{g} = \frac{2u \cos \alpha}{g}$

So, ratio of time of flight $\frac{T_1}{T_2} = \tan \alpha$.

3. Range $\propto$ horizontal component of velocity. Graph 4 shows maximum range, so football possesses maximum horizontal velocity in this case.

4. For maximum horizontal range, $\theta = 45^\circ$.

From $R = 4H \cot \theta = 4H$ [As $\theta = 45^\circ$, for maximum range.]
Speed of the particle will be minimum at the highest point of parabola. So the co-ordinate of the highest point will be $(R/2, R/4)$.



5. The maximum area will be equal to the area of the circle with radius equal to the maximum range of projectile.

Maximum area, $\pi r^2 = \pi (R_{\max})^2 = \pi \left(\frac{u^2}{g} \right)^2 = \pi \frac{u^4}{g^2}$

[As $r = R_{\max} = u^2/g$ for $\theta = 45^\circ$]

6. $R = \frac{u^2 \sin 2\theta}{g}$

$\therefore R \propto u^2 \sin 2\theta$

$\frac{R_2}{R_1} = \left(\frac{u_2}{u_1} \right)^2 \left(\frac{\sin 2\theta_2}{\sin 2\theta_1} \right)$

$\Rightarrow R_2 = R_1 \left(\frac{2u}{u} \right)^2 \left(\frac{\sin 90^\circ}{\sin 30^\circ} \right) = 8R_1$

7. For equal ranges, body should be projected with angle θ or $(90^\circ - \theta)$ from the horizontal.

And for these angles:

$h_1 = \frac{u^2 \sin^2 \theta}{2g}$ and $h_2 = \frac{u^2 \cos^2 \theta}{2g}$

By multiplication of both height,

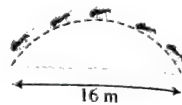
$h_1 h_2 = \frac{u^2 \sin^2 \theta \cos^2 \theta}{4g^2} = \frac{1}{16} \left(\frac{u^2 \sin 2\theta}{g} \right)^2$

$\Rightarrow 16h_1 h_2 = R^2 \Rightarrow R = 4\sqrt{h_1 h_2}$

8. Horizontal distance travelled by grasshopper will be maximum for $\theta = 45^\circ$

$R_{\max} = \frac{u^2}{g} = 1.6\text{ m}$

$\Rightarrow u = 4\text{ ms}^{-1}$



Horizontal component of velocity of grasshopper

$u \cos \theta = 4 \cos 45^\circ = 2\sqrt{2}\text{ ms}^{-1}$

Total distance covered by it in 10 s.

$S = u \cos \theta \times t = 2\sqrt{2} \times 10 = 20\sqrt{2}\text{ m}$

9. Angle of projection, $\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{b}{a}$

$\therefore \tan \theta = \frac{b}{a}$... (i)

From formula: $R = 4H \cot \theta = 2H$

$\Rightarrow \cot \theta = \frac{1}{2}$

$\therefore \tan \theta = 2$ [As $R = 2H$ given] ... (ii)

From Eqs (i) and (ii), $b = 2a$

10. Same ranges can be obtained for complementary angles, i.e., θ and $90^\circ - \theta$.

$y_1 = \frac{u^2 \sin^2 \theta}{2g}$ and $y_2 = \frac{u^2 \cos^2 \theta}{2g}$

$\therefore y_1 + y_2 = \frac{u^2 \sin^2 \theta}{2g} + \frac{u^2 \cos^2 \theta}{2g} = \frac{u^2}{2g}$

11. Both particles collide at the highest point. It means the vertical distance travelled by both the particle will be equal, i.e., the vertical component of velocity of both particle will be equal

$u_1 \sin 30^\circ = u_2 \Rightarrow \frac{u_1}{2} = u_2 \Rightarrow u_1 = 2u_2$

12. Let in 2 sec body reaches upto point A and after one more sec upto point B.

Total time of ascent for a body is given 3 s, i.e., $t = \frac{u \sin \theta}{g} = 3$
 $\therefore u \sin \theta = 10 \times 3 = 30$... (i)

Horizontal component of velocity remains always constant,
 $u \cos \theta = v \cos 30^\circ$... (ii)

For vertical upward motion between point O and A,

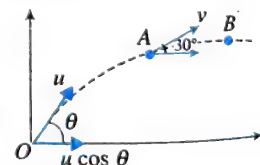
$v \sin 30^\circ = u \sin \theta - g \times 2$ [Using $v = u - gt$]

$v \sin 30^\circ = 30 - 20$ [As $u \sin \theta = 30$]

$\therefore v = 20\text{ ms}^{-1}$

Substituting this value in Eq. (ii),

$u \cos \theta = 20 \cos 30^\circ = 10\sqrt{3}$... (iii)



From Eq. (i) and (iii), $u = 20\sqrt{3}$ and $\theta = 60^\circ$.

13. Let x_1 and x_2 are the horizontal distances travelled by particle A and B, respectively, in time t .

$x_1 = \frac{u}{\sqrt{3}} \cdot \cos 30^\circ \times t$... (i)

and $x_2 = u \cos 60^\circ \times t$... (ii)

$x_1 + x_2 = \frac{u}{\sqrt{3}} \cdot \cos 30^\circ \times t + u \cos 60^\circ \times t = ut$

$\Rightarrow x = ut \Rightarrow t = x/u$

In case of projectile without wind effect, we have

Time of flight, $T = \frac{2u_y}{g}$, maximum height reached by a

projectile $H = \frac{u_y^2}{2g}$ and range of a projectile $R = u_x T$.

Due to horizontal acceleration of wind, T and H will not change as u_y is the only term that appears. If we include the effect of wind the range of the projectile will change. New range will become

$$R' = u_x T + \frac{1}{2} a T^2 = R + \frac{1}{2} a \left(\frac{4u_y^2}{g^2} \right)$$

$$R' = R + \frac{4a}{g} \left(\frac{u_y^2}{2g} \right) = R + \frac{4a}{g} H \quad \dots(i)$$

But we have given the new range with wind effect

$$R' = R + 2H$$

Comparing (i) and (ii) we get, $\frac{4a}{g} = 2 \Rightarrow a = \frac{g}{2}$... (ii)

Initial velocity of stone

$$\vec{u}_{\text{stone}} = 8\hat{i} + (12 \cos 30^\circ \hat{j}) + (12 \sin 30^\circ \hat{k}) \text{ (m/s)}$$

$$= 8\hat{i} + 6\sqrt{3}\hat{j} + 6\hat{k} \text{ (m/s)}$$

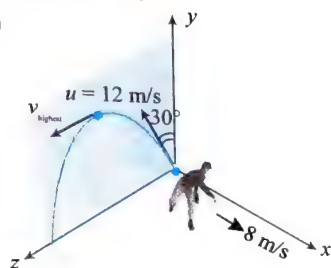
At the highest point, vertical component of velocity will become zero.

So,

$$\vec{v}_{\text{highest}} = 8\hat{i} + 6\hat{k} \text{ (m/s)}$$

Hence magnitude of the velocity at highest point

$$|\vec{v}_{\text{highest}}| = \sqrt{8^2 + 6^2} = 10 \text{ m/s}$$

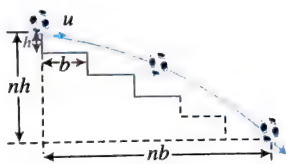


Exercise 5.3

1. By using equation of trajectory $y = \frac{gx^2}{2u^2}$ for given condition,

$$nh = \frac{g(nb)^2}{2u^2}$$

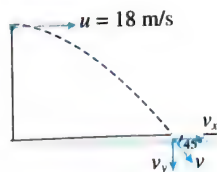
$$\text{or } n = \frac{2hu^2}{gb^2}$$



2. When the body strikes the ground,

$$\tan 45^\circ = \frac{v_y}{v_x} \Rightarrow \frac{v_y}{18} = 1$$

$$\Rightarrow v_y = 18 \text{ ms}^{-1}$$



3. For first particle: $v^2 = u^2 + 2gh$

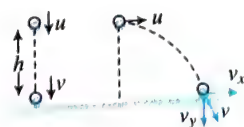
$$\Rightarrow v = \sqrt{u^2 + 2gh}$$

For second particle:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + (\sqrt{2gh})^2}$$

$$= \sqrt{u^2 + 2gh}$$

So the ratio of velocities will be 1 : 1.



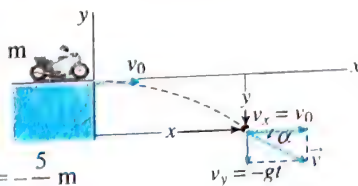
4. At $t = 0.50 \text{ s}$, the x - and y -coordinates are

$$x = v_{0x}t$$

$$= (9.0 \text{ ms}^{-1})(0.50 \text{ s}) = 4.5 \text{ m}$$

$$y = -\frac{1}{2}gt^2$$

$$= -\frac{1}{2}(10 \text{ ms}^{-2})(0.50 \text{ s})^2 = -\frac{5}{4} \text{ m}$$



The negative value of y shows that this time the motorcycle is below its starting point.

The motorcycle's distance from the origin at this time,

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{5}{4}\right)^2} = \frac{\sqrt{349}}{4} \text{ m}$$

The components of velocity at this time are

$$v_x = v_{0x} = 9.0 \text{ ms}^{-1}$$

$$v_y = -gt = (-10 \text{ ms}^{-2})(0.50 \text{ s}) = -5 \text{ ms}^{-1}$$

The speed (magnitude of the velocity) at this time is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0 \text{ ms}^{-1})^2 + (-5 \text{ ms}^{-1})^2} = \sqrt{106} \text{ ms}^{-1}$$

$$5. \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}}$$

$$t = 3 \text{ s}$$

$$R = uT$$

$$\frac{180}{3} = u \Rightarrow u = 60 \text{ ms}^{-1}$$

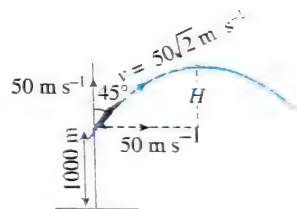
$$6. (a) \quad y = u_y t + \frac{1}{2} a_y t^2$$

$$-1000 = 50t - \frac{1}{2} \times 10 \times t^2$$

$$t^2 - 10t - 200 = 0$$

$$(t - 20)(t + 10) = 0$$

$$t = 20 \text{ s}$$



$$(b) \quad H = \frac{u_y^2}{2g} = \frac{50^2}{2 \times 10} = 125 \text{ m}$$

Hence, maximum height above ground

$$H = 1000 + 125 = 1125 \text{ m}$$

7. Equation of motion in x -direction, $100 = v \times t$

$$\Rightarrow t = \frac{100}{v}$$

... (i)

In y -direction,

$$0.1 = \frac{1}{2} \times 10 \times t^2$$

$$0.1 = \frac{1}{2} \times 10 \times (100/v)^2$$

... (ii)

From Eqs (i) and (ii), we get

$$u = 700 \text{ ms}^{-1}$$

8. (a) To calculate the time of flight (for both stone), apply equation of motion in y -direction,

$$100 = \frac{1}{2}gt^2$$

$$t = 2\sqrt{5} \text{ s}$$

$$(b) \quad x_B = 10 \times 3 = 30 \text{ m}$$

$$y_B = \frac{1}{2} \times g \times t^2$$

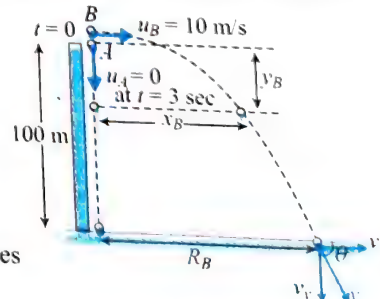
$$= \frac{1}{2} \times 10 \times 3 \times 3$$

$$= 45 \text{ m}$$

Distance between two stones

$$\text{after } 3 \text{ s, } x_B = 30 \text{ m,}$$

$$y_B = 45 \text{ m.}$$



(c) Angle of striking with ground,

$$v_y^2 = u_y^2 + 2gh = 0 + 2 \times 10 \times 100$$

$$v_y = 20\sqrt{5} \text{ ms}^{-1}; \quad v_x = 10 \text{ ms}^{-1}$$

$$\therefore \vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\tan \theta = \frac{v_y}{v_x}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{20\sqrt{5}}{10} \right) = \tan^{-1} (2\sqrt{5})$$

(d) Horizontal range of particle B,

$$x_B = 10 \times (2\sqrt{5}) = 20\sqrt{5} \text{ m}$$

9. Let stones meet at P.

$$\Rightarrow \frac{ut_2}{2} = ut_1 \Rightarrow t_2 = 2t_1$$

Given $t_1 = 3 \text{ sec}$

Hence $t_2 = 6 \text{ sec}$

It means $(t_2 - t_1) = 3 \text{ sec}$

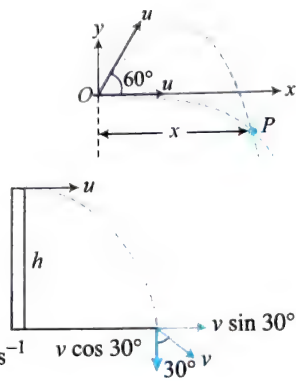
$$10. \quad h = \frac{1}{2} 10(3)^2 = 45 \text{ m}$$

$$v \cos 30^\circ = 10 \times 3$$

$$\Rightarrow v = 20\sqrt{3} \text{ ms}^{-1}$$

$$u = v \sin 30^\circ$$

$$= 20\sqrt{3} \times \frac{1}{2} = 10\sqrt{3} \text{ ms}^{-1}$$



Exercise 5.4

1. Maximum range up the inclined plane,

$$(R_{\max})_{\text{up}} = \frac{u^2}{g(1 + \sin \alpha)}$$

Maximum range down the inclined plane

$$(R_{\max})_{\text{down}} = \frac{u^2}{g(1 - \sin \alpha)}$$

$$\text{and according to: } \frac{u^2}{g(1 - \sin \alpha)} = 3 \times \frac{u^2}{g(1 + \sin \alpha)}$$

By solving $\alpha = 30^\circ$

2. Here $u = 21 \text{ ms}^{-1}$, $\alpha = 30^\circ$, $\theta = \beta - \alpha = 60^\circ - 30^\circ = 30^\circ$

$$\text{Maximum range, } R = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$

$$= \frac{2 \times (21)^2 \times \sin 30^\circ \cos 60^\circ}{9.8 \times \cos^2 30^\circ} = 30 \text{ m}$$

3. Maximum range on horizontal plane, $R = \frac{u^2}{g} = 6 \text{ km}$ (given)

$$\text{Maximum range on an inclined plane, } R_{\max} = \frac{u^2}{g(1 + \sin \alpha)}$$

Putting $\alpha = 30^\circ$

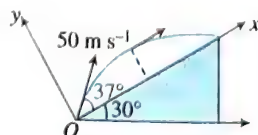
$$R_{\max} = \frac{u^2}{g(1 + \sin 30^\circ)} = \frac{2}{3} \left(\frac{u^2}{g} \right) = \frac{2}{3} \times 6 = 4 \text{ km}$$

4.(a) Taking axis system as shown in Fig.

At highest point, $v_y = 0$. Therefore,

$$v_y^2 = u_y^2 + 2a_y y$$

$$\Rightarrow 0 = (30)^2 - 2g \cos 30^\circ y$$



$$y = 30\sqrt{3} \text{ m (maximum height)}$$

(b) Again for x coordinate,

$$v_y = u_y + a_y t$$

$$0 = 30 - g \cos 30^\circ \times t \Rightarrow t = 2\sqrt{3} \text{ s}$$

$$\text{Time of flight } T = 2 \times 2\sqrt{3} \text{ s}$$

$$(c) \text{ Range, } x = u_x t + \frac{1}{2} a_x t^2$$

$$= 40 \times 4\sqrt{3} - \frac{1}{2} g \sin 30^\circ \times (4\sqrt{3})^2$$

$$= 40(4\sqrt{3} - 3) \text{ m}$$

$$(d) \theta = \frac{\pi}{4} - \frac{30^\circ}{2} = 45^\circ - 15^\circ = 30^\circ$$

$$(e) \frac{u^2}{g(1 + \sin \beta)} = \frac{50 \times 50}{10 \left(1 + \frac{1}{2} \right)} = \frac{2500}{15} = \frac{500}{3} \text{ m}$$

5. Time of flights of A and B

$$T_A = \frac{2u \sin(\alpha + \beta)}{g \cos \beta} \text{ and}$$

$$T_B = \frac{2u \sin(\alpha - \beta)}{g \cos \beta}$$

$$\frac{T_A}{T_B} = \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \cot \theta \quad (\text{given})$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \frac{\sin(90^\circ - \theta)}{\sin \theta} = \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)}$$

We can write, $\alpha + \beta = 90^\circ - \theta$

and $\alpha - \beta = \theta$

on solving (i) and (ii) we get, $\alpha = 45^\circ$

$$6. \quad v_x = u_x + a_x t$$

$$0 = u \cos \theta + (-g \sin \alpha) T$$

$$T = \frac{u \cos \theta}{g \sin \alpha} = \frac{u \operatorname{cosec} \theta}{2g}$$

$$\frac{\cos \theta}{\sin \alpha} = \frac{1}{2 \sin \theta}$$

$$\text{or } \alpha = 2\theta$$

$$7. \quad R = \frac{u^2}{g \cos^2 \theta_0} [\sin(2\alpha + \theta_0) - \sin \theta_0]$$

where $\theta_0 = 30^\circ$ and $\alpha = 45^\circ$.

$$\text{Then, } R = \frac{u^2}{g \cos^2 30^\circ} [\sin(2 \times 45^\circ + 30^\circ) - \sin 30^\circ]$$

$$= \frac{2u^2(\sqrt{3} - 1)}{3g}$$

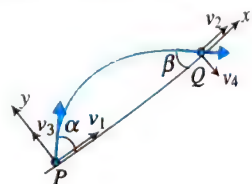
8. Let α be the angle of projection with incline and β be the angle at which projectile strikes the plane.

Then $\tan \alpha = v_3/v_1$ and $\tan \beta = v_4/v_2$. Here the magnitude of velocity component initially and finally along y-axis will be same, because $S_y = 0$. Hence, $v_3 = v_4$. But velocity component along x-axis goes on decreasing. So

$v_2 < v_1$. Hence, from above

equations,

$\tan \beta > \tan \alpha \Rightarrow \beta > \alpha$. Hence, appropriate path should be b.



Exercise 5.5

1. (a) Time taken to cross the river

$$t = \frac{d}{v_y} = \frac{400 \text{ m}}{10 \text{ m/s}} = 40 \text{ s}$$

 (b) Drift (x) = $(v_x)(t)$

$$= (2 \text{ m/s}) (40 \text{ s}) = 80 \text{ m}$$

(c) Actual direction of boat,

$$\theta = \tan^{-1} \left(\frac{10}{2} \right) = \tan^{-1} 5, \text{ (downstream) with the river flow.}$$

 2. (a) $v_{MR} = v$, $v_R = u$

$$\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$$

Therefore, velocity of man,

$$v_M = \sqrt{u^2 + v^2 + 2vu \cos \theta}$$

$$(b) \tan \phi = \frac{v \sin \theta}{u + v \cos \theta}$$

$$(c) (v \sin \theta)t = d \Rightarrow t = \frac{d}{v \sin \theta}$$

$$x = (u + v \cos \theta)t = (u + v \cos \theta) \frac{d}{v \sin \theta}$$

3. Velocity of aeroplane while flying through AB,

$$v_A = v + u$$

$$t_{AB} = \frac{a}{v + u}$$

Velocity of aeroplane while flying through BC,

$$v_A = \sqrt{v^2 - u^2}$$

$$t_{BC} = \frac{a}{\sqrt{v^2 - u^2}}$$

Velocity of aeroplane while flying through CD

$$v_A = v - u$$

$$t_{CD} = \frac{a}{v - u}$$

Velocity of aeroplane while flying through DA,

$$v_A = \sqrt{v^2 - u^2}$$

$$t_{DA} = \frac{a}{\sqrt{v^2 - u^2}}$$

 Total time = $t_{AB} + t_{BC} + t_{CD} + t_{DA}$

$$= \frac{a}{v + u} + \frac{a}{\sqrt{v^2 - u^2}} + \frac{a}{v - u} + \frac{a}{\sqrt{v^2 - u^2}}$$

$$= \frac{2a}{v^2 - u^2} \left(v + \sqrt{v^2 - u^2} \right)$$

 4. Velocity of rain w.r.t. man, $\vec{v}_{r,m} = \vec{v}_r - \vec{v}_m$

$$-4\hat{j} = \vec{v}_r - (2\hat{i} + 3\hat{j})$$

$$\vec{v}_r = 2\hat{i} - \hat{j}$$

When man starts running down,

$$\vec{v}_{r,m} = \vec{v}_r - \vec{v}_m = 2\hat{i} - \hat{j} + 2\hat{i} + 3\hat{j} = 4\hat{i} + 2\hat{j}$$

$$\text{Speed} = |\vec{v}_{r,m}| = \sqrt{16 + 4} = \sqrt{20} \text{ m/s}$$

 5. Suppose the plane is oriented at an angle α w.r.t. line AB, while the plane is moving from A to B:

Velocity of plane along


 $AB = v \cos \alpha - u \cos \theta$, and for no-drift from line AB

$$v \sin \alpha = u \sin \theta$$

$$\Rightarrow \sin \alpha = \frac{u \sin \theta}{v}$$

$$\text{Time taken from A to B: } t_{AB} = \frac{l}{v \cos \alpha - u \cos \theta}$$

 Suppose plane is oriented at an angle α' w.r.t. line AB while the plane is moving from B to A:

 Velocity of plane along BA = $v \cos \alpha' + u \cos \theta$

$$v \sin \alpha' = u \sin \theta$$

$$\Rightarrow \sin \alpha' = \frac{u \sin \theta}{v} \Rightarrow \alpha' = \alpha$$

Time taken from B to A:

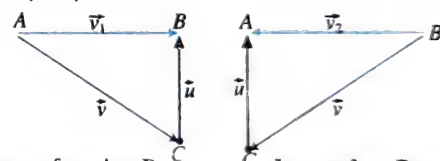
$$t_{BA} = \frac{l}{v \cos \alpha + u \cos \theta}$$

 Total time taken = $t_{AB} + t_{BA}$

$$= \frac{l}{v \cos \alpha - u \cos \theta} + \frac{l}{v \cos \alpha + u \cos \theta}$$

$$= \frac{2vl \cos \alpha}{v^2 \cos^2 \alpha + u^2 \cos^2 \theta} = \frac{2vl \sqrt{1 - \frac{u^2 \sin^2 \theta}{v^2}}}{v^2 - u^2}$$

$$6. t_0 = \frac{s}{v} + \frac{s}{v} = \frac{2s}{v}$$



Journey from A to B

Journey from B to A

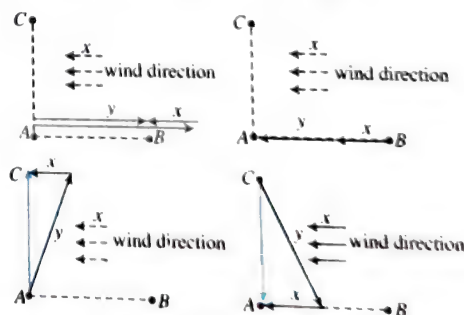
$$(a) t = \frac{s}{v + u} + \frac{s}{v - u} = \frac{2sv}{v^2 - u^2} = \frac{v^2 t_0}{v^2 - u^2} = \frac{t_0}{1 - u^2/v^2}$$

$$(b) |\vec{v}_1| = |\vec{v}_2| = \sqrt{v^2 - u^2}$$

$$t = \frac{2s}{\sqrt{v^2 - u^2}} = \frac{v t_0}{\sqrt{v^2 - u^2}} = \frac{t_0}{\sqrt{1 - \frac{u^2}{v^2}}}$$

7. Time taken from A to B than B to A,

$$t_{(AB/BA)} = \frac{AB}{y - x} + \frac{AB}{y + x} = \frac{2y(AB)}{y^2 - x^2}$$



Time taken from A to C than from C to A,

$$t_{(AC/CA)} = \frac{2AC}{\sqrt{y^2 - x^2}} R$$

$$\text{Ratio of times} = \frac{y}{\sqrt{y^2 - x^2}} = \frac{1}{\sqrt{1 - x^2/y^2}} > 1$$

Hence, the first plane takes more time.

8. Solving from the frame of particle 1,

$$\text{we get } d_{\text{short}} = 10 \cos 45^\circ \\ = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ km}$$

$$t = \frac{10 \sin 45^\circ}{|V_{21}|} = \frac{10 \times 1/\sqrt{2}}{20\sqrt{2}}$$

$$= \frac{1}{4} \text{ h} = 15 \text{ min}$$

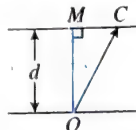
9. Let
- d
- m be the width of the river. The time to cross by quickest path (say)
- $t_1 = \frac{d}{v} = \frac{d}{17}$
- s and the time to cross the shortest path,

$$(\text{say}) t_2 = \frac{d}{\sqrt{v^2 - u^2}} = \frac{d}{\sqrt{(17)^2 - (8)^2}}$$

From the problem, $t_2 - t_1 = 6$

$$d \left[\frac{1}{\sqrt{225}} - \frac{1}{17} \right] = 6$$

$$\Rightarrow d \left[\frac{1}{15} - \frac{1}{17} \right] = 6 \Rightarrow d = \frac{6 \times 15 \times 17}{2} = 765 \text{ m}$$

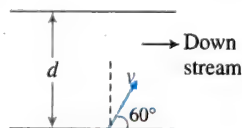


10. We know that the time taken for a man to cross the river directly is given by
- $t_1 = \frac{d}{\sqrt{v^2 - u^2}}$
- , where
- d
- is the width of the river. Now, if the man swims downstream then the resultant velocity is
- $v + u$
- . The corresponding time,
- $t_2 = \frac{d}{u + v}$
- .

$$\text{Now, } \frac{t_1}{t_2} = \frac{v + u}{\sqrt{v^2 - u^2}} = \frac{\sqrt{v + u} \sqrt{v + u}}{\sqrt{v + u} \sqrt{v - u}} \Rightarrow \frac{t_1}{t_2} = \frac{\sqrt{v + u}}{\sqrt{v - u}}$$

11. Let
- v
- be the velocity of the boat and
- d
- the width of the river. The component of velocity along the direction perpendicular to the flow of water current is
- $v \sin 60^\circ$
- . Now, if
- t_1
- be the time taken by the person to cross the river, then

$$t_1 = \frac{d}{v \sin 60^\circ} = \frac{d}{v} \left(\frac{2}{\sqrt{3}} \right)$$



Had he crossed the river in the minimum possible time, then his direction should always be perpendicular to the flow of river and the corresponding time taken,

$$t_2 (\text{say}) = \frac{d}{v}$$

$$\therefore \text{Percentage time saved} = \frac{t_1 - t_2}{t_2} \times 100$$

$$= \frac{\frac{d}{v} \left(\frac{2}{\sqrt{3}} \right) - \frac{d}{v}}{\frac{d}{v}} \times 100 = 15.47\%$$

12. Let
- v
- and
- u
- be the velocities of the person and the water current, and
- d
- the width of the river. If
- t
- be the time taken by the person to cross the river, then
- $t = \frac{d}{v \sin \alpha}$
- and distance carried downstream in this time

$$= \frac{d}{v \sin \alpha} [u + v \cos \alpha] = x (\text{say})$$

Now, eliminating α , we get

$$t^2 (u^2 - v^2) - 2xut + (x^2 + d^2) = 0$$

Clearly t will have real solutions, if "discriminant ≥ 0 "

$$\Rightarrow 4x^2 u^2 \geq 4(u^2 - v^2)(x^2 + d^2)$$

$$\Rightarrow x \geq \frac{d}{v} (\sqrt{u^2 - v^2})$$

Distance swayed away by the water current (x) has to be minimum, for the shortest path.

$$\text{Again } t = \frac{d}{v \sin \alpha} \text{ and } x = \frac{d}{v} \sqrt{u^2 - v^2} = \frac{d}{v \sin \alpha} (u + v \cos \alpha)$$

$$\Rightarrow (u^2 - v^2) \sin^2 \alpha = (u + v \cos \alpha)^2$$

$$\Rightarrow u^2 (1 - \sin^2 \alpha) + 2uv \cos \alpha + v^2 = 0$$

$$\Rightarrow u^2 \cos^2 \alpha + 2uv \cos \alpha + v^2 = 0$$

$$\Rightarrow (u \cos \alpha + v)^2 = 0 \Rightarrow \cos \alpha = -\frac{v}{u}$$

Exercises

Single Correct Answer Type

1. (3) The time taken to reach the ground depends on the height from which the bullets are fired when the bullets are fired horizontally. Here height is same for both the bullets, and hence the bullets will reach the ground simultaneously.

2. (2) The horizontal range is the same for the angles of projection
- θ
- and
- $(90^\circ - \theta)$
- .

$$t_1 = \frac{2u \sin \theta}{g}, \quad t_2 = \frac{2u \sin(90^\circ - \theta)}{g} = \frac{2u \cos \theta}{g}$$

$$t_1 t_2 = \frac{2u \sin \theta}{g} \times \frac{2u \cos \theta}{g} = \frac{2}{g} \left[\frac{u^2 \sin 2\theta}{g} \right] = \frac{2}{g} R$$

$$\text{where } R = \frac{u^2 \sin 2\theta}{g}$$

Hence, $t_1 t_2 \propto R$ (as R is constant)

$$3. (3) \quad y_1 = \frac{u^2 \sin^2 \theta}{2g}, \quad y_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$\Rightarrow y_1 + y_2 = \frac{u^2}{2g}$$

4. (4) Range is same for angles of projection
- θ
- and
- $90^\circ - \theta$
- ,

$$R = \frac{u^2 \sin^2 \theta}{g}, \quad h_1 = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{and } h_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$\text{Hence, } \sqrt{h_1 h_2} = \frac{u^2 \sin \theta \cos \theta}{2g} = \frac{1}{4} \left[\frac{u^2 \sin 2\theta}{g} \right] = \frac{R}{4}$$

5. (1) Since
- $R = 2H$

$$\text{or } \frac{v^2 \sin 2\theta}{g} = 2 \times \frac{v^2 \sin^2 \theta}{2g}$$

$$\text{or } 2 \sin \theta \cos \theta = \sin^2 \theta \quad \text{or } \tan \theta = 2$$

$$R = \frac{v^2 \sin 2\theta}{g} \\ = \frac{v^2 2 \sin \theta \cos \theta}{g} = \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

6. (4) We find that
- $H = R$
- or
- $\frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 2 \sin \theta \cos \theta}{g}$

$$\text{or } \tan \theta = 4 \text{ or } \theta = \tan^{-1}(4)$$

$$AC = R/2, PC = H$$

We have to find $h = MP$.

From the figure in previous question, we know that if $H = R$, then $\tan \theta = 4$

$$\text{Now } \tan \theta = \frac{MC}{AC} = \frac{MP + PC}{AC}$$

$$4 = \frac{h + H}{R/2} \Rightarrow 4 = \frac{(h + H)2}{H} \Rightarrow h = H$$

$$8. (3) v_x = \frac{dx}{dt} = 6 \text{ and } v_y = \frac{dy}{dt} = 8 - 10t$$

Putting $t = 0$ (Since we have to find initial velocity)

$$v_y = 8 - 10 \times 0 = 8$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10 \text{ ms}^{-1}$$

$$9. (2) \tan \theta = \frac{v_y}{v_x} = \frac{8}{6} = \frac{4}{3} \text{ or } \theta = \tan^{-1} \frac{4}{3}$$

$$10. (2) H = 100 \text{ m}, R = 2 \times 200 = 400 \text{ m}$$

$$\tan \theta = \frac{4H}{R} \Rightarrow \tan \theta = \frac{4 \times 100}{400} = 1$$

$$\Rightarrow \theta = 45^\circ \left[\because \frac{H}{R} = \frac{\tan \theta}{4} \right]$$

$$11. (4) \frac{R}{T^2} = \frac{u^2 \sin 2\theta / g}{4u^2 \sin^2 \theta / g^2} = \frac{g}{2} \cot \theta \text{ i.e., } gT^2 = 2R \tan \theta$$

If T is doubled, then R becomes 4 times.

12. (1) For the person to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e.,

$$v_0 \cos \theta = \frac{v_0}{2} \text{ or } \cos \theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

13. (1) The time of flight is given by

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 30 \times 1/2}{10} = 3 \text{ s}$$

Thus, after 1.5 s, the body will be at the highest point. So the direction of motion will be horizontal after 1.5 s, the angle with the horizontal is 0° .

$$14. (4) \text{ Let } u_x = 3 \text{ ms}^{-1}, a_x = 0$$

$$v_y = u_y + a_y t = 0 + 1 \times 4 = 4 \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2}$$

Angle made by the resultant velocity w.r.t. direction of initial velocity, i.e., x -axis, is

$$\beta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{4}{3}$$

$$15. (2) h = 150 - 27.5 = 122.5 \text{ m}$$

$$\text{Time taken, } T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 122.5}{9.8}} = 5 \text{ s}$$

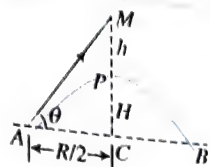
$$\text{Now } s = \mu T \text{ or } 30 = 5\mu \text{ or } \mu = 6 \text{ ms}^{-1}$$

$$16. (4) h = \frac{u^2 \sin^2 \theta}{2g} \text{ and } R = \frac{u^2 \sin 2\theta}{g}$$

$$\frac{h}{R} = \frac{45}{180} = \frac{\tan \theta}{4} \text{ or } \theta = 45^\circ$$

$$17. (2) \text{ Given } y = 12x - \frac{3}{4}x^2, u_x = 3 \text{ ms}^{-1}$$

$$v_y = \frac{dy}{dt} = 12 \frac{dx}{dt} - \frac{3}{2}x \frac{dx}{dt}$$



$$\text{At } x = 0, v_y = u_y = 12 \frac{dx}{dt} = 12u_x = 12 \times 3 = 36 \text{ ms}^{-1}$$

$$a_y = \frac{d}{dt} \left(\frac{dy}{dt} \right) = 12 \frac{d^2x}{dt^2} - \frac{3}{2} \left[\left(\frac{dx}{dt} \right)^2 + x \frac{d^2x}{dt^2} \right]$$

$$\text{But, } \frac{d^2x}{dt^2} = a_x = 0. \text{ Hence}$$

$$a_y = -\frac{3}{2} \left(\frac{dx}{dt} \right)^2 = -\frac{3}{2} u_x^2 = -\frac{3}{2} \times (3)^2 = -\frac{27}{2} \text{ ms}^{-2}$$

$$\text{Range, } R = \frac{2u_x u_y}{a_y} = \frac{2 \times 3 \times 36}{27/2} = 16 \text{ m}$$

Alternatively: We have $y = 12x - \frac{3}{4}x^2$. When projectile again comes to ground, $y = 0$ and $x = R$.

$$0 = 12R - \frac{3}{4}R^2 \Rightarrow R = 16 \text{ m}$$

$$18. (4) \text{ Range} = 150 = ut \text{ and } h = \frac{15}{100} = \frac{1}{2} \times gt^2$$

$$\text{or } t^2 = \frac{2 \times 15}{100 \times g} = \frac{30}{1000} \text{ or } t = \frac{\sqrt{3}}{10}$$

$$u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3} \text{ ms}^{-1}$$

$$19. (2) \frac{R}{T^2} = g \frac{\sin 2\theta}{4 \sin^2 \theta} = \frac{g}{2} \cot \theta = 5 \cot \theta$$

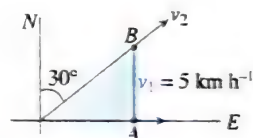
$$\text{Given } \frac{R}{T^2} = 5; \text{ Hence, } 5 = 5 \cot \theta \text{ or } \theta = 45^\circ$$

$$20. (3) \vec{v}_{r/g} = \vec{v}_r + (-\vec{v}_g)$$

$$= \vec{v}_r - \vec{v}_g = -4\hat{j} - 3\hat{i}$$

$$v_{r/g} = \sqrt{v_r^2 + v_g^2}$$

$$= \sqrt{16 + 9} \text{ km h}^{-1} = 5 \text{ km h}^{-1}$$

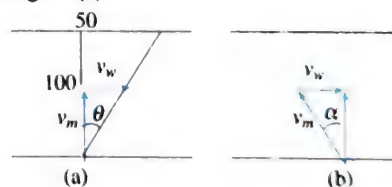


21. (1) For B always to be north of A , the velocity components of both along east should be same.

$$v_2 \cos 60^\circ = v_1$$

$$\Rightarrow v_2 = 10 \text{ km h}^{-1}$$

22. (4) Refer to figure (a),



$$\tan \theta = \frac{v_w}{v_m} = \frac{50}{100} = \frac{1}{2} \text{ or } v_m = 2v_w$$

Refer to figure (b),

$$\sin \alpha = \frac{v_w}{v_m} = \frac{v_w}{2v_w} = \frac{1}{2} \text{ or } \theta = 30^\circ$$

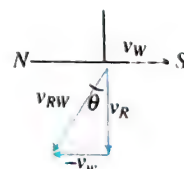
So, it is 60° upstream.

23. (1) Here, $v_R = 25 \text{ ms}^{-1}$, $v_W = 10 \text{ ms}^{-1}$

Velocity of rain w.r.t. woman:

$$v_{R/W} = v_R - v_W$$

Let $v_{R/W}$ make an angle θ with vertical.



$$\text{Then } \tan \theta = \frac{v_W}{v_R} = \frac{10}{2.5} = 0.4$$

She should hold her umbrella at an angle of $\theta = \tan^{-1}(0.4)$ with the vertical towards south.

$$24. (4) \text{ Velocity of police van } = 30 \times \frac{5}{18} = \frac{25}{3} \text{ ms}^{-1}$$

$$\text{Muzzle speed of the bullet} = 150 \text{ ms}^{-1}$$

$$\text{Speed of the bullet w.r.t. ground} = [150 + (25/3)] \text{ ms}^{-1}$$

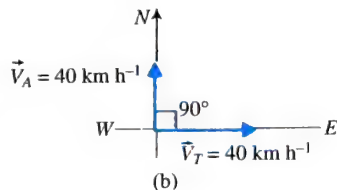
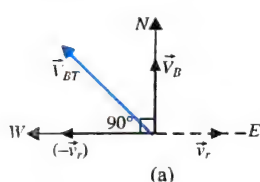
Velocity of thief's car is

$$192 \times \frac{5}{18} = \frac{32 \times 5}{3} = \frac{160}{3} \text{ ms}^{-1}$$

Relative velocity of bullet w.r.t. thief's car is

$$150 + \frac{25}{3} - \frac{160}{3} = 150 - \frac{135}{3} = 105 \text{ ms}^{-1}$$

25. (3) To find the relative velocity of bird w.r.t. train, superimpose velocity $-\vec{V}_T$ on both the objects. Now as a result of it, the train is at rest, while the bird possesses two velocities, $\vec{V}_B$ towards north and $-\vec{V}_T$ along west.



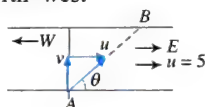
$$|\vec{V}_{BT}| = \sqrt{|\vec{V}_B|^2 + |-\vec{V}_T|^2} \quad [\text{By formula, } \theta = 90^\circ]$$

$$= \sqrt{40^2 + 40^2} = 40\sqrt{2} \text{ km h}^{-1} \text{ north-west}$$

26. (2) Finally, he will swim along B.

$$\tan \theta = \frac{v}{u} = \frac{10}{5} = 2$$

$$\Rightarrow \theta = \tan^{-1}(2) \text{ N of E}$$



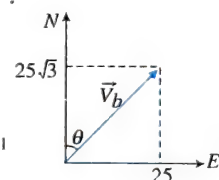
$$27. (1) \vec{v}_c = 25\hat{i}, \vec{v}_{b/c} = 25\sqrt{3}\hat{j}$$

$$\vec{v}_{b/c} = \vec{v}_b - \vec{v}_c \Rightarrow \vec{v}_b = \vec{v}_{b/c} + \vec{v}_c$$

$$\Rightarrow \vec{v}_b = 25\hat{i} + 25\sqrt{3}\hat{j}$$

$$|\vec{v}_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50 \text{ km h}^{-1}$$

$$\tan \theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$



$$28. (1) \text{ Shortest time } = \frac{d}{v} = \frac{1/2}{3} = \frac{1}{6} \text{ h} = 10 \text{ min}$$

29. (4) Relative velocity of bird with respect to train is

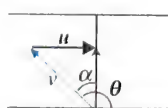
$$V_{BT} = V_B + V_T = 5 + 10 = 15 \text{ ms}^{-1}$$

[Because they are going in opposite directions]

$$\text{Time taken by the bird to cross the train is } \frac{150}{5} = 10 \text{ s}$$

$$30. (2) \sin \alpha = \frac{u}{v} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60^\circ$$

$$\Rightarrow \theta = 90^\circ + \alpha = 150^\circ$$

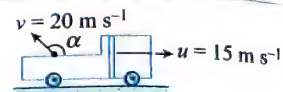


31. (1) Let the stone be projected at an angle α to the direction of motion of truck with a speed of $v = 20 \text{ ms}^{-1}$. Since the resultant displacement along horizontal is zero, the velocity along horizontal is 0.

$$15 + 20 \cos \alpha = 0$$

$$\Rightarrow \cos \alpha = -\frac{3}{4}$$

$$\Rightarrow \alpha = \cos^{-1}\left(-\frac{3}{4}\right) = 138^\circ 35'$$



32. (4) We know that to cross the river by the shortest path,

$$\sin \alpha = \frac{u}{v}$$

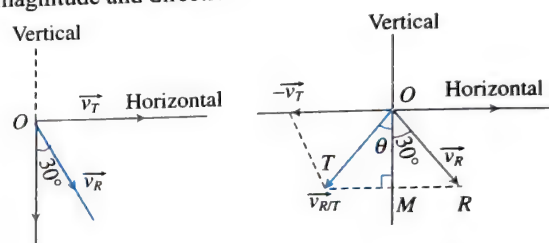
But $u > v \Rightarrow \sin \alpha > 1$, which is not possible.

$$33. (2) \text{ Speed of train } = 108 \times \frac{5}{18} = 30 \text{ ms}^{-1}$$

Let $\vec{V}_R$ and $\vec{V}_T$ represent the respective velocities of rain and train. Now, the relative velocity of rain w.r.t. person (train) is given

$$\text{by } v_{R,T} = \vec{V}_R - \vec{V}_T \Rightarrow \vec{V}_R + (-\vec{V}_T)$$

Let $\vec{OR}$ and $\vec{RT}$ represent the vectors, respectively, in magnitude and direction.

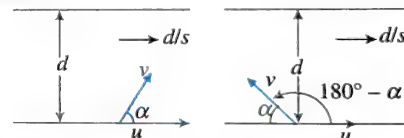


$$OT^2 = OR^2 + RT^2 + 2 OR \cdot RT \cos 120^\circ$$

$$= 20^2 + 30^2 - 2 \times 20 \times 30 \times \frac{1}{2}$$

$$= 400 + 900 - 600 = 700 = \sqrt{700} \text{ ms}^{-1} = 10\sqrt{7} \text{ ms}^{-1}$$

34. (1) Motion of the person making an angle (say α) with the downstream.



$$\text{The time taken to cross the river} = \frac{d}{v \sin \alpha}$$

The distance carried away downstream in the same time = speed $\times$ time.

$$x_1 = (u + v \cos \alpha) \frac{d}{v \sin \alpha} \quad \dots(i)$$

Motion of the person making α angle with upstream.

$$\text{The time taken to cross the river is equal to } \frac{d}{v \sin \alpha}$$

Distance carried away downstream in the same time

$$x_2 = [u + v \cos(180^\circ - \alpha)] \frac{d}{v \sin \alpha}$$

$$\Rightarrow x_2 = (u - v \cos \alpha) \frac{d}{v \sin \alpha} \quad \dots(ii)$$

$$\text{Given } \frac{(u + v \cos \alpha) \frac{d}{v \sin \alpha}}{(u - v \cos \alpha) \frac{d}{v \sin \alpha}} = \frac{2}{1}$$

$$\frac{(u + v \cos \alpha)}{(u - v \cos \alpha)} = \frac{2}{1} \Rightarrow 3v \cos \alpha = u$$

$$\Rightarrow \frac{v}{u} = \frac{\sec \alpha}{3}$$

$$\sec \alpha \geq 1 \Rightarrow \frac{\sec \alpha}{3} \geq \frac{1}{3} \quad \dots(iii)$$

From Eq. (iii), $\frac{v}{u} \geq \frac{1}{3}$

So, v/u cannot be less than $1/3$.

35. (1) The motion of the train will affect only the horizontal component of the velocity of the ball. Since, vertical component is same for both observers, h_m will be same, but R will be different.

$$\frac{h_2}{h_1} = \frac{u^2 \sin^2 \theta_2}{2g} \times \frac{2g}{u^2 \sin^2 \theta_1} = \frac{\sin^2 \theta_2}{\sin^2 \theta_1} = \frac{\sin^2 \pi/6}{\sin^2 \pi/3} = \frac{1}{3}$$

$$\Rightarrow h_2 = \frac{h_1}{3}$$

37. (3) Suppose the angle made by the instantaneous velocity with the horizontal be α . Then

$$\tan \alpha = \frac{v_y}{v_x} = \frac{u \sin \theta - gt}{u \cos \theta}$$

Given that $\alpha = 45^\circ$, when $t = 1$ s; $\alpha = 0^\circ$, when $t = 2$ s

This gives $u \cos \theta = u \sin \theta - g$

and $u \sin \theta - 2g = 0$

...(i)

Solving Eqs. (i) and (ii), we find $u \sin \theta = 2g$ and $u \cos \theta = g$.

Squaring and adding,

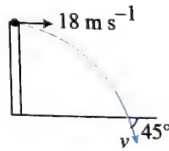
$$u = \sqrt{5}g = 10\sqrt{5} \text{ ms}^{-1}$$

38. (3) $v \cos 45^\circ = u = 18 \text{ ms}^{-1}$

$$\Rightarrow v = 18\sqrt{2} \text{ ms}^{-1}$$

Vertical component

$$v \sin 45^\circ = 18\sqrt{2} \times \frac{1}{\sqrt{2}} = 18 \text{ ms}^{-1}$$



39. (4) Angle of projection from B is 45° . As the body is able to cross the well of diameter 40 m, hence

$$R = \frac{v^2}{g} \text{ or } v = \sqrt{gR} = \sqrt{10 \times 40} = 20 \text{ ms}^{-1}$$

On the inclined plane, the retardation is

$$g \sin \alpha = g \sin 45^\circ = 10/\sqrt{2} \text{ ms}^{-2}$$

Using $v^2 - u^2 = 2ax$

$$(20)^2 - u^2 = 2 \times \left(-\frac{10}{\sqrt{2}}\right) \times 20\sqrt{2}$$

$$u = 20\sqrt{2} \text{ ms}^{-1}$$

40. (2) Time taken by bullet to reach the target is

$$\frac{\text{Distance}}{\text{Velocity}} = \frac{\text{Distance}}{u \cos \theta} \quad [\text{As } \theta \text{ is very small, } \cos \theta = 1]$$

$$\text{Time} = \frac{\text{Distance}}{u} = \frac{400}{400} = 1 \text{ s}$$

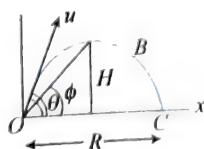
Vertical deflection of bullet is

$$= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (1)^2 = 5 \text{ m}$$

41. (3) From figure

$$\tan \phi = \frac{H}{R/2}$$

$$= \frac{u^2 \sin^2 \theta / 2g}{u^2 \sin 2\theta / 2g} = \frac{\sin^2 \theta}{\sin 2\theta} = \frac{1}{2} \tan \theta$$



42. (3) $H = \frac{u^2 \sin^2 \theta}{2g}$ or $80 = \frac{u^2 \sin^2 \theta}{2 \times 10}$

$$\text{or } u^2 \sin^2 \theta = 1600 \text{ or } u \sin \theta = 40 \text{ ms}^{-1}$$

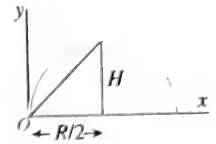
$$\text{Horizontal velocity} = u \cos \theta = 3 \times 30 = 90 \text{ ms}^{-1}$$

$$\frac{u \sin \theta}{u \cos \theta} = \frac{40}{90} \text{ or } \tan \theta = \frac{4}{9} \text{ or } \theta = \tan^{-1} \left(\frac{4}{9} \right)$$

$$43. (3) \text{ Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\sqrt{H^2 + R^2/4}}{T/2}$$

Putting the required values, we get

$$v_{av} = \frac{v}{2} \sqrt{1 + 3 \cos^2 \theta}$$



$$44. (2) \frac{u^2 \sin 2\theta}{g} = \frac{(u/2)^2 \sin 30^\circ}{g} = \frac{u^2}{8g}$$

$$\therefore \sin 2\theta = \frac{1}{8} \text{ or } \theta = \frac{1}{2} \sin^{-1} \left(\frac{1}{8} \right)$$

$$45. (2) T = \frac{100}{25} = 4 \text{ s} \Rightarrow \frac{2u \sin \theta}{g} = 4 \Rightarrow u \sin \theta = 20 \text{ ms}^{-1}$$

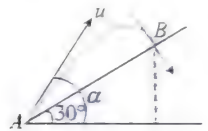
$$46. (4) H = \frac{u^2 \sin^2 \theta}{2g}, R = \frac{u^2 \sin 2\theta}{g}$$

Maximum height will be same because acceleration $a = g/4$ is in horizontal direction

$$R' = u \cos \theta T + \frac{1}{2}aT^2 = R + \frac{1}{2} \frac{g}{4} \left(\frac{2u \sin \theta}{g} \right)^2 = R + H$$

47. (1) t_{AB} = time of flight of projectile

$$= \frac{2u \sin(\alpha - 30^\circ)}{g \cos 30^\circ}$$



Now component of velocity along the plane becomes zero at point B.

$$0 = u \cos(\alpha - 30^\circ) - g \sin 30^\circ \times T$$

$$\text{or } u \cos(\alpha - 30^\circ) = g \sin 30^\circ \times \frac{2u \sin(\alpha - 30^\circ)}{g \cos 30^\circ}$$

$$\text{or } \tan(\alpha - 30^\circ) = \frac{\cot 30^\circ}{2} = \frac{\sqrt{3}}{2}$$

$$\text{or } \alpha = 30^\circ + \tan^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

48. (1) Horizontal component of velocity, $u_H = u \cos 60^\circ = \frac{u}{2}$

$$AC = u_H \times t = \frac{ut}{2}$$

$$\text{and } AB = AC \sec 30^\circ = \left(\frac{ut}{2} \right) \left(\frac{2}{\sqrt{3}} \right) = \frac{ut}{\sqrt{3}}$$

49. (4) Speed in horizontal direction remains constant during whole journey because there is no acceleration in this direction. So, $v_h = 5 \text{ ms}^{-1}$

In vertical direction, loss in gravitation potential energy = gain in KE, i.e.,

$$mgh = \frac{1}{2}mv_v^2$$

$$v_v^2 = 2gh = 2 \times 10 \times (70 - 60) = 200$$

Hence, the speed with which he touches the cliff B is:

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{25 + 200} = \sqrt{225} = 15 \text{ ms}^{-1}$$

$$50. (3) x = 6t, u_x = \frac{dx}{dt} = 6$$

$$y = 8t - 5t^2, v_y = \frac{dy}{dt} = 8 - 10t$$

$$v_y(t=0) = 8 \text{ ms}^{-1}$$

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{6^2 + 8^2} = 10 \text{ ms}^{-1}$$

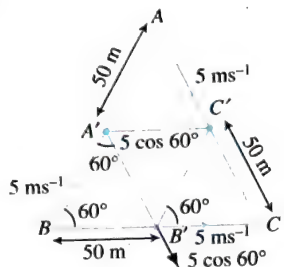
51. (3) Vertical component of both should be same.

$$v_2 = v_1 \sin 30^\circ \Rightarrow \frac{v_2}{v_1} = \frac{1}{2} = 0.5$$

52. (4) Applying equation of trajectory, we get

$$0.5 = \frac{\sqrt{3}}{2} \tan 30^\circ - \frac{g \left(\frac{\sqrt{3}}{2} \right)^2}{2v_0^2 \cos^2 30^\circ} \Rightarrow v_0 = \infty$$

53. (4) At
- $t = 10$
- s,



Along the line joining any two, $v_r = 5 \cos 60^\circ - 5 \cos 60^\circ = 0$

54. (1) Let
- d
- be the river width and
- u
- and
- v
- the speeds of water current.

For P , time taken

$$t_1 = \frac{2d}{\sqrt{v^2 - u^2}} \quad (i)$$

$$\text{For } Q, \text{ time taken, } t_2 = d \left[\frac{1}{v+u} + \frac{1}{v-u} \right] \quad (ii)$$

Dividing Eq. (i) by (ii), we get

$$\frac{t_1}{t_2} = \sqrt{1 - \left(\frac{u}{v} \right)^2} < 1 \Rightarrow t_1 < t_2$$

55. (1)
- $\vec{v}_m$
- = Velocity of man

$\vec{v}_{re}$ = Velocity of rain w.r.t. earth

$\vec{v}_{rm}$ = Velocity of rain w.r.t. man

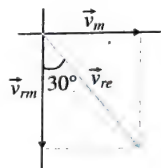
Velocity of man $|\vec{v}_m| = 10 \text{ ms}^{-1}$

Using $\sin 30^\circ = \frac{v_m}{v_{re}}$

$$v_{re} = \frac{v_m}{\sin 30^\circ} = \frac{10}{1/2} = 20 \text{ ms}^{-1},$$

$$\cos 30^\circ = \frac{v_{rm}}{v_{re}}$$

$$v_{rm} = v_{re} \cos 30^\circ = \frac{20 \times \sqrt{3}}{2} = 10\sqrt{3} \text{ ms}^{-1}$$



56. (3) Case I: Let
- $OP = 3\hat{i}$
- be the velocity of

man. $\overrightarrow{OQ}$ be the velocity of rain.

PQ is the velocity of rain relative to man.

Case II: $\overrightarrow{OR} = 6\hat{i}$ is the new velocity of man.

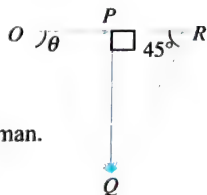
$\overrightarrow{RQ}$ = new velocity of rain relative to man

$$OP = PR = PQ = 3$$

$$\text{Now } OQ^2 = OP^2 + PQ^2, \text{ i.e., } OQ^2 = 3^2 + 3^2$$

$$\text{i.e., } OQ = 3\sqrt{2} \text{ km h}^{-1}$$

$$\text{and } \tan \theta = \frac{PQ}{OP} = \frac{3}{3} = 1, \text{ i.e., } \theta = 45^\circ$$



57. (3) The velocity of motor boat is given as

$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$

$$\frac{5}{\sin \theta} = \frac{5\sqrt{3}}{\sin 120^\circ} \Rightarrow \sin \theta = 1/2 \Rightarrow \theta = 30^\circ$$

58. (3) The magnitude will decrease till the direction of the velocity with respect to man becomes vertical. It will increase thereafter.

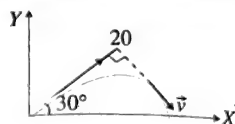
$$59. (4) \vec{u} = 20 \cos 30^\circ \hat{i} + 20 \sin 30^\circ \hat{j}$$

$$\text{Now, } \vec{v} = 20 \cos 30^\circ \hat{i} + (20 \sin 30^\circ - gt) \hat{j}$$

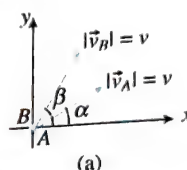
Let $\vec{v}$ be perpendicular to $\vec{u}$ at time t , then

$$\vec{v} \cdot \vec{u} = 0 \Rightarrow t = 4 \text{ s}$$

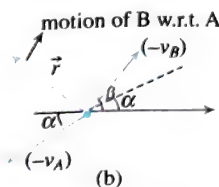
Here time of flight, $T = 2 \text{ s}$. So it is not possible at any instant.



60. (3)



(a)



(b)

A is fixed and B moves in the direction as shown in figure. w.r.t. A with constant velocity.

Direction of $\vec{r}$ remains same and magnitude changes with time linearly.

Here velocity of B w.r.t. A is constant because relative acceleration is zero.

61. (1) In a direction along the inclined plane,

$$0 = V_0 \cos 30^\circ - g \sin 30^\circ t \Rightarrow t = \sqrt{3} V_0 / g$$

In a direction perpendicular to incline,

$$-H \cos 30^\circ = -V \sin 30^\circ t - \frac{1}{2} g \cos 30^\circ t^2$$

Putting the value of t and solving, we get $V_0 = \sqrt{\frac{2gH}{5}}$

62. (1) Time taken by particles to collide
- $t = \sqrt{\frac{2H}{g}}$

$$\text{Then } u \sqrt{\frac{2H}{g}} + v \sqrt{\frac{2H}{g}} = d$$

$$\Rightarrow u + v = d \sqrt{\frac{g}{2H}} \quad \text{or} \quad v = d \sqrt{\frac{g}{2H}} - u$$

63. (3)
- $a_x = -g \sin \theta$
- ;
- $a_y = -g \cos \theta$

$$u_x = 0; u_y = v$$

$$\text{Along } y\text{-axis, } S_y = u_y t + \frac{1}{2} a_y t^2$$

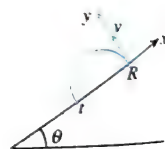
$$0 = vt - \frac{1}{2} g \cos \theta t^2$$

$$t = \frac{2v}{g \cos \theta}$$

Along x -axis,

$$S_x = u_x t + \frac{1}{2} a_x t^2 - R = 0 \times t - \frac{1}{2} g \sin \theta \left(\frac{2v}{g \cos \theta} \right)^2$$

$$R = \frac{2v^2 \tan \theta \sec \theta}{g}$$



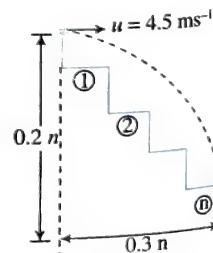
64. (1)
- $0.2n = \frac{1}{2} g t^2$
- or
- $t = \sqrt{\frac{2 \times 0.2n}{g}}$

$$0.3n = ut = 4.5 \sqrt{\frac{2 \times 0.2n}{g}}$$

$$\Rightarrow n = 9$$

65. (2) For the maximum range,
- $\theta = 45^\circ$
- .

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2}{g} \sin 90^\circ = \frac{u^2}{g} \quad \text{or} \quad 500 = \frac{u^2}{g}$$



The distance covered along the inclined plane can be obtained using the equation,

$$v^2 - u^2 = 2as$$

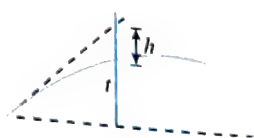
$$\text{or } 0 - u^2 = 2(-g \sin 30^\circ)s \quad \text{or } s = \frac{u^2}{g} = 500 \text{ m}$$

66. (3) In general, $h = \frac{1}{2}gt^2$

$$X = \frac{1}{2}g(1)^2 = 5 \text{ m,}$$

$$Y = \frac{1}{2}g(2)^2 = 20 \text{ m,}$$

$$Z = \frac{1}{2}g(3)^2 = 45 \text{ m}$$



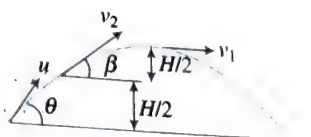
67. (4) $v_1 = v_2 \cos \beta = u \cos \theta$

$$0 = (v_2 \sin \beta)^2 - 2g(H/2)$$

$$\Rightarrow v_1^2 = v_2^2 - gH$$

$$\text{Also } \frac{v_1}{v_2} = \sqrt{\frac{2}{5}}$$

$$\text{From above } v_1 = \sqrt{\frac{2gH}{3}}, v_2 = \sqrt{\frac{5gH}{3}}$$



$$H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \sin^2 \theta = \frac{2gH \cos^2 \theta}{v_1^2}$$

$$\text{or } \tan^2 \theta = \frac{2gH}{v_1^2} \quad \text{or } \tan^2 \theta = \frac{2gH \times 3}{2gH}$$

$$\text{or } \tan \theta = \sqrt{3} \quad \text{or } \theta = 60^\circ$$

68. (2) $\frac{y}{x} = \tan \beta$

$$\Rightarrow \frac{u \sin \alpha t - \frac{1}{2}gt^2}{u \cos \alpha t} = \tan \beta$$

$$\text{or } u \sin \alpha - \tan \beta u \cos \alpha = \frac{gt}{2} \quad \text{or } u = \frac{gt \cos \beta}{2 \sin(\alpha - \beta)}$$

69. (3) $u_1 \cos \alpha = u_2 \cos \beta$

$$30\sqrt{3} = \frac{u_2^2 \sin 2\beta}{g} \Rightarrow 30\sqrt{3} = \frac{u_2^2 \sqrt{3}}{2g}$$

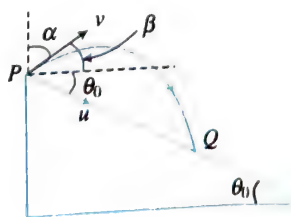
$$\Rightarrow u_2^2 = 600 \quad \text{or } u_2 = 10\sqrt{6}$$

$$t_2 = \frac{2u_2 \sin 60^\circ}{g} = \frac{2(10\sqrt{6})\sqrt{3}/2}{10} \Rightarrow t_2 = \sqrt{18}$$

$$u_1 = \frac{10\sqrt{6} \cos 60^\circ}{\cos 30^\circ} = 10\sqrt{2}$$

$$-h = 10\sqrt{2} \sin 30^\circ \sqrt{18} - \frac{1}{2}10(\sqrt{18})^2$$

$$= 30 - 90 \Rightarrow h = 60 \text{ m}$$



70. (1) If the particles collide at Q, it means they travel same displacement along the plane in same time. So their velocity components along the plane should be same.

$$v \cos(\beta + \theta_0) = u$$

$$\text{and } \beta = 90^\circ - \alpha$$

$$\text{Solve to get } v \sin(\alpha - \theta_0) = u$$

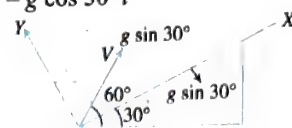
71. (1) Acceleration w.r.t. platform is $g + a$.

$$\text{So time taken } T = \frac{2u \sin \theta}{g + a}$$

72. (4) If the particles collide in mid air, they travel same displacement in horizontal direction. So their velocity components along horizontal should be same, i.e., $v_1 \cos \theta_1 = v_2 \cos \theta_2$.

73. (2) $v_y = v \sin 30^\circ - g \cos 30^\circ t$

$$t = \frac{v \sin 30^\circ}{g \cos 30^\circ} = \frac{v}{g} \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ s}$$



74. (2) Initially: $\vec{u}_P = 20 \cos 60^\circ \hat{i} + 20 \sin 60^\circ \hat{j} = 10\hat{i} + 10\sqrt{3}\hat{j}$

$$\vec{u} = 20\sqrt{2} [\cos 45^\circ (-\hat{i}) + \sin 45^\circ \hat{j}] = -20\hat{i} + 20\hat{j}$$

Initial relative velocity:

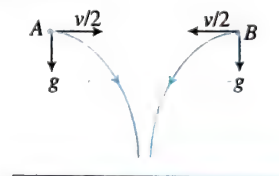
$$\vec{u}_{P/Q} = \vec{u}_P - \vec{u}_Q = 30\hat{i} + (10\sqrt{3} - 20)\hat{j}$$

$$u_{P/Q} = \sqrt{30^2 + (10\sqrt{3} - 20)^2} = 20\sqrt{4 - \sqrt{3}} \text{ ms}^{-1}$$

This relative velocity will remain same till both the particles are in air, because relative acceleration is zero.

75. (2) Time of flight depends upon maximum height and maximum height for A is large in comparison to B.

76. (3) With respect to man,



77. (2) $0 = (200)^2 - 2(a_{\text{rel}})(1000)$

$$\text{or } a_{\text{rel}} = 20 \text{ ms}^{-2}$$

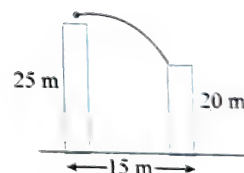
So to avoid the hit,

$$a_{\text{rel}} > 20 \text{ ms}^{-2}$$

$$\text{or } a_p > 10 \text{ ms}^{-2}$$

78. (3) $t = \sqrt{\frac{2 \times 5}{10}} = 1 \text{ ms}^{-1}$

$$\text{Speed} = \frac{15}{1} = 15 \text{ ms}^{-1}$$



79. (3) $\frac{u \sin \theta}{g} = \frac{2u \sin(\theta - 37^\circ)}{g \cos 37^\circ}$

$$\sin \theta = \frac{2(\sin \theta) \left(\frac{4}{5} \right) - 2 \cos \theta \left(\frac{3}{5} \right)}{\frac{4}{5}}$$

$$4 \sin \theta = 8 \sin \theta - 6 \cos \theta$$

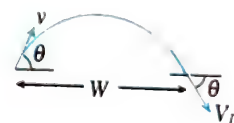
$$4 \sin \theta = 6 \cos \theta \Rightarrow \tan \theta = \frac{3}{2}$$

80. (3) Due to symmetry, the maximum of the trajectory lies halfway across the gap.

Therefore, range of projectile motion = W

$$\Rightarrow W = \frac{2V_L^2 \sin \theta \cos \theta}{g}$$

$$\therefore V_L = \sqrt{\frac{Wg}{2 \sin \theta \cos \theta}}$$



Multiple Correct Answers Type

1. (2), (3)

$y = \frac{x}{2}$ implies that the particle is moving in a straight line passing through the origin.

$$v_x = 4 - 2t, \quad v_x = u_x + a_x t,$$

$$u_x = 4, \quad a_x = -2$$

$$\text{Now, } y = \frac{x}{2} \Rightarrow \frac{dy}{dt} = \frac{1}{2} \frac{dx}{dt}$$

$$v_y = \frac{1}{2} v_x = 2 - t, \quad v_y = u_y + a_y t,$$

$$u_y = 2 \quad \text{and} \quad a_y = -1$$

2. (2), (3), (4)

If the particle is projected with velocity u at an angle θ , then equation of its trajectory will be

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

We know slope is given by $m = \frac{dy}{dx}$

$$\text{Therefore, slope } m = \tan \theta - \frac{gx}{u^2 \cos^2 \theta}$$

It implies that the graph between slope and x will be straight line having negative slope and a non-zero positive intercept on y -axis.

But x is directly proportional to the time t ; therefore, the shape of graph between slope and time will be same as that of the graph between slope and x . Hence, only option (1) is correct, i.e., options (2), (3) and (4) are incorrect.

3. (1), (2)

$$\text{Time of ascent} = 2 + 1 = 3 \text{ s}$$

$$\Rightarrow \frac{u \sin \theta}{g} = 3 \Rightarrow u \sin \theta = 30$$

$$\text{and } \tan \beta = \frac{u \sin \theta - gt}{u \cos \theta} \Rightarrow \tan 30^\circ = \frac{30 - 10 \times 2}{u \cos \theta}$$

$$\Rightarrow u \cos \theta = 10\sqrt{3}$$

$$\text{From here } u = \sqrt{(10\sqrt{3})^2 + 10^2} = 20\sqrt{3} \text{ ms}^{-1}$$

$$\text{and } \tan \theta = \frac{30}{10\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60^\circ$$

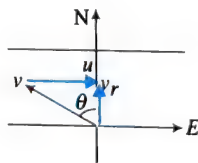
4. (1), (3)

For angle θ in west of north,

$$\sin \theta = \frac{5}{10} = 0.5, \text{ i.e., } \theta = 30^\circ$$

V_r = resultant velocity

$$= \sqrt{(10^2) - (5)^2} = 5\sqrt{3} \text{ ms}^{-1}$$



So, options (1) and (3) are correct.

5. (1), (3), (4)

$(KE + PE)_f = (KE + PE)_i$ in all situations.

Hence, KE_f is also equal as $PE_f = 0$. Hence, all the particles collide with the same speed.

$$-h = vt_1 - \frac{1}{2}gt_1^2 \quad [\text{for first particle}] \quad \dots(i)$$

$$-h = -vt_2 - \frac{1}{2}gt_2^2 \quad [\text{for second particle}] \quad \dots(ii)$$

From Eq. (i) and Eq. (ii), $t_2 > t_1$

t_2 = maximum, t_1 = minimum

i.e., options (3) and (4) are correct.

6. (3), (4)

Distance between two buses on road is $V_b T$.

For A to B direction:

Distance = Relative velocity $\times$ Time

$$V_b T = (V_b - V_c) T_1 \Rightarrow T_1 = \frac{V_b T}{V_b - V_c}$$

For B to A direction:

$$V_b T = (V_b + V_c) T_2 \Rightarrow T_2 = \frac{V_b T}{V_b + V_c}$$

7. (1), (2)

$$\vec{v}_{AW} = -20\hat{j}$$

$$\vec{v}_{BW} = -32\hat{i} + 24\hat{j}$$

$$\vec{v}_{AB} = \vec{v}_{AW} - \vec{v}_{BW} = -32\hat{i} - 44\hat{j}$$

$$\frac{d\vec{r}_{AB}}{dt} = \vec{v}_{AB} = -32\hat{i} - 44\hat{j}$$

$$\int_{3\hat{i}+4\hat{j}}^{\vec{r}_{AB}} d(\vec{r}_{AB}) = - \int_a^t 32 dt \hat{i} - \int_0^t 44 dt \hat{j}$$

$$\vec{r}_{AB} = (3 - 32t)\hat{i} + (4 - 44t)\hat{j}$$

At $t = 1/11$ h, $\hat{j}$ component of $\vec{r}_{AB}$ is zero. At this time, its $\hat{i}$

component is $3 - 32t = 3 - \frac{32}{11} = \frac{1}{11}$ km

It means at $t = 1/11$ h, A will be east of B. So at no time, A will be west of B.

8. (2), (3)

$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 10 \times \frac{1}{2}}{10} = 1 \text{ s}$$

9. (1), (3)

$$\frac{dx}{dt} = t^2$$

$$y = \frac{1}{2} t^3 \Rightarrow \frac{dy}{dt} = \frac{t^2}{2}$$

$$t = 1, v_x = 1, v_y = \frac{1}{2} \Rightarrow \vec{v} = \hat{i} + \frac{1}{2}\hat{j}$$

$$\frac{d^2x}{dt^2} = 2t$$

$$\frac{d^2y}{dt^2} = t$$

$$\text{At } t = 1 \text{ s, } a_x = 2 \quad \text{and} \quad a_y = 1$$

$$\Rightarrow \vec{a} = 2\hat{i} + \hat{j}$$

10. (1), (2)

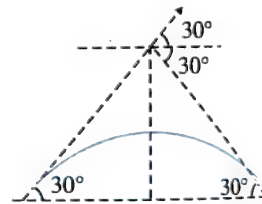
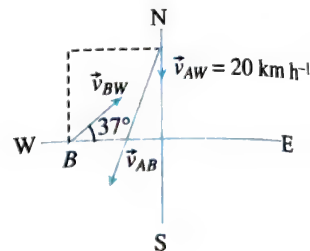
$$T = \frac{2u \sin \theta}{g}, h = \frac{u^2 \sin^2 \theta}{2g}, R = \frac{u^2 \sin 2\theta}{g}$$

$$y = u \sin \theta t - \frac{1}{2}gt^2$$

$$= 4ht \left[\frac{u \sin \theta}{4h} - \frac{gt}{8h} \right]$$

$$= 4ht \left[\frac{u \sin \theta}{4(u^2 \sin^2 \theta / 2g)} - \frac{gt}{8(u^2 \sin^2 \theta / 2g)} \right]$$

$$= 4ht \left[\frac{1}{(2u \sin \theta / g)} - \frac{t}{(2u \sin \theta / g)^2} \right]$$



$$= 4ht \left[\frac{1}{T} - \frac{t}{T^2} \right] = 4h \left(\frac{1}{T} \right) \left(1 - \frac{t}{T} \right)$$

$$\text{Now } x = u \cos \theta t, R = u \cos \theta T$$

$$\Rightarrow \frac{t}{T} = \frac{x}{R}, \text{ so } y = 4h \left(\frac{x}{R} \right) \left(1 - \frac{x}{R} \right)$$

11. (2), (4)

$$\vec{V}_{S/G} = \vec{V}_{S/A} + \vec{V}_{A/G}$$

Horizontal component of velocity of shell will be in the direction of velocity of air plane

12. (1), (3)

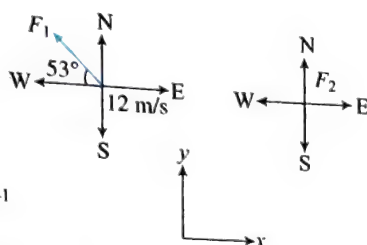
$$\text{Velocity of air} = v \hat{j}$$

$$\vec{V}_{AB} = v \hat{j} - 12 \hat{i}$$

$$\tan 53^\circ = \frac{v}{12} = \frac{4}{3}$$

$$v = \frac{4 \times 12}{3} = 16 \text{ m s}^{-1}$$

$$v_{AB} = \sqrt{(16)^2 + (12)^2} = 20 \text{ m s}^{-1}$$



13. (1), (2)

Since acceleration is in x-direction only, velocity in y-direction will not change.

$$\text{When speed} = 5 \text{ m s}^{-1}$$

$$5^2 = V_x^2 + V_y^2 = V_x^2 + 4^2$$

$$\Rightarrow V_x = \pm 3 \text{ m s}^{-1}$$

$$\therefore V_x = u_x + a_x t \Rightarrow t = \frac{V_x - u_x}{a_x}$$

$$\text{or } t_1 = \frac{3 - 4}{-0.4} = 2.5 \text{ s}$$

$$\text{and } t_2 = \frac{(3 + 4)}{0.4} = 17.5 \text{ s}$$

14. (1), (2), (3)

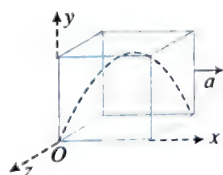
$$H = \frac{u_y^2}{2g} \Rightarrow u_y = V_2 = 5 \text{ m s}^{-1}$$

$$T = \frac{2u \sin \theta}{g} = \frac{2u_y}{g} = 1 \text{ s}$$

$$L = u_x t - \frac{1}{2} a_x t^2$$

$$\frac{5}{4} = u_x \times 1 - \frac{1}{2} \times (0.5) \times (1)^2 \Rightarrow u_x = V_1 = 3/2 \text{ m s}^{-1}$$

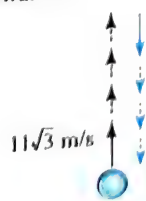
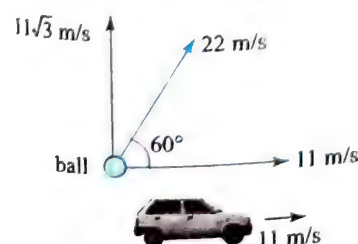
$$\text{Along } z \text{ axis } \frac{5}{4} = u_z \times 1 \Rightarrow v_z = \frac{5}{4} \text{ m s}^{-1}$$



15. (1), (3)

w.r.t. to observer (V_x) football/observer = 11 - 11 = 0

So motion of the ball as seen from observer will be purely vertical with initial velocity $11\sqrt{3}$ upwards.



16. (1), (2), (4)

Since maximum height attained by all is same, so their time of flight and vertical component of velocity will be same. If vertical component of velocity is same, their height or y-coordinate at any time will be same. Now range of C is greatest, so its horizontal component of velocity is greatest. Hence, overall launch speed of C is also greatest.

17. (1), (3), (4)

$$R_1 = \frac{24^2 \sin \alpha \cos (\alpha + \theta)}{g \cos^2 \theta}$$

$$R_2 = \frac{24^2 \sin \alpha \cos (\alpha - \theta)}{g \cos^2 \theta}$$

$$R_1 - R_2 = g \sin \theta T_2^2$$

$$R_1 - R_2 = g \sin \theta T_1^2$$

18. (1), (4)

So, velocity of first particle

$$= 3 \cos 30^\circ \hat{i} + 3 \sin 30^\circ \hat{j} = \frac{12}{5} \hat{i} + \frac{9}{5} \hat{j}$$

Velocity of second particle

$$= 4 \cos 53^\circ \hat{i} + 4 \sin 53^\circ \hat{j} = \frac{12}{5} \hat{i} + \frac{16}{5} \hat{j}$$

So, relative horizontal velocity is zero. So their relative velocity is vertical only. Since both particles are moving under gravity, so that relative acceleration is zero.

$$\text{Their relative velocity} = \frac{16}{5} - \frac{9}{5} = \frac{7}{5} = 1.4 \text{ m/s}$$

19. (2), (4)

$$(y_B - y_A) = 20t - 10t = 10t \quad \dots (i)$$

Difference in the height of building

$$(X_B - X_A) = \int_0^t (0.5y_B) dt - \int_0^t (0.5y_A) dt \quad \dots (ii)$$

Separation between the buildings = 250 m

$$\text{Hence } \int_0^t 0.5 \times 20t \cdot dt - \int_0^t 0.5 \times 10t \cdot dt = 250$$

$$5 \int_0^t t \cdot dt = 250$$

$$\text{Hence } 5 \frac{t^2}{2} = 250 \Rightarrow t = 10 \text{ s}$$

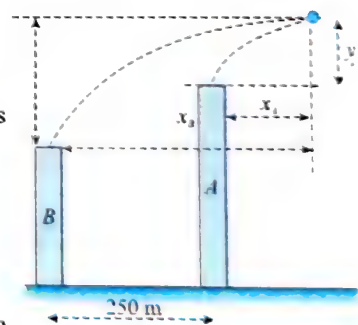
Difference in the height

of building = 10t

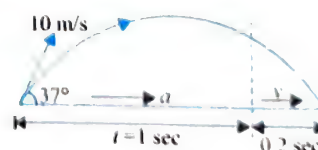
$$= 10 \times 10 = 100 \text{ m}$$

Difference in heights

$$= y_2 - y_1 = 20t - 10t = 100 \text{ m}$$



20. (1), (3)



$$\text{Time of flight } (T) = \frac{2(10) \sin 37^\circ}{10} = 1.2 \text{ sec.}$$

$$\text{Range, } R = \frac{10^2 \sin 2(37^\circ)}{10} = 9.6 \text{ m}$$

$$\text{Now } \frac{1}{2} a(1)^2 + v(0.2) = 9.6$$

$$0.5a + 0.2v = 9.6 \quad \dots(i)$$

$$\text{Also } v = u + at$$

$$v = 0 + a(1) = a \quad \dots(ii)$$

$$\text{From (i) and (ii): } v = a = \frac{96}{7} \approx 13.7$$

21. (1), (3), (4)

$$R = \frac{u^2 \sin 2\theta}{g} \text{ or } \frac{Rg}{u^2} = \sin 2\theta < 1$$

Therefore, there shall be two values of θ , that is θ_1 and $90^\circ - \theta_1$ for same range.

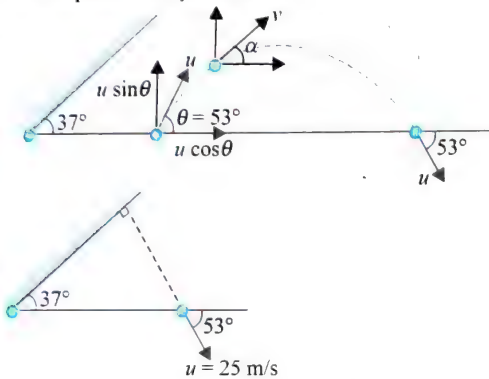
It is obvious θ_1 and $90^\circ - \theta_1$ are complementary.

$$T_1 = \frac{2u \sin \theta_1}{g} \text{ and } T_2 = \frac{2u \sin(90^\circ - \theta_1)}{g} = \frac{2u \cos \theta_1}{g}$$

$$\Rightarrow T_1 T_2 = \frac{2u^2 \sin 2\theta_1}{g^2} = \frac{2}{g} R$$

22. (1), (2), (3), (4)

Relative speed will be minimum (zero) when velocity of object will be parallel to plane mirror.



$$\tan \alpha = \frac{u \sin \theta - gt}{u \cos \theta} \Rightarrow \frac{3}{4} = \frac{25 \cdot \frac{4}{5} - 10t}{25 \cdot \frac{3}{5}}$$

$$\Rightarrow \frac{3}{4} = \frac{20 - 10t}{15} \Rightarrow 45 = 80 - 40t$$

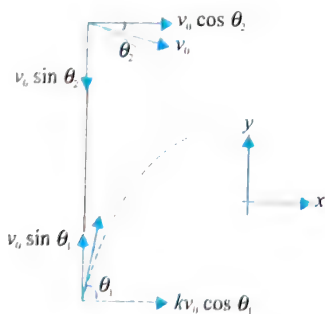
$$\Rightarrow 40t = 35 \Rightarrow t = \frac{7}{8} \text{ s}$$

At maximum relative speed, the object moves normal to mirror.

And it occurs when the particle hits the ground.

Hence, maximum relative speed = 50 m/s

23. (2), (3)



$$\text{In } x\text{-direction: } d = v_0 \cos \theta_1 t = (kv_0 \cos \theta_1) t$$

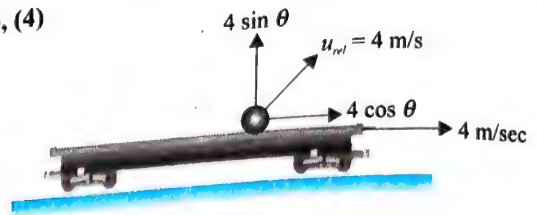
$$\cos \theta_2 = k \cos \theta_1 \Rightarrow \left[k = \left(\frac{\cos \theta_2}{\cos \theta_1} \right) \right]$$

Motion of A w.r.t. B in y direction

$$h = v_r t \text{ here } v_r = (kv_0 \sin \theta_1 + v_0 \sin \theta_2)$$

$$t = \frac{h}{v_r} = \left(\frac{h}{kv_0 \sin \theta_1 + v_0 \sin \theta_2} \right)$$

24. (1), (4)



Range with respect to ground

$$R = \frac{2u_x u_y}{g} = \frac{2(4 + 4 \cos \theta)(4 \sin \theta)}{g}$$

$$R = \frac{32 \sin \theta (1 + \cos \theta)}{g} = \frac{32}{g} (\sin \theta + \sin \theta \cos \theta)$$

$$R_{\max} \text{ when } \sin \theta + \sin \theta \cos \theta \text{ will be max}$$

$$\Rightarrow \frac{d}{d\theta} (\sin \theta + \sin \theta \cos \theta) = 0$$

$$\Rightarrow \cos \theta + 2 \cos^2 \theta - 1 = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

Since the |velocity| of the ball w.r.t. to the cart is constant So range w.r.t. to the cart will be max when $\theta = 45^\circ$

25. (1), (3), (4)

$$R = \frac{u^2 \sin 2\theta}{g} \text{ is same for angles } \theta \text{ and } 90^\circ - \theta.$$

$$t_1 = \frac{2u \sin \theta_1}{g} = \frac{2u \sin(90^\circ - \theta_2)}{g}$$

$$t_2 = \frac{2u \sin \theta_2}{g} = \frac{2u \sin(90^\circ - \theta_1)}{g}$$

Linked Comprehension Type

For Problems 1–3

$$1. (2) y = ax - bx^2$$

$$\text{or } \frac{dy}{dt} = a \frac{dx}{dt} - 2bx \frac{dx}{dt} = (a - 2bx) \frac{dx}{dt}$$

$$\text{Initially } x = 0, v_y = av_x$$

$$\frac{d^2 y}{dt^2} = (a - 2bx) \frac{d^2 x}{dt^2} + \left[-2b \frac{dx}{dt} \right]$$

$$= (a - 2bx) \frac{d^2 x}{dt^2} - 2b \left(\frac{dx}{dt} \right)^2$$

$$\text{At } t = 0, \frac{d^2 y}{dt^2} = -\alpha, x = 0, \frac{d^2 x}{dt^2} = 0$$

$$\Rightarrow -\alpha = -2b \left(\frac{dx}{dt} \right)^2 = -2b(v_x)^2 \text{ or } v_x = \sqrt{\frac{\alpha}{2b}}$$

$$\Rightarrow v = \sqrt{v_x^2 + v_y^2} = v_x \sqrt{1 + a^2} = \sqrt{\frac{\alpha(1 + a^2)}{2b}}$$

$$2. (4) \vec{v} = a\hat{i} + bx\hat{j}, v_x = a \text{ or } dx = a dt \Rightarrow x = at$$

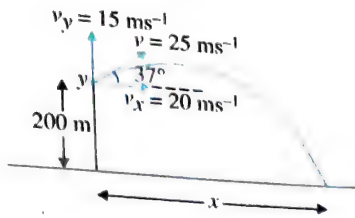
$$v_y = bx = bat \text{ or } y = \frac{bat^2}{2}$$

$$\Rightarrow y = \frac{ba}{2} \left(\frac{x}{a} \right)^2 \Rightarrow y \propto x^2 \rightarrow \text{Parabolic}$$

$$3. (4) \text{ Ranges will be same. } \frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2}$$

For Problems 4 and 5

$$\begin{aligned}
 4. (4) \quad s_y &= v_{iy}t - \frac{1}{2}gt^2 \\
 &\Rightarrow -200 = 15t - 5t^2 \\
 &\Rightarrow t = 8 \text{ s} \\
 x &= 20 \times 8 = 160 \text{ m} \\
 5. (1) \quad x &= 20 \times 8 = 160 \text{ m} \\
 y &= 200 + 15 \times 8 = 320 \text{ m}
 \end{aligned}$$


For Problems 6–8

$$\begin{aligned}
 6. (1) \quad \vec{V}_{R,A} &= \vec{V}_B - \vec{V}_A \\
 &= -14 \cos 45^\circ \hat{i} + 14 \sin 45^\circ \hat{j} - (2 \cos 45^\circ \hat{i} - 2 \sin 45^\circ \hat{j}) \\
 &= -\frac{16}{\sqrt{2}} \hat{i} + \frac{12}{\sqrt{2}} \hat{j} = -8\sqrt{2} \hat{i} + 6\sqrt{2} \hat{j}
 \end{aligned}$$

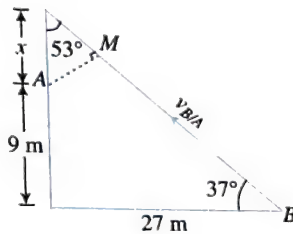
$$7. (2) \quad \tan \phi = \frac{6\sqrt{2}}{8\sqrt{2}} \Rightarrow \tan \phi = \frac{3}{4} \Rightarrow \phi = 37^\circ$$

$$8. (1) \quad \frac{x+9}{27} = \tan 37^\circ = \frac{3}{4}$$

$$\Rightarrow x = \frac{45}{4} \text{ m}$$

Shortest distance

$$AM = x \sin 53^\circ = \frac{45}{4} \times \frac{4}{5} = 9 \text{ m}$$


For Problems 9–13

9. (4), 10. (4), 11. (2), 12. (3), 13. (1)

Sol. Two given planes are mutually perpendicular and the particle is projected perpendicularly from plane OA . It means $\vec{u}$ is parallel to plane OB .

At the instant of collision of the particle with OB , its velocity is perpendicular to OB or velocity component parallel to OB is zero.

First considering motion of particle parallel to plane OB ,

$$\begin{aligned}
 u &= 10\sqrt{3} \text{ m s}^{-1}, \\
 \text{acceleration} &= -g \sin 60^\circ = -5\sqrt{3} \text{ m s}^{-2} \\
 v &= 0, \quad t = ? \quad s = ?
 \end{aligned}$$

$$\text{Using } v = u + at \Rightarrow 0 = 10\sqrt{3} - 5\sqrt{3}t \Rightarrow t = 2 \text{ s}$$

$$\begin{aligned}
 s &= ut + \frac{1}{2}at^2 \Rightarrow OQ = 10\sqrt{3} \times 2 - \frac{1}{2}5\sqrt{3}(2)^2 \\
 &\Rightarrow OQ = 10\sqrt{3} \text{ m}
 \end{aligned}$$

Now considering motion of the particle normal to plane OB .

$$\begin{aligned}
 \text{Initial, velocity} &= 0, \text{ acceleration} = g \cos 60^\circ = 5 \text{ m s}^{-2} \\
 t &= 2 \text{ s}, \quad v = ? \quad s = PO = ?
 \end{aligned}$$

$$\text{Using } v = u + at, \text{ we get } v = 10 \text{ m s}^{-1}$$

$$s = ut + \frac{1}{2}at^2 \quad \text{or} \quad PO = 10 \text{ m}$$

$$h = PO \sin 30^\circ = 10 \times \sin 30^\circ = 5 \text{ m}$$

Inclination of velocity at point P with the vertical is 30° ,

therefore its vertical component is,

$$u \cos 30^\circ = 15 \text{ m s}^{-1} \text{ (upwards)}$$

Considering vertically upward motion of the particle from P ,

$$\text{initial velocity} = 15 \text{ m s}^{-1},$$

$$\text{Acceleration} = -g = -10 \text{ m s}^{-2}, \quad v = 0, \quad s = H = ?$$

$$\text{Using } v^2 = u^2 + 2as, \text{ we get } H = 11.25 \text{ m}$$

Maximum height reached by particle above O is

$$h + H = 16.25 \text{ m}$$

$$\text{Distance } PQ = \sqrt{(PO)^2 + (OQ)^2} = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ m}$$

For Problems 14–16

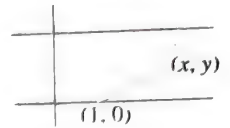
$$\begin{aligned}
 14. (2) \quad a &= 5t \Rightarrow \frac{dv_y}{dt} = 5t \Rightarrow v_y = \frac{5t^2}{2} \\
 &\Rightarrow \frac{dy}{dt} = \frac{5}{2}t^2 \Rightarrow y = \frac{5}{6}t^3 \\
 \text{Putting } y &= d, \text{ we get } d = \frac{5}{6}t^3 \Rightarrow t = \left(\frac{6d}{5}\right)^{1/3}
 \end{aligned}$$

15. (3) Let at any time the position coordinates of boat be (x, y) .

$$y = \frac{5}{6}t^3, \quad x - 1 = ut$$

$$\Rightarrow x - 1 = u \left(\frac{6y}{5}\right)^{1/3}$$

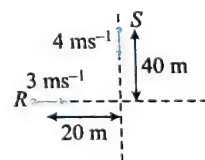
$$16. (1) \quad y = d/2, \text{ drift: } x - 1 = u(3d/5)^{1/3}$$


For Problems 17–19

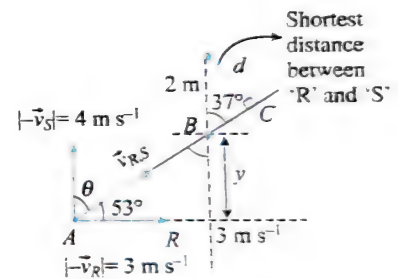
$$\begin{aligned}
 17. (3) \quad \vec{v}_{R,S} &= \vec{v}_R - \vec{v}_S - \vec{v}_R + (-\vec{v}_S) \\
 |\vec{v}_{R,S}| &= \sqrt{3^2 + 4^2} = 5 \text{ m s}^{-1} \\
 \tan \theta \frac{|\vec{v}_R|}{|\vec{v}_S|} &= \frac{3}{4}
 \end{aligned}$$

Hence, $\theta = 37^\circ$

18. (4)



At $t = 0$: Position of Ram and Shyam



$$\tan 53^\circ = \frac{y}{20}$$

$$y = 20 \times \frac{4}{3} = \frac{80}{3} \text{ m}$$

$$\sin 37^\circ = \left(\frac{d}{40 - \frac{80}{3}}\right); \Rightarrow d = \frac{3}{5} \left[\frac{40}{3}\right] = 8 \text{ m}$$

$$19. (1) \quad \tan 37^\circ = \frac{8}{BC}$$

$$\frac{3}{4} = \frac{8}{BC}$$

$$BC = \frac{32}{3} \text{ m}$$

$$\sin 53^\circ = \frac{80}{3AB}, \quad \frac{4}{5} = \frac{80}{3 \times AB}$$

$$AB = \frac{100}{3} \text{ m}$$

$$t = \frac{\left(\frac{32}{3} + \frac{100}{3}\right)}{5} = \frac{132}{15} = 8.8 \text{ s}$$

For Problems 20–22

20. (2), 21. (3), 22. (1)

Sol. Time of flight (T) = 1 s

$$\frac{2u_v}{g} = 1 \text{ s} \Rightarrow u_v = 5 \text{ m s}^{-1}$$

$$\text{Range (R)} = 4 \text{ m}$$

$$\frac{2(u_x)(u_y)}{g} = 4 \Rightarrow u_x = 4 \text{ m s}^{-1}$$

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{41} \text{ m s}^{-1}$$

$$\theta = \tan^{-1} \left(\frac{u_y}{u_x} \right) = \tan^{-1} \left(\frac{5}{4} \right)$$

$$H = \frac{u_y^2}{2g} = \frac{(5)^2}{20} = \frac{5}{4} = 1.25 \text{ m}$$

For Problems 23–25

23. (1) Let us be the velocity of ball A is 2.0 m.

$$2.0 = (u \sin \theta)t - \frac{1}{2}gt^2$$

For bullet C, vertical distance travelled $= u \sin 30^\circ t + \frac{1}{2}gt^2 \dots(i)$
 For bullet C, vertical distance travelled

$$d = u't - \frac{1}{2}gt^2 \dots(ii)$$

From equations (i) and (ii),

$$u \sin 30^\circ t + u't = 1500$$

$$50t + 100t = 1500 \text{ which gives } t = 10 \text{ sec}$$

24. (3) CO = greatest height for bullet A

$$CO = u't - \frac{1}{2}gt^2 = 100(10) - \frac{1}{2}(10)^3 = 500$$

$$AC = \frac{1}{2}(\text{Range for bullet at A})$$

$$AC = u \cos 30^\circ t = 500\sqrt{3}$$

$$\tan \phi = \frac{CO}{AC} = \frac{1}{\sqrt{3}} \Rightarrow \phi = 30^\circ$$

25. (2) If θ is the angle of projection at A,

$$\tan \theta = 2 \tan \phi = 2\sqrt{3}$$

$$\cos \theta = \sqrt{\frac{3}{7}}$$

For bullet at A, $u_A \cos \theta t = 500\sqrt{3}$

$$u_A = \frac{500\sqrt{3}(\sqrt{7})}{\sqrt{3} \times 10} = 50\sqrt{7} \text{ m/s}$$

For Problems 26–28

26. (2) To reach the port B, the x-component of the total velocity must be zero: $v \sin \theta - u = 0$.

$$\text{So } \sin \theta = 1/2 \text{ or } \theta = 30^\circ$$

Take positive x-axis along east and positive y-axis along north.

27. (2) To cross the river the fastest, we need to maximize the y-component of the total velocity is $v_B \cos \theta$. So $\theta = 0$. The boat should head straight to the north.

Take positive x-axis along east and positive y-axis along north.

28. (4) The trip takes time $t = \frac{D}{v}$. The y-component of the total velocity

is u . So the boat is at a distance $u \frac{D}{v} = \frac{D}{2}$ downstream from the port B after crossing.

Take positive x-axis along east and positive y-axis along north.

For Problems 29–33

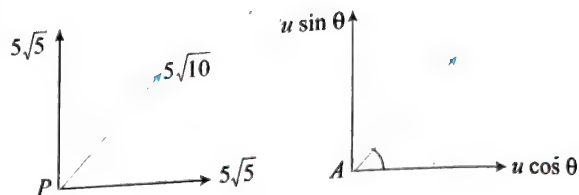
29. (1) Let speed at P = v

$$\therefore \text{Range } PQ = \sqrt{15^2 + 20^2} = 25$$

$$25 = \frac{v^2 \sin^2 90^\circ}{g} = \frac{v^2}{10}$$

$$v = \sqrt{250} = 5\sqrt{10} \text{ m/sec}$$

30. (3)



$$u^2 \sin^2 \theta - 2 \times 10 \times 12.5 = (5\sqrt{5})^2$$

$$u \sin \theta = \sqrt{125 + 250} = \sqrt{375}$$

$$\text{and } u \cos \theta = 5\sqrt{5} = \sqrt{125}$$

$$\tan \theta = \sqrt{\frac{375}{125}} = \sqrt{3} \text{ or } \theta = 60^\circ$$

$$31. (2) u \cos \theta = 5\sqrt{5}$$

$$\text{and } \theta = 60^\circ$$

$$\Rightarrow u = \frac{5\sqrt{5}}{\frac{1}{2}} = 10\sqrt{5} \text{ m/sec}$$

32. (4) At the top of trajectory speed $= u \cos \theta = 5\sqrt{5} \text{ m/sec}$

$$33. (2) \text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{(10\sqrt{5})^2 \sin 120^\circ}{10}$$

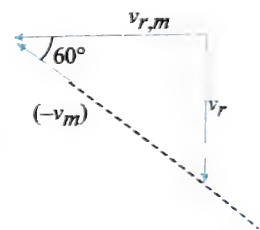
$$= \frac{100 \times 5 \times \sqrt{3}}{2 \times 10} = 25\sqrt{3} \text{ m}$$

For Problems 34 and 35

$$34. (1) \vec{v}_{r,m} = \vec{v}_r - \vec{v}_m = \vec{v}_r + (-\vec{v}_m)$$

$$\cos 60^\circ = \frac{|\vec{v}_{r,m}|}{|\vec{v}_m|} = \frac{|\vec{v}_r|}{6}$$

$$\frac{1}{2} = \frac{|\vec{v}_{r,m}|}{6} \Rightarrow |\vec{v}_{r,m}| = 3 \text{ m/s}$$



$$35. (2) \sin 60^\circ = \frac{|\vec{v}_r|}{|\vec{v}_m|} = \frac{|\vec{v}_r|}{6}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{|\vec{v}_r|}{6} \Rightarrow |\vec{v}_r| = 3\sqrt{3} \text{ m/s}$$

Matrix Match Type

1. i $\rightarrow$ b, d; ii $\rightarrow$ c; iii $\rightarrow$ a; iv $\rightarrow$ c

Distance will be minimum because man will reach from A to B directly.

$$\sin \theta = \frac{v_\omega}{v_{m\omega}} \text{ and } \vec{v}_m \perp \vec{v}_\omega$$

Hence, i $\rightarrow$ b, d

Time taken is minimum if $\vec{v}_{m\omega}$ is perpendicular to $\vec{v}_\omega$.

Hence, ii $\rightarrow$ c, iv $\rightarrow$ c

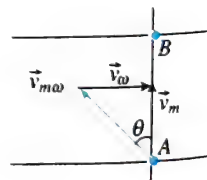
If $\vec{v}_{m\omega} < \vec{v}_\omega$, then drift or distance is shortest if $\sin \theta = \frac{v_{m\omega}}{v_\omega}$.
 Hence, iii $\rightarrow$ a

2. i $\rightarrow$ a; ii $\rightarrow$ b; iii $\rightarrow$ c; iv $\rightarrow$ d

i $\rightarrow$ a Range is maximum, when the angle of projection is 45° .

$$H = \frac{v^2}{2g} \sin^2 45^\circ = \frac{v^2}{4g} \dots(i)$$

Velocity, at half of the maximum height is v .



$$v'^2 = v^2 \sin^2 45^\circ - 2g \frac{H}{2} = \frac{v^2}{2} - \frac{v^2}{4} \Rightarrow v' = \frac{\sqrt{3}v}{2}$$

ii $\rightarrow$ b Velocity at the maximum height

$$v' = v \cos 45^\circ \Rightarrow v' = \frac{v}{\sqrt{2}}$$

[Because vertical component of velocity is zero at the highest point]

iii $\rightarrow$ c Projection velocity

At projection point, $\vec{v}_i = v \cos 45^\circ \hat{i} + v \sin 45^\circ \hat{j}$

At the point, when the body strikes the ground

$$\vec{v}_f = v \cos 45^\circ \hat{i} - v \sin 45^\circ \hat{j}$$

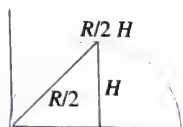
$$\Delta \vec{v} = \vec{v}_f + (-\vec{v}_i) = 2 v \sin 45^\circ (-\hat{j})$$

$$|\Delta \vec{v}| = 2 v \sin 45^\circ = v\sqrt{2}$$

iv $\rightarrow$ d Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

$$\text{Displacement} = \sqrt{\left(\frac{R}{2}\right)^2 + H^2}$$

$$\vec{v}_{av} = \frac{\sqrt{\frac{R^2}{4} + H^2}}{\frac{v \sin \theta}{g}} = \frac{\sqrt{R^2 + 4H^2}}{2v \sin \theta}$$



$$v_{av} = \frac{\sqrt{\left(\frac{v^2}{g}\right)^2 + 4\left(\frac{v^2}{4g}\right)^2}}{\frac{\sqrt{2}v}{g}} = \frac{\frac{v^2}{g} \sqrt{1 + \frac{1}{4}}}{\frac{v\sqrt{2}}{g}} = \frac{v^2 \sqrt{5}}{2v\sqrt{2}} = \frac{v}{2} \sqrt{\frac{5}{2}}$$

3. i $\rightarrow$ a, c; ii $\rightarrow$ b, d; iii $\rightarrow$ a, c; iv $\rightarrow$ a, b, c, d

i. In uniform circular motion, acceleration and velocity are perpendicular to each other, but in non-uniform circular motion, angle between velocity and acceleration lies between zero and $\pi/2$.

ii. In straight line motion, acceleration vector and velocity vector are collinear to each other, i.e., angle between them is either zero or 180° .

iii. In projectile motion, angle θ between velocity and acceleration can vary from $0 < \theta < \pi$.

iv. In space, angle between velocity and acceleration may be $0 \leq \theta \leq \pi$.

4. i $\rightarrow$ d, ii $\rightarrow$ d, iii $\rightarrow$ c, iv $\rightarrow$ a

Here, maximum height for all the particles is same.

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u_y^2}{2g}$$

So, all three particles have same u_y .

$$T = \frac{2u \sin \theta}{g} = \frac{2u_y}{g}$$

So all three particles have the same time period.

Range (R) is maximum for C.

R = horizontal component of velocity $\times T$

So, horizontal component of velocity is greater for C.

$$u = \sqrt{u_x^2 + u_y^2}$$

u_x is least for A and u_y are same for all, so u is least for A.

5. i $\rightarrow$ a, ii $\rightarrow$ c, iii $\rightarrow$ d, iv $\rightarrow$ b

$$y = Px - Qx^2$$

Equation of trajectory of projectile motion is given by

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

On comparing, $\tan \theta = P$

So iv $\rightarrow$ b.

$$\frac{g}{2u^2 \cos^2 \theta} = Q; u \cos \theta = \sqrt{\frac{g}{2Q}}$$

$$\begin{aligned} \text{Range } R &= \frac{2u^2}{g} \sin \theta \cos \theta = \frac{2u^2 \cos^2 \theta}{g} \tan \theta \\ &= \frac{2}{g} \frac{g}{2Q} P = \frac{P}{Q} \end{aligned}$$

So i $\rightarrow$ a.

$$\text{Maximum height } H = \frac{u^2}{2g} \sin^2 \theta$$

$$= \frac{1}{2g} \frac{g}{2Q} \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{2Q} P^2 = \frac{P^2}{4Q^2}$$

So ii $\rightarrow$ c.

$$\text{Time of flight } T = \frac{2u \sin \theta}{g} = \frac{2}{g} \sqrt{\frac{g}{2Q}} P = \sqrt{\frac{2}{Qg}} P$$

So iii $\rightarrow$ d.

6. i $\rightarrow$ c; ii $\rightarrow$ d; iii $\rightarrow$ a; iv $\rightarrow$ b

Initial velocity $\vec{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$

$$\vec{u} = 60 \frac{\sqrt{3}}{2} \hat{i} + 60 \left(\frac{1}{2}\right) \hat{j} = 30\sqrt{3} \hat{i} + 30 \hat{j}$$

Velocity after 3 s

$$\begin{aligned} \vec{v} &= u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j} \\ &= 30\sqrt{3} \hat{i} (30 - 10 \times 3) \hat{j} = 30\sqrt{3} \hat{i} \end{aligned}$$

Displacement after 2 s:

$$\begin{aligned} \vec{s} &= u \cos \theta t \hat{i} + \left(u \sin \theta t - \frac{1}{2} gt^2\right) \hat{j} \\ &= 30\sqrt{3} \times 2 \hat{i} + \left(30 \times 2 - \frac{1}{2} \times 10 \times (2)^2\right) \hat{j} \\ &= 60\sqrt{3} \hat{i} + 40 \hat{j} \end{aligned}$$

Velocity after 2 s $\vec{v} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$

$$\begin{aligned} &= 30\sqrt{3} \hat{i} + (30 - 10 \times 2) \hat{j} \\ &= 30\sqrt{3} \hat{i} + 10 \hat{j} \end{aligned}$$

7. i $\rightarrow$ a, c; ii $\rightarrow$ c, iii $\rightarrow$ d

Considering y-axis along the inclined plane time of flight

$$= \frac{2u \sin 37^\circ}{g \sin 37^\circ} = 2s$$

speed is minimum at $t = 1$ s

velocity makes an angle 37° with x-axis at $t = 2$ s

$$y = 10 \sin 37^\circ t - \frac{1}{2} g \sin 37^\circ t^2$$

substituting $y = 2.25$ we get $t = 0.5$ and 1.5 s

8. (1)

(i) Initial velocity $\vec{v}_i = 5\sqrt{3} \hat{i} + 5 \hat{j}$

Final velocity (for half time of flight) $\vec{v}_f = 5\sqrt{3} \hat{i}$

$$\Rightarrow \text{Average velocity } \vec{v}_a = \frac{\vec{v}_i + \vec{v}_f}{2} = 5\sqrt{3} \hat{i} + \frac{5}{2} \hat{j}$$

$$\Rightarrow \text{Magnitude} = \frac{5}{2}\sqrt{13} \text{ m/s, and Angle} = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

(ii) Initial velocity $\vec{v}_i = 5\sqrt{3} \hat{i} + 5 \hat{j}$

Acceleration $\vec{a} = -10 \hat{j}$

Velocity after time t $\vec{v}_f = 5\sqrt{3} \hat{i} + (5 - 10t) \hat{j}$

Since angle between acceleration vector and velocity vector after time t is $\pi/2$

$$\Rightarrow \vec{a} \cdot \vec{v} = 0 = 5 - 10t \Rightarrow t = \frac{1}{2} \text{ sec}$$

(iii) Horizontal tangential

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{100 \times \sqrt{3}}{20} = 5\sqrt{3} \text{ m}$$

(iv) Initial momentum $\vec{p}_i = 15 \hat{i} + 5\sqrt{3} \hat{j}$

Final momentum (for half time of flight) $\vec{p}_f = 15 \hat{i}$

$$\Rightarrow \text{Change in linear momentum } \Delta \vec{p} = \vec{p}_f - \vec{p}_i = -5\sqrt{3} \hat{j}$$

$$\Rightarrow \Delta p = 5\sqrt{3} \text{ N-s}$$

9. (2) Calculate the velocity of the two projectiles in vector form.

$$\vec{V}_A = V_{Ax} \hat{i} + V_{Ay} \hat{j}; \vec{V}_B = V_{Bx} \hat{i} + V_{By} \hat{j}$$

$$|\vec{V}_B - \vec{V}_A| = \sqrt{(V_{Bx} - V_{Ax})^2 + (V_{By} - V_{Ay})^2}$$

Calculate the coordinates of the projectiles after 2 seconds taking point of projection as origin.

$$x = u_x t, y = u_y t + \frac{1}{2} a_y t^2 \quad \vec{r}_A = u_{Ax} t \hat{i} + \left(u_{Ay} t + \frac{1}{2} a_{Ay} t^2\right) \hat{j}$$

$$\vec{r}_B = u_{Bx} t \hat{i} + \left(u_{By} t + \frac{1}{2} a_{By} t^2\right) \hat{j}$$

Compute $|\vec{r}_B - \vec{r}_A|$, it gives the distance between the particles.

10. (3) Maximum height from ground of projectile is

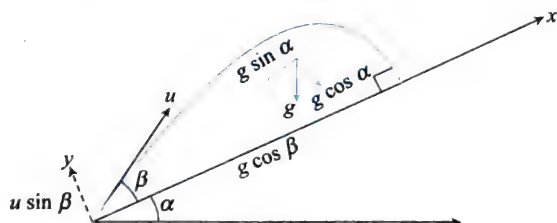
$$\frac{u^2 \sin^2(\alpha + \beta)}{2g}$$

Maximum height from inclined plane is

$$\frac{u_{\perp}^2}{2g_{\perp}}$$

$$u_{\perp} = u \sin \beta \text{ and } g_{\perp} = g \cos \alpha.$$

$$\therefore h \text{ from inclined plane} = \frac{u^2 \sin^2 \beta}{2g \cos \alpha}$$



Acceleration of the projectile is always g downward and horizontal component will obviously be zero.

11. (1) The initial velocity of A relative to B is

$$\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = (8\hat{i} - 8\hat{j}) \text{ m/s}$$

$$\therefore u_{AB} = 8\sqrt{2} \text{ m/s}$$

Acceleration of A relative to B is

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = (-2\hat{i} + 2\hat{j}) \text{ m/s}^2$$

$$\therefore a_{AB} = 2\sqrt{2} \text{ m/s}^2$$

Since B observes initial velocity and constant acceleration of A in opposite directions, Hence, B observes A moving along a straight line.

From frame of B

$$\text{Hence time when } v_{AB} = 0 \text{ is } t = \frac{u_{AB}}{a_{AB}} = 4 \text{ sec}$$

The distance between A and B when $v_{AB} = 0$ is

$$S = \frac{u_{AB}^2}{2a_{AB}} = 16\sqrt{2} \text{ m}$$

The time when both are at same position is

$$T = \frac{2u_{AB}}{a_{AB}} = 8 \text{ sec}$$

Magnitude of relative velocity when they are at same position in

$$u_{AB} = 8\sqrt{2} \text{ m/s}$$

12. (2) Equation of path is given as

$$y = ax - bx^2$$

Comparing it with standard equation of projectile;

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\tan \theta = a, \quad \frac{g}{2u^2 \cos^2 \theta} = b$$

Horizontal component of velocity

$$= u \cos \theta = \sqrt{\frac{g}{2b}}$$

$$\text{Time of flight } T = \frac{2u \sin \theta}{g} = \frac{2(u \cos \theta) \tan \theta}{g}$$

$$= \frac{2 \left(\sqrt{\frac{g}{2b}} \right) a}{g} = \sqrt{\frac{2a^2}{bg}}$$

$$\text{Maximum height } H = \frac{u^2 \sin^2 \theta}{2g} = \frac{[u \cos \theta \cdot \tan \theta]^2}{2g}$$

$$= \frac{\left[\sqrt{\frac{g}{2b}} \cdot a \right]^2}{2g} = \frac{a^2}{4b}$$

Horizontal range

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2(u \sin \theta)(u \cos \theta)}{g} = \frac{2 \left[\sqrt{\frac{g}{2b}} \cdot a \right] \left[\sqrt{\frac{g}{2b}} \right]}{g} = \frac{a}{b}$$

Numerical Value Type

1. (4) Initial velocity

$$u = u \cos \theta \hat{i} + u \sin \theta \hat{j}$$

Velocity after time t is

$$v = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$$

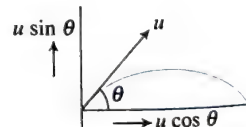
Since u and v are perpendicular to each other, therefore,

$$\vec{u} \cdot \vec{v} = 0$$

$$(u \cos \theta \hat{i} + u \sin \theta \hat{j}) \times [u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}] = 0$$

$$u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta gt = 0$$

$$t = \frac{u}{g \sin \theta} = \frac{20}{10 \times \frac{1}{2}} = 4 \text{ s}$$



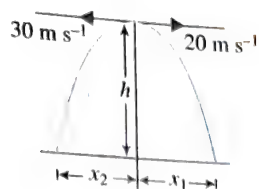
$$1. (2) 80 = \frac{1}{2} \times 10$$

$$t^2 = \frac{80}{5} \Rightarrow t = 4$$

$$x_1 = 20 \times 4 = 80$$

$$x_2 = 30 \times 4 = 120$$

$$\text{Separation } x_1 + x_2 = 80 + 120 = 200 \text{ m}$$



$$1. (2) AB = 2R \cos \theta$$

$$AB = \frac{1}{2} g \cos \theta t^2 \Rightarrow 2R \cos \theta = \frac{1}{2} g \cos \theta t^2$$

$$2\sqrt{\frac{R}{g}} = t \Rightarrow 2\sqrt{\frac{10}{10}} = t = 2 \text{ s}$$

$$4. (2) y = u \sin \theta t - \frac{1}{2} g t^2$$

$$15 = 52 \times \frac{5}{13} t - \frac{1}{2} \times 10 t^2$$

$$5t^2 - 20t + 15 = 0$$

$$t^2 - 4t + 3 = 0$$

$$t^2 - 3t + t + 3 = 0 \Rightarrow t(t-3) - 1(t-3) = 0$$

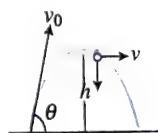
$$t_1 = 1 \text{ s} \Rightarrow t_2 = 3 \text{ s}$$

$$\text{Thus, } t_2 - t_1 = 3 - 1 = 2 \text{ s}$$

5. (9) Radius of curvature at the highest point

$$r = \frac{v}{g} \Rightarrow r = \frac{(v_0 \cos \theta)^2}{g}$$

$$v_0 \cos \theta = \sqrt{gr} \quad \dots(i)$$



$$\text{Maximum height, } h = \frac{v_0^2 \sin^2 \theta}{g} \Rightarrow v_0 \sin \theta = \sqrt{gh} \quad \dots(ii)$$

By using Eqs (i) and (ii), we get

$$v_0^2 \sin^2 \theta + v_0^2 \cos^2 \theta = gh + gr = g(h+r)$$

$$\Rightarrow v_0 = \sqrt{g(h+r)} = \sqrt{10(5+3)} = 9 \text{ m s}^{-1}$$

6. (2) Let the velocity of car be u when the ball is thrown.

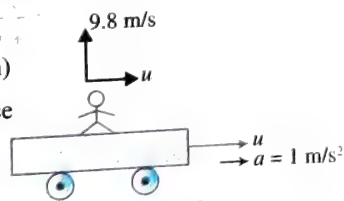
Initial velocity of car = Horizontal velocity of ball.

Distance travelled by ball B,

$$S_b = ut \text{ (in horizontal direction)}$$

Car has travelled extra distance

$$= S_c = S_b = \frac{1}{2} at^2$$



Ball can be considered a projectile having $\theta = 90^\circ$

$$t = \frac{2u \sin \theta}{g} = \frac{2 \times 9.8}{9.8} = 2 \text{ s}$$

$$S_c - S_b = \frac{1}{2} at^2 = \frac{1}{2} (1) 2^2 = 2 \text{ m}$$

Hence, the ball will drop 2 m behind the boy.

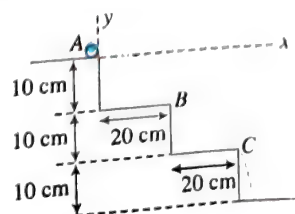
7. (2) At minimum velocity, it will move just touching point E reaching the ground. A is the origin of reference co-ordinate.

If u is the minimum speed,

$$x = 40, y = 20, \theta = 0^\circ$$

$$y = \tan \theta - g \frac{x^2 \sec^2 \theta}{2u^2}$$

$$\text{where } g = 10 \text{ ms}^{-2} = 1000 \text{ cm s}^{-2}$$



$$-20 = -\frac{800000}{2u^2}$$

$$u = 200 \text{ cm s}^{-1} = 2 \text{ m s}^{-1}$$

$$8. (2) (a) T = \frac{2u \sin(\alpha - \beta)}{g \cos \beta}$$

Now we shall consider the motion of particle along OA.



(b) When the particle strikes the plane horizontally

$$\text{In this case, } t = \frac{2u \sin(\alpha - \beta)}{g \cos \beta}$$

$$\text{and } 0 = u \sin \alpha - gt$$

$$u \sin \alpha = g \left[\frac{2u \sin(\alpha - \beta)}{g \cos \beta} \right]$$

$$2 \tan \beta = \tan \alpha \Rightarrow \tan \alpha : \tan \beta = 2$$

9. (7) At $t = T/6$, particle will travel only $1/6$ of the circle.

Average speed

$$= \frac{\text{Arc } AB}{\text{Time}} = \frac{(2\pi R)/6}{T/6} = \frac{2\pi R}{T}$$

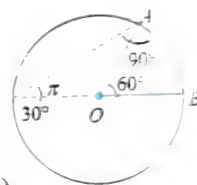
Average velocity

$$= \frac{\text{Chord } AB}{\text{Time}} = \frac{2R \sin 30^\circ}{T/6} = \frac{6R}{T}$$

Difference

$$= \frac{R}{T} \left[\frac{44}{7} - 6 \right] = \frac{22}{7T} = 2 \text{ ms}^{-1} \text{ (given)}$$

$$R = 7T = 7(1) = 7 \text{ m}$$



$$10. (5) v_x = \frac{dx}{dt} = 3 \text{ and } v_y = \frac{dy}{dt} = 4 - 10t = 4 - 10(0) = 4$$

$$v = [v_x^2 + v_y^2]^{1/2} = [3^2 + 4^2]^{1/2} = 5 \text{ ms}^{-1}$$

$$11. (5) v_x = u_x + a_x t$$

$$\Rightarrow 0 = u \cos 30^\circ - g \sin 30^\circ t$$

$$\Rightarrow t = \frac{u \sqrt{3}}{g}$$

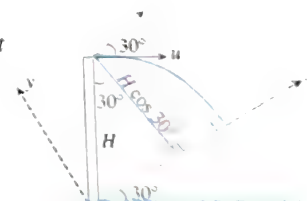
$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow -H \cos 30^\circ = -u \sin 30^\circ t$$

$$-\frac{1}{2} g \cos 30^\circ t^2$$

$$\Rightarrow -H \frac{\sqrt{3}}{2} = \frac{-u u \sqrt{3}}{2g} - \frac{1}{2} g \frac{\sqrt{3}}{2} \frac{u^2 3}{g^2}$$

$$\Rightarrow u = \sqrt{2gH/5} = \sqrt{2 \times 10 \times 6.25} = 5 \text{ ms}^{-1}$$



$$12. (9) v_x = v \cos 60^\circ = 2\sqrt{15} \text{ ms}^{-1}$$

$$\therefore v = 4\sqrt{15} \text{ ms}^{-1}$$

$$\therefore v_y = v \sin 60^\circ = 4\sqrt{15} \frac{\sqrt{3}}{2} = 2\sqrt{45} \text{ ms}^{-1}$$

$$H_{\max} = \frac{v_y^2}{2g} = \frac{4 \times 45}{20} = 9 \text{ m}$$

13. (3) The horizontal and vertical components of the velocity are the same, let it be

$$u = v \cos 45^\circ$$

From A to B:

$$1 = \frac{u^2}{2g} \Rightarrow u^2 = 2g$$

$$\text{At B: } d = ut_1 \Rightarrow t_1 = d/u$$

$$1 = ut_1 - \frac{g}{2}t_1^2 = u \frac{d}{u} - \frac{g}{2} \frac{d^2}{u^2}$$

$$\Rightarrow 1 = d - \frac{g}{2} \frac{d^2}{u^2}$$

$$\Rightarrow 1 = d - \frac{gd^2}{4g} \Rightarrow 4 = 4d - d^2$$

$$\Rightarrow d^2 - 4d + 4 = 0 \Rightarrow d = 2 \text{ m}$$

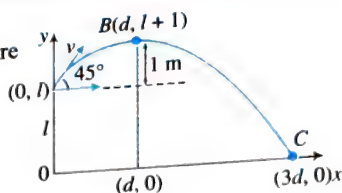
$$\text{At C: } 3d = ut_2 \Rightarrow t_2 = \frac{3d}{u}$$

$$-1 = ut_2 - \frac{1}{2}gt_2^2$$

$$= u \cdot \frac{3d}{u} - \frac{g}{2} \frac{9d^2}{u^2} = 3d - \frac{9gd^2}{4g}$$

$$= 3d - \frac{9d^2}{4} = 3 \times 2 - \frac{9}{4} \times 4 = 6 - 9 = -3$$

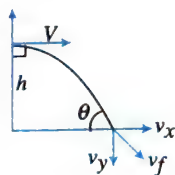
$$\Rightarrow l = 3 \text{ m}$$



14. (5) Minimum θ implies minimum V_{fy} and maximum V_{fx} . In order to have the aforementioned situation, the rock has to be launched horizontally.

$$v_y = \sqrt{2gh}$$

$$\tan 30^\circ = \frac{v_y}{v} = \frac{\sqrt{2 \times 10 \times h}}{10\sqrt{3}}, h = 5 \text{ m}$$



15. (4) Let time of flight is T ,

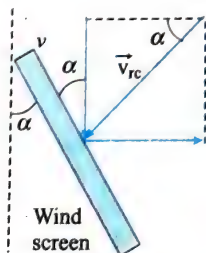
$$T = \frac{2v_0 \sin \theta}{g \cos \theta} = \frac{2 \times 10 \times 3/5}{10 \times 4/5} = \frac{3}{2} \text{ s}$$

Let x is the distance travelled by cart in time T .

$$x + 6 = v_0 \cos \theta T$$

$$\Rightarrow \frac{3}{2} + 6 = 10 \times \frac{4}{5} \times \frac{3}{2} \Rightarrow v = 4 \text{ m/s}$$

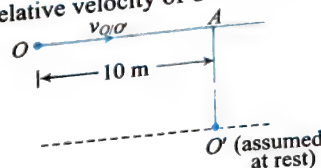
16. (3) $v_r = 6 \text{ m/s}$, $v_c = 2 \text{ m/s}$, $\vec{v}_{rc} = \vec{v}_r - \vec{v}_c$



where $\vec{v}_{rc}$ is relative velocity of rain w.r.t. car. It should be perpendicular to wind screen.

$$\text{So } \tan \alpha = \frac{v_r}{v_c} = \frac{6}{2} = 3$$

17. (1) Let us find relative velocity of O w.r.t. O' .



$$\vec{v}_{O/O'} = \vec{v}_O - \vec{v}_{O'} = (10 \cos 60^\circ \hat{i} + 10 \sin 60^\circ \hat{j}) - [10 \cos 60^\circ (-\hat{i}) + 10 \sin 60^\circ \hat{j}] = 10 \hat{i}$$

This is along horizontal direction.

If we assume O' to be at rest, then O will move along OA w.r.t. O' and minimum separation will be when particle O is at A .

For this, relative displacement travelled = $OA = 10 \text{ m}$

$$\text{Time taken} = \frac{10}{v_{O/O'}} = \frac{10}{10} = 1 \text{ s}$$

18. (2) Instantaneous speed of particle is

$$v = \sqrt{v_x^2 + v_y^2} \text{ or } v^2 = v_x^2 + v_y^2 \quad \dots(1)$$

Differentiating (1) with respect to time we get:

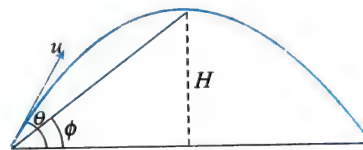
$$2v \cdot \frac{dv}{dt} = 2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}$$

$$\text{or } \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{v}$$

$$\text{or rate of change of speed } \frac{dv}{dt} = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}}$$

$$\text{substituting the given values, we get } \frac{dv}{dt} = 2 \text{ m/s}^2$$

19. (2)



$$\tan \phi = \frac{2H}{R} = \frac{2u^2 \sin^2 \theta / 2g}{2u^2 \sin \theta \cos \theta / g}$$

$$\Rightarrow \tan \phi = \frac{\tan \theta}{2} \Rightarrow \frac{\tan \theta}{\tan \phi} = 2$$

20. (6)

$$\text{Time of flight: } T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 20}{10}} = 2 \text{ s}$$

$$\text{Range: } D + (D - x_0) = v_0 T$$

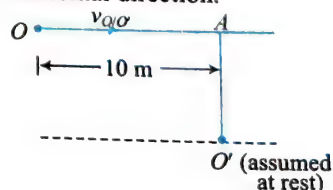
$$\Rightarrow 16 - x_0 = 5 \times 2 \Rightarrow x_0 = 6 \text{ m}$$

21. (1) Let us find relative velocity of O w.r.t. O' .

$$\vec{v}_{O/O'} = \vec{v}_O - \vec{v}_{O'} = (10 \cos 60^\circ \hat{i} + 10 \sin 60^\circ \hat{j})$$

$$- [10 \cos 60^\circ (-\hat{i}) + 10 \sin 60^\circ \hat{j}] = 10 \hat{i}$$

This is along horizontal direction.

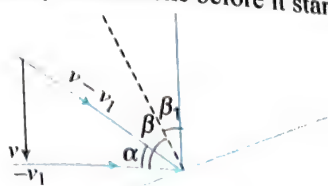


If we assume O' to be at rest, then O will move along OA w.r.t. O' and minimum separation will be when particle O is at A .

For this, relative displacement travelled = $OA = 10 \text{ m}$

$$\text{Time taken} = \frac{10}{v_{O/O'}} = \frac{10}{10} = 1 \text{ s}$$

Let us suppose that hail falls along the vertical at a velocity v . In the reference frame fixed to the motorcar the angle of incidence of hailstone on the windscreen is equal to the angle of reflection. The velocity of hailstone before it starts is $\vec{v} - \vec{v}_1$.



Since hailstones are turned vertically (from view point of driver) after reflection, the angle of reflection, the angle of incidence is equal to β_1 . Consequently $\alpha + 2\beta_1 = \frac{\pi}{2}$ and $\tan \alpha = \frac{v}{v_1}$.

$$\text{Hence, } \tan \alpha = \tan \left(\frac{\pi}{2} - 2\beta_1 \right) = \cot 2\beta_1$$

$$\frac{v}{v_1} = \cot 2\beta_1$$

Similarly for the other car, $\frac{v}{v_2} = \cot 2\beta_2$

$$\frac{v_1}{v_2} = \frac{\cot 2\beta_2}{\cot 2\beta_1} = \frac{\cot 30^\circ}{\cot 60^\circ} = 3 \text{ or } v_1 : v_2 = 3 : 1$$

Archives

Main

Single Correct Answer Type

1. (C) Total area around the fountain that gets wet is equal to area of a circle whose radius is equal to maximum range of sprinkle water.

$$R_{\max} = \frac{v^2 \sin 90^\circ}{g} = \frac{v^2}{g}$$

$$\text{Area} = \pi (R_{\max})^2 = \frac{\pi v^4}{g^2}$$

2. (1) Given initial velocity of the particle, $u = \hat{i} + 2\hat{j}$

$$\therefore u_x = 1 \text{ m/s and } u_y = 2 \text{ m/s}$$

$$\text{Also } x = u_x t \therefore x = t \quad \dots(i)$$

$$\text{and } y = u_y t - \frac{1}{2} g t^2$$

$$\text{and } y = 2t - \frac{1}{2} \times 10 \times t^2 = 2t - 5t^2 \quad \dots(ii)$$

From (i) and (ii)

$$\text{Equation of trajectory is } y = 2x - 5x^2.$$

3. (1) When a particle is thrown vertically upwards with a speed u , at highest point the velocity of the particle will be zero.

Using $v = u + at$, we get

$$0 = u - gt \Rightarrow t = \frac{u}{g} \quad \dots(i)$$

The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. Let $t_1 = nt$.

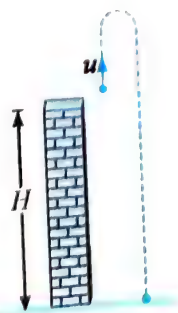
Using, $s = ut_1 + \frac{1}{2} at_1^2$, we get

$$\Rightarrow -H = u(nt) - \frac{1}{2} g (nt)^2$$

$$\Rightarrow -H = u \times n \left(\frac{u}{g} \right) - \frac{1}{2} g n^2 \left(\frac{u}{g} \right)^2 \text{ [using (i)]}$$

$$\Rightarrow 2gH = 2nu^2 - n^2 u^2$$

$$\Rightarrow 2gH = nu^2 (n - 2)$$



4. (1) Time to reach the maximum height, $t_1 = \frac{u}{g}$
 If t_2 be the time taken to hit the ground, then $-H = ut_2 - \frac{1}{2} g t_2^2$
 But $t_2 = nt_1$ (given)
 $\Rightarrow -H = u \frac{nu}{g} - \frac{1}{2} g \frac{n^2 u^2}{g}$
 $\Rightarrow 2gH = nu^2 (n - 2)$

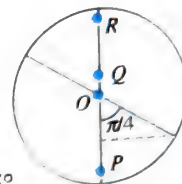
JEE Advanced

Single Correct Answer Type

$$1. (3) \text{ At } t = \frac{1}{8} \times \frac{2\pi}{\omega} = \frac{\pi}{4\omega}$$

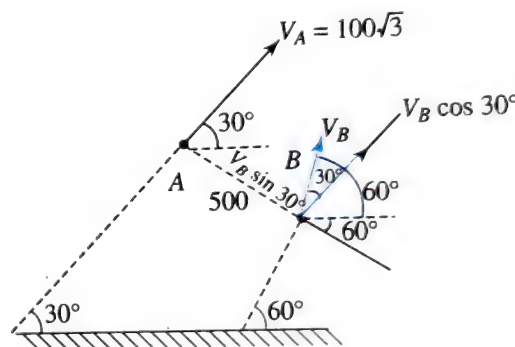
$$x\text{-coordinate of } P = \omega R \left(\frac{\pi}{4\omega} \right) = \frac{\pi R}{4} > R \cos 45^\circ$$

Therefore, both particles P and Q land in unshaded region.



Numerical Value Type

1. (5)



$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 10 \times \sqrt{3}}{10 \times 2} = \sqrt{3} \text{ s}$$

$$R = u \cos \theta T - \frac{1}{2} a T^2$$

$$1.15 = 10 \times \frac{1}{2} \sqrt{3} - \frac{1}{2} a (\sqrt{3})^2$$

$$\frac{3}{2} a = 5\sqrt{3} - 1.15 = 8.65 - 1.15 = 7.5$$

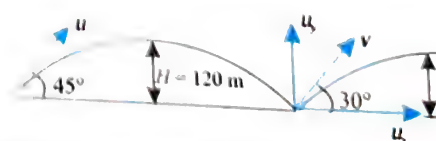
$$a = 7.5 \times \frac{2}{3} = 5 \text{ ms}^{-2}$$

2. (5) For relative motion perpendicular to line of motion of A
 $V_A = 100\sqrt{3} = V_B \cos 30^\circ$

$$\Rightarrow V_B = 200 \text{ m/s}$$

$$t_0 = \frac{500}{V_B \sin 30^\circ} = \frac{500}{200 \times \frac{1}{2}} = 5 \text{ sec}$$

3. (30) Initial kinetic energy of the ball, $K_i = \frac{1}{2} mu^2$



The ratio of the components of the ball velocity just after bounce,

$$\frac{u_v}{u_v} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{or } \sqrt{3} u_v = u_x$$

...(i)

$$H = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} = 120\text{m}$$

Final kinetic energy of the ball,

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}m(u_x^2 + u_y^2)$$

Given, $K_f = \frac{1}{2}K_i$

$$\Rightarrow \frac{1}{2}m(u_x^2 + u_y^2) = \frac{1}{2} \times \frac{1}{2} \times mu^2$$

$$\Rightarrow u_x^2 + u_y^2 = \frac{u^2}{2}$$

$$\text{From (i) and (ii), } 3u_y^2 + u_y^2 = \frac{u^2}{2} \Rightarrow u_y^2 = \frac{u^2}{8}$$

Hence maximum height reached by ball after bounce,

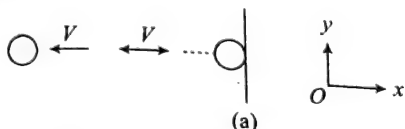
$$h = \frac{u_y^2}{2g} = \frac{u^2}{16g} = \frac{1}{4} \left(\frac{u^2}{4g} \right) = \frac{H}{4} = \frac{120}{4} = 30\text{m}$$

Chapter 6

Concept Application Exercises

Exercise 6.1

1. (a) Force on a body acts in a direction of change in its momentum. In both cases change in momentum of ball is along XO . Hence, in both the cases force will be along x -direction on the wall.
- (b) Case (a)



Momentum of the ball before reflection = mv (along OX)

Momentum after reflection = $-mv$ (along XO)

Now,

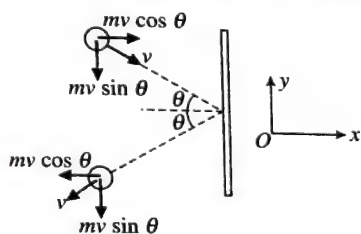
I = change in momentum of ball

$$= -mv - mv = -2mv \text{ (along } OX) = 2mv \text{ (along } x)$$

Hence, impulse on wall will be $2mv$ (along ox)

Case (b)

Similarly $I' = -mv \cos \theta - mv \cos \theta = -2mv \cos \theta$



$$\text{Ratio} = \frac{I}{I'} = \frac{2mv}{2mv \cos \theta}; \frac{I}{I'} = \frac{1}{\cos \theta}$$

2. Just before $t = 2$ s, the velocity of the particle is

$$u = \frac{2-0}{2-0} = 1 \text{ cm s}^{-1} = 0.01 \text{ m s}^{-1}$$

Just after $t = 2$ s, the velocity of the particle is

$$v = \frac{0-2}{4-2} = -1 \text{ cm s}^{-1} = -0.01 \text{ m s}^{-1}$$

The magnitude of impulse, $J = |m(v - u)|$

$$= |0.04(-0.01 - 0.01)| = 8 \times 10^{-4} \text{ N s}$$

The given $x-t$ graph may represent the repeated rebounding of a particle between two elastic walls at $x = 0$ and $x = 2$ cm. The particle will get an impulse of 8×10^{-4} N s after every 2 s.

3. Velocity of the ball just before collision: $v^2 = 0 + 2gh$

$$v = v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m s}^{-1}$$

Therefore, momentum of the ball before collision,

$$\vec{P}_i = m\vec{v}_i$$

$$= 0.050 \times (-10) \hat{j} \text{ N s} = -0.50 \hat{j} \text{ N s}$$

Velocity of the ball just after collision,

$$v_f = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.25} = 5.0 \text{ m s}^{-1}$$

Therefore, momentum of the ball just after collision,

$$\vec{P}_f = m\vec{v}_f = 0.050 \times (5.0 \hat{j}) = 0.25 \hat{j} \text{ N s}$$

Now impulse imparted by the ground on the ball

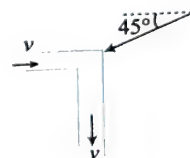
$$\vec{I} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i = 0.25 \hat{j} - (-0.50 \hat{j}) = 0.75 \hat{j} \text{ N s}$$

$$\text{Required force, } F = \frac{\Delta P}{\Delta t} = \frac{0.75}{0.1} = 7.5 \text{ N}$$

4. Impulse, $I = 2mu \cos 30^\circ$

$$= 2 \times 3 \times 10 \times \frac{\sqrt{3}}{2} = 30\sqrt{3} \text{ N s}$$

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{30\sqrt{3}}{0.2} = 150\sqrt{3} \text{ N}$$



5. (a) The impulse is to the right and equal to the area under the $F-t$ graph

$$I = \left\{ \frac{5+1}{2} \right\} \times 4 = 12 \text{ N s}$$

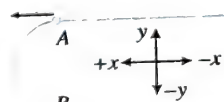
$$(b) m\vec{v}_i + \vec{F}t = m\vec{v}_f$$

$$\Rightarrow 2.5 \times 0 + 12 = (2.5)v \Rightarrow v = 4.8 \text{ m s}^{-1}$$

$$(c) \text{ From the same equation,}$$

$$\Rightarrow 2.5 \times (-2) + 12 = (2.5)v \Rightarrow v = 2.8 \text{ m s}^{-1}$$

$$(d) F_{\text{avg}} \Delta t = 12.0 \Rightarrow F_{\text{avg}} = 2.40 \text{ N}$$



6. Force applied by hail stone,

$$|F| = \frac{\Delta p}{\Delta t} = \left| \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} \right| = \left| \frac{0 - m\vec{v}_i}{\Delta t} \right| = \frac{mv_i}{\Delta t}$$

$$|F| = \frac{2000 \times 10 \times 10 \times \frac{4}{3} \pi \left(\frac{1}{2 \times 100} \right)^3 \times 900 \times 20}{1} = 1900 \text{ N}$$

7. Choosing the positive X - Y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$.

Therefore, the magnitude of the change in momentum between A and B is

$$\Delta \vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$$

i.e. $\Delta p = 2mv$ along positive X -axis.

The time interval taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi \cdot d / 2}{v} = \frac{\pi d}{2v}$$

Therefore, the average force exerted by the bead on the wire is

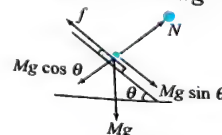
$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{\pi d}{2v}} = \frac{4mv^2}{\pi d}$$

8. Force exerted by the air on the wall is

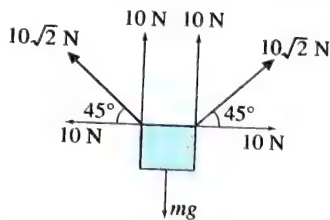
$$Av^2 \rho = 108 \times \left(\frac{100 \times 1000}{3600} \right)^2 \times 1.2 \approx 10^5 \text{ N}$$

Exercise 6.2

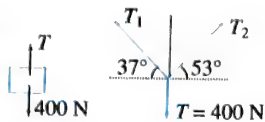
- Incorrect. F is not acting on M .
- No force is acting in horizontal direction on m , as there is no friction between blocks. No relative motion of m . Acceleration of m is zero.
- Weight shown by machine is $N = Mg \cos \theta$. So the mass shown by the machine will be $N/g = M \cos \theta$



4. $10 + 10 = mg \Rightarrow m = 2 \text{ kg}$



5. From the FBD of the block it is clear that tension in lower cord is 400 N. Figure shows the junction where the three cords meet.



As the system is in equilibrium, net sum of all the forces at the junction must be zero. In horizontal direction, we get

$$\cos 53^\circ T_2 - \cos 37^\circ T_1 = 0 \quad \dots(i)$$

In vertical direction, we get

$$\sin 37^\circ T_1 + \sin 53^\circ T_2 - 400 = 0 \quad \dots(ii)$$

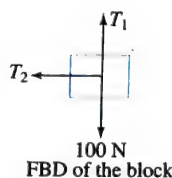
On solving the above equations, we get

$$T_1 = 240 \text{ N and } T_2 = 320 \text{ N}$$

6. A horizontal string cannot balance a vertical weight.

$$\sum F_y = 0 \Rightarrow T_1 = 100 \text{ N,}$$

$$\sum F_x = 0 \Rightarrow T_2 = 0$$



7. The free body diagram is drawn so that only the string T_4 is cut, as shown in Figure

$$\sum F_y = 0$$

$$\Rightarrow T_4 \cos 60^\circ = 100$$

$$\Rightarrow T_4 = 200 \text{ N}$$



8. The FBD for the two cases is shown in Figure.

In first case, let the force exerted by the man on the floor is N_1 . we have

$$N_1 = T + 50g$$

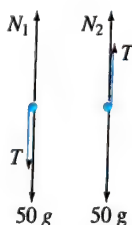
Block is to be raised without acceleration, so $T = 25g$

$$\therefore N_1 = 25g + 50g = 75g = 75 \times 10 = 750 \text{ N}$$

In second case, let the force exerted by the man on the floor is N_2 . We have $N_2 = 50g - T$ and $T = 25g$

$$\therefore N_2 = 50g - 25g = 25g = 25 \times 10 = 250 \text{ N}$$

As the floor yields to a downward force of 700 N, so the man should adopt the second case.

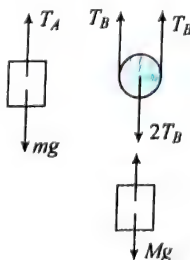


9. From diagram it is clear

$$T_A = Mg \quad \dots(i)$$

$$\text{and } 2T_B = Mg \quad \dots(ii)$$

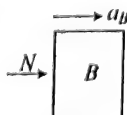
$$\text{Hence, } \frac{T_A}{T_B} = 2$$



Exercise 6.3

1. Block C will not move and blocks A and B will move with common acceleration

$$a_A = a_B = \frac{50}{(1+2)} = \frac{50}{3} \text{ ms}^{-2}$$



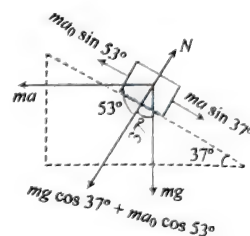
From free body diagram of B

$$N = m_B a_B = 2 \times \frac{50}{3} = \frac{100}{3} \text{ N}$$

2. (a) Acceleration of dominoes does not depend upon the number of blocks as acceleration $a = \frac{F}{\text{Total mass}} = \frac{10}{7} \text{ ms}^{-2}$
- (b) To maximize the force on column C due to column B, the mass of column C should be maximum. But at least one block should be present in each column. Hence, 5 blocks should be on column C and one each on column A and B.
- (c) Similarly, to maximize the force on column B, maximum mass should be on column B. Hence column A—one, column B—five, and column C—one.
- (d) To maximize force on column B due to column A, only one block should be on column A and remaining six can be placed in any arrangement.

3. For discussion of the block inclined plane is taken as reference frame. From the force diagram,

$$\begin{aligned} N &= mg \cos 37^\circ + ma_0 \cos 53^\circ \\ &= 5 \times 10 \times 0.8 + 5 \times 5 \times 0.6 \\ &= 40 + 15 = 55 \text{ N} \end{aligned}$$



4. When there is no sliding at any contact surface, we may take complete system as a single body.

Considering motion of the system

From FBD of m

$$F = (M + m)a$$

$$N \cos \theta = mg$$

$$\text{and } F - N \sin \theta = ma$$

$$\text{From Eq. (ii) } N = \frac{mg}{\cos \theta}$$

Solving Eqs. (i), (ii) and (iii), we get

$$F = \frac{mg}{M} (m + M) \tan \theta$$

5. Normal to inclined the object is at rest

$$N = mg \cos 37^\circ - ma \cos 37^\circ$$

$$= m(g - a) \cos 37^\circ$$

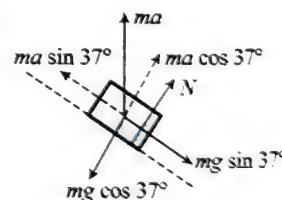
$$= 1 \times 7 \times \frac{4}{5} = \frac{28}{5} \text{ N}$$

$$mg \sin 37^\circ - ma \sin 37^\circ = ma_{\text{rel}}$$

$$a_{\text{rel}} = (g - a) \sin 37^\circ$$

$$= (10 - 3) \times \frac{3}{5} = \frac{21}{5} \text{ ms}^{-2}$$

$$s_{\text{rel}} = u_{\text{rel}} t + \frac{1}{2} a_{\text{rel}} t^2 \Rightarrow 2.1 = 0 + \frac{1}{2} \left(\frac{21}{5} \right) t^2 \Rightarrow t = 1 \text{ s}$$



6. From $t = 0$ to 2 s, the lift has a uniform acceleration

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{3.6 - 0}{2 - 0} = 1.8 \text{ ms}^{-2}$$

From $t = 2$ to 10 s, the lift has a constant speed and hence acceleration a_2 during the period is 0 ms^{-2} .

From $t = 10$ s to 12 s, the lift has a uniform acceleration. If a_3 is the acceleration during the period, then

$$a_3 = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{0 - 3.6}{12 - 10} = -1.8 \text{ ms}^{-2}$$

(a) Here $m = 1500 \text{ kg}$, $g = 9.8 \text{ ms}^{-2}$

(i) At $t = 1 \text{ s}$, $a = a_1 = 1.8 \text{ ms}^{-2}$

$$\text{Tension } T_1 = m(g + a) = 1500(10 + 1.8) = 17700 \text{ N}$$

(ii) At $t = 6 \text{ s}$, $a = a_2 = 0$

$$\text{Tension } T_2 = m(g + a_2) = mg = 1500 \times 10 = 15000 \text{ N}$$

(iii) At $t = 11 \text{ s}$ $a = a_3 = -1.8 \text{ ms}^{-2}$

$$\text{Tension } T_3 = m(g + a_3) = 1500(10 - 1.8) = 12300 \text{ N}$$

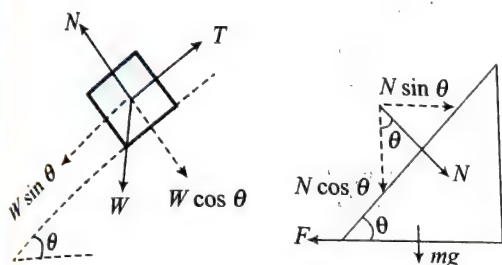
(b) Height reached by the lift

= Area of enclosed by $v-t$ curve and t -axis

= Area of quadrilateral $OABC = 36 \text{ m}$

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{26}{12} = 3 \text{ ms}^{-1}$$

$$\text{Average acceleration} = \frac{\text{Net change in velocity}}{\text{Total time}} = \frac{0}{12} = 0 \text{ ms}^{-2}$$



$$N = W \cos \theta \text{ and } N \sin \theta = F$$

$$\Rightarrow W \cos \theta \sin \theta = F$$

$$\text{or } F = W \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \Rightarrow F = \frac{\sqrt{3}W}{4}$$

Exercise 6.4

1. By Newton's second law, we have

$$1g - T_1 = 1a$$

$$(T_1 + 2g) - T_2 = 2a$$

$$\text{and } T_2 - 2g = 2a$$

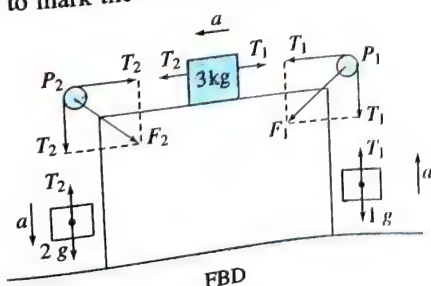
Solving Eqs (i), (ii), and (iii), we get $a = 2 \text{ ms}^{-2}$, $T_1 = 8 \text{ N}$, and $T_2 = 24 \text{ N}$

By shortcut method:

$$a = \frac{[(2+1)-2]g}{(1+2+2)} = 2 \text{ ms}^{-2}$$

$$T_1 = m(g - a) = 1(10 - 2) = 8 \text{ N}$$

2. Note: As the motion of the block is on smooth horizontal surface, so no need to mark the forces along vertical direction.



By Newton's second law, we have

$$T_1 - 1g = 1a \quad \dots(i)$$

$$T_2 - T_1 = 3a \quad \dots(ii)$$

$$\text{and } 2g - T_2 = 2a \quad \dots(iii)$$

Solving above equations, we get

$$a = \frac{g}{6} \text{ ms}^{-2}, \quad T_1 = \frac{7g}{6} \text{ N}, \quad \text{and} \quad T_2 = \frac{5g}{3} \text{ N}$$

$$\text{Force on pulley } P_1, F_1 = \sqrt{T_1^2 + T_1^2} = \sqrt{2} T_1$$

$$\text{Force on pulley } P_2, F_2 = \sqrt{T_2^2 + T_2^2} = \sqrt{2} T_2$$

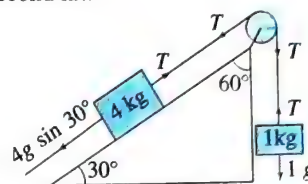
By shortcut method:

$$a = \frac{\text{Unbalanced load}}{\text{Total mass}} = \frac{(2-1)g}{(1+3+2)} = \frac{g}{6} \text{ ms}^{-2}$$

$$T_1 = m_{\text{up}}(g + a) = 1\left(g + \frac{g}{6}\right) = \frac{7g}{6} \text{ N}$$

$$T_2 = m_{\text{down}}(g - a) = 2\left(g - \frac{g}{6}\right) = \frac{5g}{3} \text{ N}$$

3. By Newton's second law



$$4g \sin 30 - T = 4a \quad \dots(i)$$

$$\text{and } T - 1g = 1a \quad \dots(ii)$$

Solving Eqs (i) and (ii), we get

$$a = 2 \text{ ms}^{-2} \text{ and } T = 12 \text{ N}$$

By shortcut method:

$$a = \frac{\text{Unbalanced load}}{\text{Total mass}} = \frac{4g \sin 30 - 1g}{4+1} = 2 \text{ ms}^{-2}$$

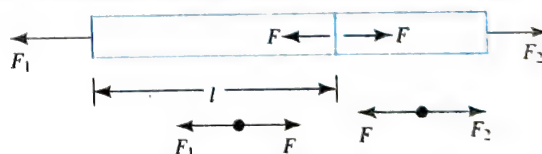
$$T = M_{\text{up}}(g + a) = 1(10 + 2) = 12 \text{ N}$$

4. Let tension in the rope is T . By Newton's second law, we have

$$T - (10g + 8g) = 10 \times 2 + 8 \times 0$$

$$\text{or } T = 18g + 20 = 200 \text{ N}$$

5. Suppose $F_2 > F_1$. Then the motion of the rod will take place in the direction of the force F_2 with acceleration, say, a . Considering the motion of the entire rod, we have $F_2 - F_1 = \text{net force on the rod} = mL \times a$ where m is the mass of the rod per unit length of it.



Considering the motion of the length l from the first end $F - F_1 = mla$. Dividing, we get

$$\frac{F - F_1}{F_2 - F_1} = \frac{l}{L} \text{ or } F = \frac{(F_2 - F_1)l}{L} + F_1$$

6. The point to this problem is that the monkey and the bananas have the same weight, and the tension in the string is the same at the point where the bananas are suspended and where the monkey is pulling. In all the cases, the monkey and the bananas will have the same net force and hence the same acceleration, direction, and magnitude.

(a) The bananas move up.

(b) The monkey and bananas always move at the same velocity, so the distance between them stays the same.

(c) Both the monkey and the bananas are in free fall, and as they have the same initial velocity, the distance between them does not change.

(d) The bananas will slow down at the same rate as the monkey; if the monkey comes to a stop, so will the bananas.

7. Equation of motion for M :

Since M is stationary, $T - Mg = 0$

$$\Rightarrow T = Mg$$

...(i)

Since the boy moves up with an acceleration a ,

$$T - mg = ma$$

$$\Rightarrow T = m(g + a)$$

...(ii)

Equating (i) and (ii), we obtain

$$Mg = m(g + a)$$

$$\Rightarrow a = \left(\frac{M}{m} - 1 \right) g$$

That means, if $a > \left(\frac{M}{m} - 1 \right) g$, block M can be lifted from floor.

8. From force diagram of pulley,

$$P + P \cos \theta = T$$

From force diagram of block

$$T > mg \sin \theta$$

$$\text{or } P + P \cos \theta > mg \sin \theta$$

$$\text{or } P > \frac{mg \sin \theta}{1 + \cos \theta}$$

$$\therefore P_{\min} = \frac{mg \sin \theta}{1 + \cos \theta}$$

9. Here, $T_0 = 8g = 80 \text{ N}$

$$\text{Also, } T_0 = 2T$$

$$\therefore T = 40 \text{ N}$$

From force diagram of 5 kg block,

$$50 - T = 5a$$

$$50 - 40 = 5a$$

$$10 = 5a$$

$$a = 2 \text{ ms}^{-2}$$

From force diagram of m_1 ,

$$T - m_1 g = m_1 a$$

$$\text{or } 40 - m_1 g = m_1 \times 2$$

$$\text{or } 40 = 10m_1 + 2m_1 = 12m_1$$

$$\therefore m_1 = \frac{40}{12} = \frac{10}{3} \text{ kg}$$

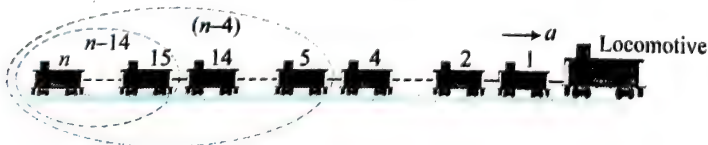
10. Taking man and platform together on system with constant velocity.

$$T = 950 \text{ N} = \text{force applied by man}$$

From FBD of man only,

$$N = T + 550 = 950 + 550 = 1500 \text{ N}$$

11. Acceleration of each cart will be same



Tension in connection between 4 and 5 cart

$$T_{4,5} = m(n - 4)a \quad \dots(i)$$

Tension in connection between 14 and 15

$$T_{14,15} = m(n - 14)a \quad \dots(ii)$$

$$\text{Given, } T_{4,5} = 3T_{14,15}$$

$$M(n - 4)a = 3m(n - 14)a \Rightarrow n = 19$$

12. Let the tension in the string be T at any angular position θ . The acceleration of each ball along x and y axes be a and a_1 , respectively. Writing the equation of motion of m , we obtain

$$\Sigma F_x = ma \Rightarrow T \cos \theta = ma \quad \dots(i)$$

$$\Sigma F_y = ma_1 \Rightarrow T \sin \theta = ma_1 \quad \dots(ii)$$

At point P , as it is accelerating with an acceleration a , therefore $F - 2T \cos \theta = m_p a$

where m_p = mass of the string at the point $P \cong 0$

$$\Rightarrow F = 2T \cos \theta$$

$$(ii) + (iii) \Rightarrow \tan \theta = \frac{a_1}{a}$$

$$\Rightarrow a_1 = a \tan \theta, \text{ where } a = \frac{T \cos \theta}{m} \text{ from (i).}$$

$$\text{Putting } T = \frac{F}{2 \cos \theta} \text{ from (3), we obtain } a_1 = \frac{F}{2m} \tan \theta$$

13. (a) When the lift moves upward with an acceleration, then the apparent weight is

$$W' = W + ma \text{ or } W' = W + \frac{W}{g} a = W \left(1 + \frac{a}{g} \right)$$

$$\text{Here } W' = 50 \text{ N, } a = 2.45 \text{ ms}^{-2}, g = 9.8 \text{ ms}^{-2}$$

$$50 = \left(1 + \frac{2.45}{9.8} \right) W \Rightarrow W = \frac{4}{5} \times 50 = 40 \text{ N}$$

(b) Again if a is upward acceleration, then

$$W' = W \left(1 + \frac{a}{g} \right) \text{ gives } 30 = 40 \left(1 + \frac{a}{g} \right)$$

$$\Rightarrow \frac{a}{g} = \frac{30}{40} - 1 = -\frac{1}{4} \Rightarrow a = -\frac{g}{4}$$

i.e., lift has downward with acceleration $g/4$.

(c) When the elevator cable breaks, the lift falls freely with acceleration g .

$$\text{Therefore, apparent weight } W' = W \left(1 - \frac{g}{g} \right) = 0 \text{ N}$$

Exercise 6.5

1. Pseudo force = ma in backward direction.

$$2. T_1 \cos 60^\circ - T_2 \cos 60^\circ = ma$$

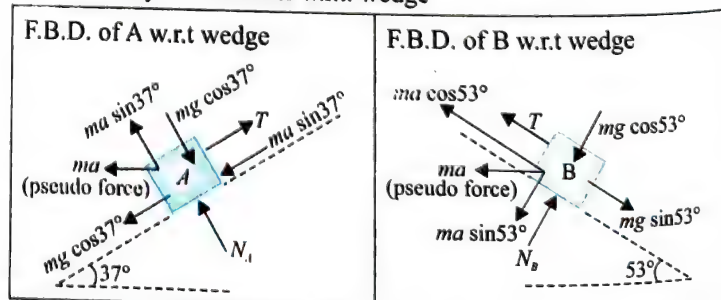
$$T_1 \sin 60^\circ + T_2 \sin 60^\circ = mg$$

$$T_1 = 2T_2$$

On solving equations, we get

$$T_1 = \frac{4mg}{3\sqrt{3}}, T_2 = \frac{2mg}{3\sqrt{3}}, \text{ and } a = \frac{g}{3\sqrt{3}}$$

3. Let us analyse the blocks w.r.t. wedge



(i) In frame of reference of the wedge, the block A is not moving normal to the surface of inclined plane. Hence from F.B.D of A

$$\Rightarrow N_A = mg \cos 37^\circ - ma \sin 37^\circ$$

$$\Rightarrow N_A = mg \times \frac{4}{5} - ma \times \frac{3}{5} = \frac{m}{5} (4g - 3a)$$

(ii) Similarly analyzing block B, w.r.t wedge

$$N_B = mg \cos 53^\circ + ma \sin 53^\circ = mg \times \frac{3}{5} + ma \times \frac{4}{5}$$

$$\Rightarrow N_B = \frac{m}{5}(3g + 4a)$$

(iii) Let the magnitude of the acceleration of the blocks w.r.t wedge be a' . Writing equation of motion of the blocks parallel to slopping surface.

$$\text{For A: } mg \sin 37^\circ + ma \cos 37^\circ - T = md$$

$$\text{For B: } T + ma \cos 53^\circ - mg \sin 53^\circ = md \quad \dots(i)$$

$$\text{From (i) and (ii), we get } a' = \frac{1}{10}(7a - g) \quad \dots(ii)$$

(iv) For maximum acceleration of the wedge for normal reaction on the block A to be non zero,

$$N_A > 0$$

$$\frac{m}{5}(4g - 3a) > 0 \quad \text{or} \quad a > \frac{4}{3}g$$

Hence the maximum value of acceleration of the wedge is $\frac{4}{3}g$.

4. Drawing F.B.D of the person w.r.t elevator



F.B.D when elevator is accelerating
Case-I



F.B.D when lift is moving with constant velocity
Case-II



F.B.D when elevator is retarding
Case-III

Equation of motion,

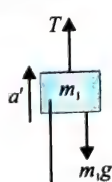
$$\text{For Case-I: } N_1 = mg + ma = 72 \times 10 \text{ N} \quad \dots(i)$$

$$\text{For Case-III: } N_3 = mg - ma = 60 \times 10 \text{ N} \quad \dots(ii)$$

From (i) and (ii) $mg = \text{true weight} = 660 \text{ N}$
or true weight recorded by machine will be 66 kg.

$$\text{Again from (i), } a = \frac{720 - 660}{66} = \frac{10}{11} \text{ m/s}^2$$

5. Analysing the block w.r.t elevator



$m_1 a_0$ (pseudo force)



$m_2 a_0$ (pseudo force)

Writing equation of motion w.r.t elevator $\dots(i)$

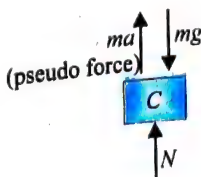
$$\text{For } m_1: T - m_1 a_0 - m_1 g = m_1 a \quad \dots(ii)$$

$$\text{For } m_2: m_2 g - m_2 a_0 - T = m_2 a$$

$$\text{From (i) and (ii) we get } a = \frac{(m_2 - m_1)}{(m_2 + m_1)}(g - a_0)$$

6. When the block C is placed on the block B, the system will start accelerating with acceleration.

$$a = \frac{[(M + m) - M]}{[(M + m) + M]}g = \frac{mg}{(2M + m)}$$



The block 'B + C' combined will move down with acceleration ' a '

Now analyzing the block C w.r.t block B.

As the block C is at rest w.r.t block B

$$\text{Hence } N = mg - ma$$

$$\Rightarrow N = mg - m\left(\frac{mg}{2M + m}\right) = \frac{2Mmg}{(2M + m)}$$

Exercise 6.6

1. Sum of change in length of segments of string (1)

$$\text{Should be zero} \Rightarrow (-2x_A) + (2x_{p1}) = 0$$

$$\text{or } x_A = x_{p1} \Rightarrow a_A = a_{p1} \quad \dots(i)$$

Also sum of the change in segment length should also be zero

$$(-x_{p1} + x_B) + (x_B) = 0$$

$$2x_B - x_{p1} = 0 \Rightarrow 2a_B = a_{p1} \quad \dots(ii)$$

$$\text{From (1) and (2) } a_A = 2a_B$$

$$\Rightarrow a_B = \frac{a_A}{2} = 1 \text{ m/s}^2$$

2. $v_A = v_{p2} = 10 \text{ m/s}^{-1}$

Using shortcut method

For pulley P_2

$$v_{P2} = \frac{V_B + (-V_C)}{2}$$

$$10 = \frac{V_B - 2}{2} \Rightarrow v_B = 22 \text{ m/s}^{-1}$$

3. Sum of segment lengths should be zero.

$$(-x_A) + (2x_B) + x_C = 0$$

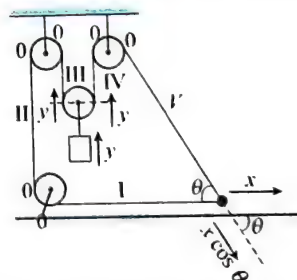
$$\text{or } x_A = 2x_B + x_C$$

$$\text{or } a_A = 2a_B + a_C$$

$$5 = 2 \times 2 + a_C$$

$$\Rightarrow a_C = 1 \text{ m/s}^2$$

4. Sum of total change in segment lengths should be zero.



$$(+x) + (0) + (-y) + (-y) + (0 + x \cos \theta) = 0$$

$$2y = x + x \cos \theta$$

$$y = \frac{x}{2}(1 + \cos 60^\circ) = \frac{x}{2}\left(1 + \frac{1}{2}\right)$$

$$y = \frac{3x}{4} \Rightarrow v_{\text{block}} = \frac{3}{4}v_{\text{truck}}$$

5. Speed of C and B will be same as shown. Let it be v . $|x_B| = |x_C|$

Using constraint for string passing over C:

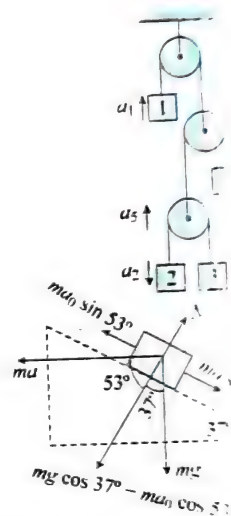
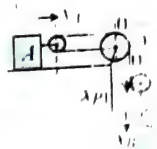
$$(x_A - x_C) + (x_C - x_B) + (-x_B) = 0$$

$$x_A - 2x_C - 2x_B = 0$$

$$\text{But } x_B = x_C \Rightarrow x_A - 4x_B = 0$$

$$\text{or } v_A = 4v_B$$

$$\text{Hence, } v_B = \frac{v_A}{4} = \frac{2}{4} = 0.5 \text{ m/s}^{-1}$$



Let A be the acceleration of block M and pulley, and a be the acceleration of m .

Let x, X be the coordinates of m and pulley as shown.

Length of string passing over the pulley is

$$L = X + (X - x) \Rightarrow L = 2X - x$$

Differentiating twice with respect to time,

$$0 = 2A - a \Rightarrow a = 2A$$

From force diagrams of m and M , we have

$$T = ma$$

$$\text{and } F - 2T = MA$$

$$\text{Comparing (i), (ii), and (iii), we get } A = \frac{F}{M + 4m} \text{ and } a = \frac{2F}{(M + 4m)} \quad \dots(iii)$$

7. Let in a given time displacement of A is x_1 , of B is x_2 , and that of Y is x_3 in downward direction.

As the length of string will remain constant, so we can write

$$x_1 + (x_1 - x_3) + x_3 + (x_2 - x_3) + x_2 = 0$$

$$\Rightarrow 2x_1 + 2x_2 - x_3 = 0$$

(a) Differentiating, we get $2v_1 + 2v_2 - v_3 = 0$

$$\Rightarrow 2v_1 + 2(-1) - (-2) = 0 \Rightarrow v_1 = 0$$

(b) Again differentiating, we get $2a_1 + 2a_2 - a_3 = 0$

$$\Rightarrow 2a_1 + 2(-3) - 1 = 0 \Rightarrow a_1 = 7/2 \text{ ms}^{-2}$$

$$8. (a) V_p = \frac{V_A + V_B}{2} \Rightarrow 5 = \frac{V_A + 10}{2} \Rightarrow V_A = 0$$

$$(b) 5 = \frac{V_A - 20}{2} \Rightarrow V_A = 30 \text{ ms}^{-1}$$

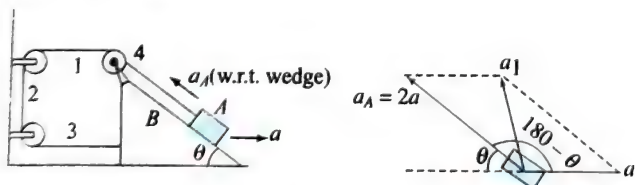
9. Let the speed of the ring is u , then $u \cos \theta = v$

Because components of velocity along the string should be same.

$$\Rightarrow u = v / \cos \theta = v \sec \theta$$

10. (a) Length of (1) and (3) will increase at the cost of part 4 (Fig). Hence decrease in length of (4) will be twice of increase in lengths of either 1 or 3.

$$\text{So, } a_A = 2a$$



(b) Acceleration of block w.r.t. ground :

$$a_1 = \sqrt{a^2 + a_A^2 + 2a \cdot a_A \cos(180 - \theta)} = a\sqrt{5 - 4\cos\theta}$$

11. (a) (i) Length of (5) will decrease at the cost of increase in length of (1) and (3).

Hence, acceleration of B w.r.t.

$$A: a_{BA} = 2a$$

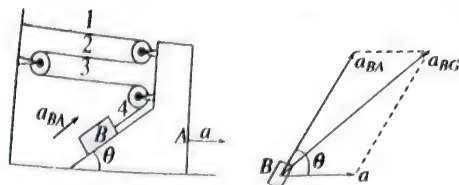
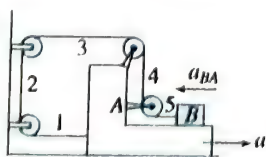
(ii) Acceleration of B w.r.t. ground

$$a_{BA} = a_B - a_A \Rightarrow -2a = a_B - a$$

$$\Rightarrow a_B = -a$$

(b) (i) Length of 4 will decrease at the cost of 1, 2 and 3, hence

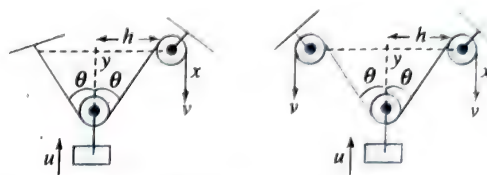
$$a_{BA} = 3a$$



$$(ii) a_{BG} = \sqrt{a^2 + A^2 + 2aA \cos \theta} = a\sqrt{10 + 6 \cos \theta}$$

12. (a) Length of the string: $l = x + 2\sqrt{h^2 + y^2}$

$$\text{Differentiating, we get } \frac{dl}{dt} = \frac{dx}{dt} + 2 \frac{2y}{2\sqrt{h^2 + y^2}} \frac{dy}{dt}$$



$$\Rightarrow 0 = v + \frac{2y}{\sqrt{h^2 + y^2}} (-u)$$

$$\Rightarrow 0 = v - 2u \cos \theta \Rightarrow u = v / (2 \cos \theta)$$

(b) Length of string, $l = 2x + 2\sqrt{h^2 + y^2}$

Again differentiate to get $u = v / \cos \theta$

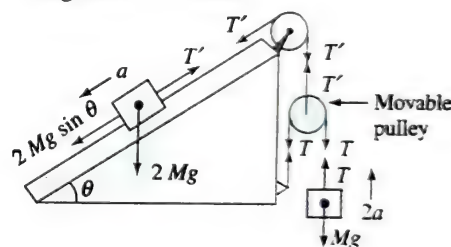
13. (a) For block $2M$, $2T = 2Ma$

For block M , $Mg - T = M2a$

Solving equations (i) and (ii), we get $a = g/3$.

(b) For block 2,

$$2Mg \sin \theta - T' = 2Ma$$



For movable pulley,

$$T' - 2T = 0 \times a \Rightarrow T' = 2T$$

For block M , $T - Mg = M2a$

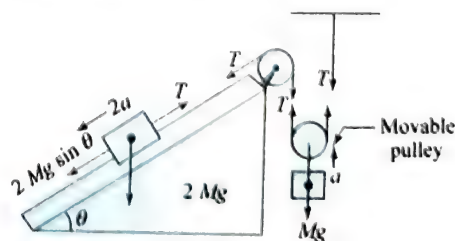
$$\text{Solving Eq. (i), (ii), and (iii), we get } a = \frac{g(\sin \theta - 1)}{3}$$

This is negative, hence directions of accelerations will be opposite to those shown in figure.

Note: Here we assume that the block $2M$ is moving down the plane. You may assume that it is moving up the plane. The magnitude of the acceleration found will remain the same.

(c) For block $2M$,

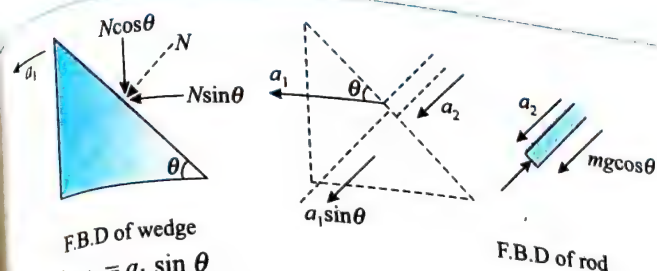
$$2Mg \sin \theta - T = 2M(2a)$$



For block M , $2T - Mg = Ma$

$$\text{Solving Eqs (i) and (ii), we get } a = \frac{g[4 \sin \theta - 1]}{9}$$

14. Components of acceleration of M and m perpendicular to the incline should be same as both always remain in contact.



F.B.D of wedge

F.B.D of rod

For this $a_2 = a_1 \sin \theta$ For M : $N \sin \theta = Ma_1$... (i)For m : $mg \cos \theta - N = ma_2$... (ii)

Solving equations (i), (ii), and (iii), we get ... (iii)

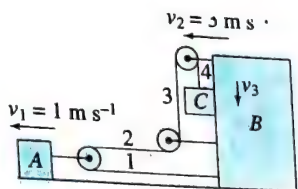
$$a_1 = \frac{mg \sin \theta \cos \theta}{M + m \sin^2 \theta}, \quad a_2 = \frac{mg \sin^2 \theta \cos \theta}{M + m \sin^2 \theta}$$

Let $v_1 = 1 \text{ m s}^{-1}$, $v_2 = 3 \text{ m s}^{-1}$ Let the velocity of C is v_3 downwardw.r.t. B . Then

$$(v_1 - v_2) + (v_1 - v_2) + v_3 = 0$$

$$\Rightarrow v_3 = 2(v_2 - v_1) = 2(3 - 1) = 4 \text{ m s}^{-1}$$

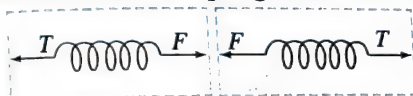
$$\text{Net velocity of } C = \sqrt{v_2^2 + v_3^2} = \sqrt{3^2 + 4^2} = 5 \text{ m s}^{-1}$$



Exercise 6.7

1. In all the three cases the spring balance reads 10 kg.

Let us cut a section inside the spring as shown in figure.

As each part of the spring is at rest, so $F = T$. As the block is stationary, so $T = F = 100 \text{ N}$.2. Let T be the reading of the spring balance. Then

For 20-kg block: $20g - T = 20a$... (i)

For 10-kg block: $T - 10g = 10a$... (ii)

Solving Eqs (i) and (ii), we get $a = \frac{g}{3}$, $T = \frac{40g}{3}$.

So the spring balance reading is $\frac{40}{3} \text{ kg}$.

3. The spring force on all springs will be same as springs are ideal. As spring constants of all three springs are same; hence, stretch of all three springs will be equal.

4. Given spring constant $k = 20 \text{ N m}^{-1}$

Initial length = 0.25 m

Final length = 0.30 m

$$\Delta x = 0.30 - 0.25 = 0.05 \text{ m} (\rightarrow)$$

Mass of the block = m

$$\bar{F} = -k\bar{x} \Rightarrow ma = -k\Delta x$$

$$a = -\frac{k\Delta x}{m} = \frac{-20 \times 0.05}{m} = -\frac{1}{m}$$

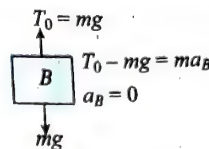
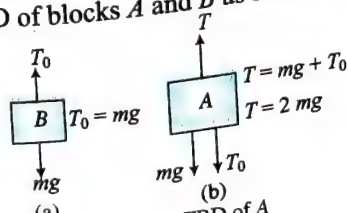
Acceleration = $1/m (\leftarrow)$ 5. When blocks A and B are in equilibrium position. When string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B , again draw FBD of blocks A and B as shown in Fig.

Fig. (a)

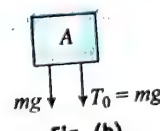


Fig. (b)

$$mg + T_0 = ma_A$$

$$2mg = ma_A$$

$$a_A = 2g \text{ (downwards)}$$

As forces acting on block B are not changing, hence, block B will remain at equilibrium. It means $a_B = 0$.

6. For calculating the reading, first we draw FBD of 20-kg block.

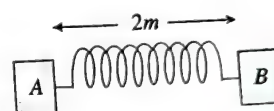
F.B.D of 20 kg.

$$mg - T = 0$$

$$T = 20g = 200 \text{ N}$$

Since both balances are light so, both the scales will read 20 kg.

7. (a) Extension in spring = 1 m.



$$F_{\text{spring}} = K \times x = 200 \times 1 = 200 \text{ N}$$

(b) Same Extension same spring force in both directions,

$$F_{\text{spring}} = 200 \text{ N}$$

(c) Both displacements of A or B are compressing the spring total compressing = $0.75 + 0.25 = 1 \text{ m}$

$$\therefore F_{\text{spring}} = kx = 200 \times 1 = 200 \text{ N}$$

8. $T = 100 \text{ N} \Rightarrow mg = 100 \text{ N}$

$$m = \frac{100}{g} = 10 \text{ kg}$$

9. When string is not cut:

FBD of A block

$$kx = mg + T$$

FBD of B block

$$T = mg$$

When string is cut:

FBD of A block

$$kx - mg = ma_A$$

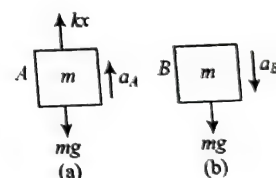
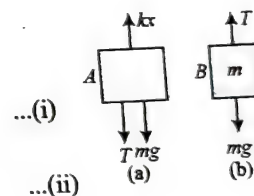
$$2mg - mg = ma_A$$

$$= a_A = g \text{ (upwards)}$$

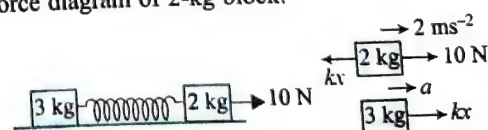
FBD of B block

$$ma_B = mg$$

$$a_B = g \text{ (downwards)}$$



10. From force diagram of 2-kg block:



$$10 - kx = 2a = 4$$

$$\therefore kx = 6$$

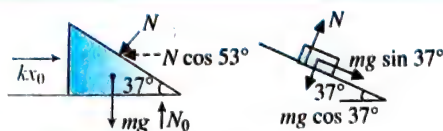
From force diagram of 3kg block,

$$kx = 3a$$

$$\text{or } 6 = 3a$$

$$\therefore a = \frac{6}{3} = 2 \text{ m s}^{-2}$$

11.



From force diagram of wedge,

$$N \cos 53^\circ = kx_0$$

From force diagram of smaller block:

$$N = mg \cos 37^\circ$$

$$kx_0 = N \cos 53^\circ$$

$$kx_0 = mg \cos 37^\circ \cos 53^\circ$$

$$kx_0 = 2.5 \times 10 \times 0.8 \times 0.6 = 12 \text{ N}$$

Hence, reading is $kx_0 = 12 \text{ N}$

12. FBD: Let the spring forces be $F = kx$ just after cutting the spring. Hence, at that instant, the forces acting on m_1 are $T = kx \rightarrow m_1 g \downarrow$ and $N \uparrow$; on m_2 the forces are $m_2 g \downarrow$ and $T \uparrow$.

Force equation: Initially, all the particles are stationary; hence, $a_1 = a_2 = 0$. Applying Newton's second law,

$$\text{For } m_1: \Sigma F = T - kx = 0 \quad \dots(i)$$

$$\text{For } m_2: \Sigma F = T - m_2 g = 0 \quad \dots(ii)$$

$$\text{For spring: } \Sigma F = T - kx = 0 \quad \dots(iii)$$

Solving Eqs. (i), (ii) and (iii), we have

$$T = T = kx = m_2 g$$

Just after the string S is cut, tension T' vanishes immediately, but the spring cannot regain its shape and size instantaneously. Therefore, the spring force remains as it is. Then, just after cutting the string, the net force acting on m_1 is equal to kx , whereas the net force acting on m_2 is equal to $T - m_2 g$. Hence, the acceleration of m_1 and m_2 just after cutting the string is given by

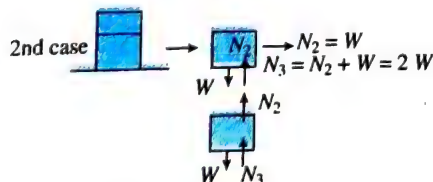
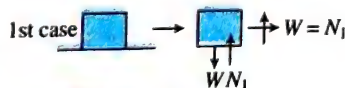
$$\Sigma F = m_1 a_1 = kx = m_2 g, \Sigma F = m_2 a_2 = T - m_2 g = 0$$

$$\text{This gives } a_1 = \left(\frac{m_2}{m_1} \right) g \text{ and } a_2 = 0$$

Exercises

Single Correct Answer Type

1. (3) Let the weight of each block be
- W
- (Figure)



$$\text{So, } N_1 = N_2 = \frac{N_3}{2}$$

$$2. (1) a = \frac{\sqrt{R_1^2 + R_2^2}}{m} = \frac{R_3}{m} \quad \left[\because R_3 = \sqrt{R_1^2 + R_2^2} \right]$$

3. (1) Acceleration of the skaters will be in the ratio

$$\frac{F}{4} : \frac{F}{5} \text{ or } 5 : 4$$

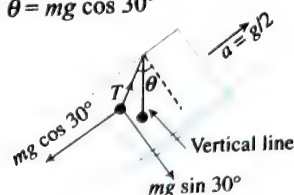
Now according to the problem, $s = 0 + \frac{1}{2} at^2$, we get

$$\frac{s_1}{s_2} = \frac{a_1}{a_2} = \frac{5}{4}$$

$$4. (2) T \sin \theta - mg \sin 30^\circ = ma$$

$$\Rightarrow T \sin \theta = mg \sin 30^\circ + mg/2$$

$$T \cos \theta = mg \cos 30^\circ$$



Dividing Eqs. (i) by (ii), we get

$$\tan \theta = \frac{2}{\sqrt{3}}$$

5. (2) Acceleration of cylinder down the plane is

$$a = (g \sin 30^\circ)(\sin 30^\circ)$$

$$= 10 \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = 2.5 \text{ m s}^{-2}$$

$$\text{Time taken } t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 5}{2.5}} = 2 \text{ s}$$

6. (3) Given (
- $V = 10 \text{ m s}^{-1}$
-)

$$\text{After 2s: } V_x = \frac{10}{\sqrt{2}} - \frac{10}{\sqrt{2}} \times 2 \Rightarrow V_x = -\frac{10}{\sqrt{2}} \text{ m s}^{-1}$$

$$V_x = -\frac{10}{\sqrt{2}} \text{ m s}^{-1} \text{ and } V_y = -\frac{10}{\sqrt{2}} \text{ m s}^{-1}$$

$$V = \sqrt{\frac{100}{2} + \frac{100}{2}} = 10 \sqrt{\frac{1}{2} + \frac{1}{2}} = 10 \text{ m s}^{-1}$$

7. (2)
- $\vec{a}_{b,l} = \vec{a}_b - \vec{a}_l = (g - a) \downarrow \Rightarrow \vec{a}_b = g \downarrow$

$$8. (4) \vec{a} = \frac{\vec{F}}{m} = -10 \hat{j} (\text{m s}^{-2})^2$$

Displacement in y-direction

$$y = ut + \frac{1}{2} at^2 \Rightarrow 0 = 4 \times t \times -\frac{1}{2} \times 10 \times t^2$$

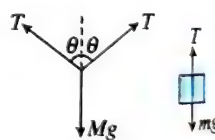
$$t = \frac{4}{5} \text{ s} \Rightarrow x = 4t = 4 \times \frac{4}{5} = 3.2 \text{ m}$$

9. (3) At equation
- $2T \cos \theta = Mg$
- ,
- $T = mg$

$$\cos \theta = \frac{Mg}{2mg} = \frac{M}{2m}$$

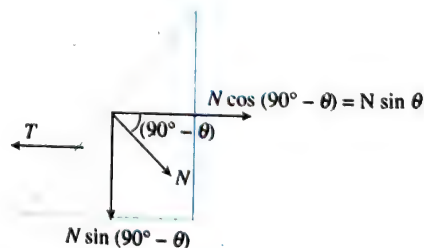
$$\Rightarrow \cos \theta < 1$$

$$\frac{M}{2m} < 1 \Rightarrow M < 2m$$

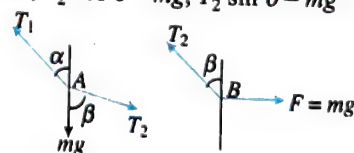


10. (3)
- $T = N \sin \theta$
- and
- $N = mg \cos \theta$

$$T = mg \cos \theta \sin \theta = \frac{mg}{2} \sin 2\theta$$



11. (3) From figure,
- $T_2 \cos \theta = mg$
- ,
- $T_2 \sin \theta = mg$



$$T_1 \sin \alpha = mg \sqrt{2} \sin 45^\circ$$

$$T_1 \sin \alpha = mg \Rightarrow \frac{\cos \alpha}{\sin \alpha} = \frac{2mg}{mg}$$

$$\tan \alpha = \frac{1}{2} \Rightarrow \cos \alpha = \frac{2}{\sqrt{5}}$$

$$T_1 \frac{2}{\sqrt{5}} = 2mg \Rightarrow T_1 = mg\sqrt{5}$$

$$\frac{T_1}{T_2} = \frac{\sqrt{5}}{\sqrt{2}} \Rightarrow \sqrt{2} T_1 = \sqrt{5} T_2$$

$$(2) \tan \beta = \frac{12}{5} \therefore \cos \beta = \frac{5}{13}$$

$$T_1 \cos \beta + T_2 \cos \beta = mg$$

$$T_1 \sin \beta = T_2 \sin \beta$$

$$\therefore T_1 = T_2 = T$$

$$\therefore 2T \cos \beta = mg \Rightarrow T = \frac{mg}{2 \cos \beta} \Rightarrow T = \frac{13}{10} mg$$

$$(4) \text{ Speed, } v = \sqrt{v_x^2 + v_y^2}$$

Rate of change of speed,

$$\frac{dv}{dt} = \frac{2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}}{2\sqrt{v_x^2 + v_y^2}} = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}}$$

$$(4) \text{ Let at any time, their velocities are } v_1 \text{ and } v_2, \text{ respectively. Then } v_1 = v_2 \cos \theta$$

$$\text{Differentiating: } a_1 = a_2 \cos \theta - v_2 \sin \theta \frac{d\theta}{dt}$$

Hence, none of them is correct.

[Note: Option (1) is correct initially, because initially $v_2 = 0$.]

$$(4) \text{ The acceleration of block-rope system is } a = \frac{F}{(M+m)},$$

where M is the mass of block and m is the mass of rope.

So the tension in the middle of the rope will be

$$T = \{M + (m/2)\} a = \frac{M + (m/2)F}{M + m}$$

Given that $m = M/2$

$$\therefore T = \left[\frac{M + (M/4)}{M + (M/2)} \right] F = \frac{5F}{6}$$

$$(4) \text{ The reading of the spring scale is the normal reaction between man and spring scale.}$$

As the reading is decreasing, it means normal reaction is decreasing. Firstly, the lift must be moving upwards with constant velocity and decelerated to rest.

$$(2) \text{ Acceleration of box } = 10 \text{ m s}^{-2}$$

Inside the box forces acting on bob (see figure)

$$T = \sqrt{(mg)^2 + (ma)^2}$$

$$= 10\sqrt{2} \text{ N}$$

$$(3) \text{ Let } T \text{ be the tension in the rope and } a \text{ the acceleration of rope.}$$

The absolute acceleration of man is, therefore, $\left(\frac{5g}{4} - a\right)$.

Equations of motion for mass and man gives:

$$T - 100g = 100a$$

$$T - 60g = 60\left(\frac{5g}{4} - a\right) \quad \dots(ii)$$

$$\text{Solving Eqs (i) and (ii), we get } T = \frac{4875}{4} \text{ N}$$

$$19. (4) \text{ Let spring does not get elongated, then net pulling force on the system is } Mg + mg - mg \text{ or simply } Mg. \text{ Total mass being pulled is } M + 2m. \text{ Hence, acceleration of the system is}$$

$$a = \frac{Mg}{M + 2m}$$

Now since $a < g$, there should be an upward force on M so that its acceleration becomes less than g . It means there is some tension developed in the string. Hence, for any value of M spring will be elongated.

$$20. (2) \text{ Suppose } F = \text{upthrust due to buoyancy}$$

Then while descending, we find

$$Mg - F = Ma \quad \dots(i)$$

When ascending, we have

$$F - (M - m)g = (M - m)a \quad \dots(ii)$$

$$\text{Solving Eqs. (i) and (ii), we get } m = \left[\frac{2\alpha}{\alpha + g} \right] M$$

$$21. (1) \text{ Acceleration of the system:}$$

$$a = \frac{P}{M + m} \quad \dots(i)$$

The FBD of mass m is shown in the figure.

$$R \sin \beta = ma \quad \dots(ii)$$

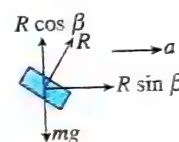
$$R \cos \beta = mg \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$a = g \tan \beta$$

Putting the value of a in (i), we get

$$P = (M + m)g \tan \beta$$



$$22. (2)$$

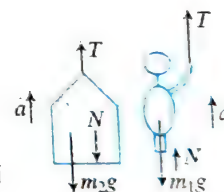
$$23. (4) \text{ Same solution for both}$$

$$m_1 = 100 \text{ kg}, m_2 = 50 \text{ kg}, a = 5 \text{ m s}^{-2}$$

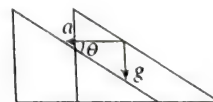
$$T + N - m_1 g = m_1 a,$$

$$T - N - m_2 g = m_2 a$$

$$\text{Solving these: } T = 1125 \text{ N and } N = 375 \text{ N}$$



$$24. (3) \tan \theta = \frac{g}{a} \Rightarrow a = g \cot \theta$$



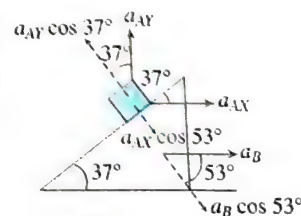
$$25. (4) \text{ From constraint, the acceleration of both block and wedge should be same in a direction perpendicular to the inclined plane as shown in Fig.}$$

$$(a_A)_\perp = (a_B)_\perp,$$

$$a_{AX} = 15, a_{AY} = 15$$

$$a_{AX} \cos 53^\circ - a_{AY} \cos 37^\circ = a_B \cos 53^\circ$$

$$\text{or } a_B = -5 \text{ m s}^{-1} \text{ or } \vec{a}_B = -5\hat{i}$$



$$26. (4) \text{ From constraint relations we can see that the acceleration of block B in upward direction is}$$

$$a_B = \left(\frac{a_C + a_A}{2} \right) \text{ with proper signs}$$

$$\text{So } a_B = \left(\frac{3 - 12t}{2} \right) = 1.5 - 6t$$

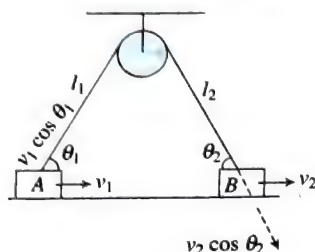
$$\text{or } \frac{dv_B}{dt} = 1.5 - 6t \quad \text{or } \int_0^B dv_B = \int_0^1 (1.5 - 6t) dt$$

$$\text{or } v_B = 1.5t - 3t^2 \quad \text{or } v_B = 0 \text{ at } t = 1/2 \text{ s}$$

27. (1) As m_2 moves with constant velocity, there is no acceleration in the centre of mass. Net force should be zero. For this

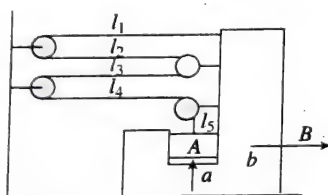
$$N = m_1 g + m_2 g.$$

28. (3) From Figure, $l_1 + l_2 = C$ or $\frac{dl_1}{dt} + \frac{dl_2}{dt} = 0$



$$-v_1 \cos \theta_1 + v_2 \cos \theta_2 = 0 \quad \text{or} \quad \frac{v_1}{v_2} = \frac{\cos \theta_2}{\cos \theta_1}$$

29. (1)

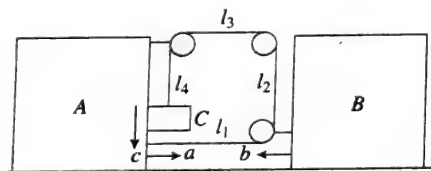


Acceleration of A in horizontal direction = the acceleration of B = b rightwards.

Acceleration of A in vertical direction = the acceleration of A with respect to b in upwards direction = $a = 4b$.

Hence, net acceleration of A = $b\hat{i} + 4b\hat{j}$.

30. (1)



From length constraint $l_1 + l_2 + l_3 + l_4 = C$

$$l_1' + l_2' + l_3' + l_4' = 0$$

$$(-a - b) + 0 + (-a - b) + c = 0$$

$$c = 2a + 2b$$

From wedge constraints, acceleration of C is right side is a .
acceleration of C w.r.t. ground = $a\hat{i} - 2(a + b)\hat{j}$

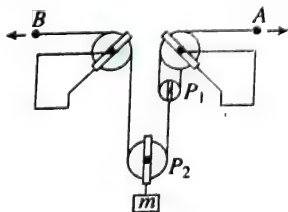
31. (3) $Tx(\text{Hanged part}) = 2T'(\text{Sliding part})$

$$\therefore x = 3x' \Rightarrow x = 3 \times 0.6 = 1.8 \text{ ms}^{-1}$$

32. (1) $V_A = 2 \text{ ms}^{-1}$ (towards right)

$$\therefore V_P = \frac{V_A}{2} = 1 \text{ ms}^{-1} \quad (\text{upwards})$$

$$V_B = 2 \text{ ms}^{-1} \quad (\text{towards left})$$



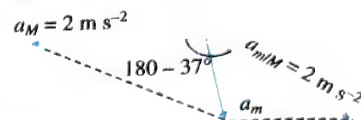
$$\text{Now, } 2V_{P_2} = V_B + V_{P_1}$$

$$\therefore V_{P_2} = \frac{V_B + V_{P_1}}{2} = \frac{2 + 1}{2} = 1.5 \text{ ms}^{-1}$$

33. (4) For A: $5g - T = 5(2C)$

$$\text{For C: } 2T - 8g = 8C \Rightarrow C = \frac{10}{14} = \frac{5}{7} \text{ ms}^{-2}$$

34. (3) If acceleration of M is 2 ms^{-2} , then acceleration of m w.r.t. M will be 2 ms^{-2}



Acceleration of m w.r.t. ground

$$a_m = \sqrt{2^2 + 2^2 + 2 \times 2 \times 2 \cos(180 - 37^\circ)} = \sqrt{8/5} \text{ ms}^{-2}$$

35. (3) From 0 to 2 s: at any time t , $F = 10t$

$$\Rightarrow a = F/m = 10 t/m$$

$$\Rightarrow \int_0^v dv = \int_0^t \frac{10t}{m} dt \Rightarrow v = \frac{5t^2}{m}$$

$$\text{Momentum: } P = mv = 5t^2$$

$$\text{At } t = 2 \text{ s, } P = 5(2)^2 = 20 \text{ kg ms}^{-1}, v = 20/m$$

$$\text{From 2 to 4 s: } F = 40 - 10t$$

$$\int_{20/m}^v dv = \int_2^t \frac{40 - 10t}{m} dt \Rightarrow v = \frac{1}{m} [40t - 5t^2]$$

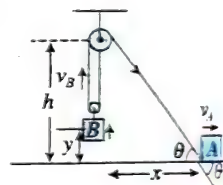
$$P = mv = 40t - 5t^2$$

36. (3) From figure, $L = 2h - 2y + \sqrt{x^2 + h^2}$

Differentiating the equation, we get

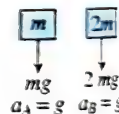
$$\frac{dy}{dt} = \frac{x}{2\sqrt{h^2 + x^2}} \frac{dx}{dt}$$

$$\Rightarrow v_B = \frac{xv_A}{2\sqrt{h^2 + x^2}}$$



37. (1) In this case, spring force is zero initially. FBD of A and B are shown in figure.

Tension in the string and spring will be zero just after release.



38. (4) $a_1 = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$ and $a_2 = \frac{(m_2 - m_1)g}{m_1}$

$$\text{Hence } \frac{a_1}{a_2} = \frac{m_1}{(m_1 + m_2)} = \frac{1}{\left(1 + \frac{m_2}{m_1}\right)} \Rightarrow \frac{a_1}{a_2} < 1$$

$$\text{As } T_1 = \frac{2m_1 m_2 g}{(m_1 + m_2)} \text{ and } T_2 = m_2 g$$

$$\text{Hence, } \frac{T_1}{T_2} = \frac{2m_1}{(m_1 + m_2)} = \frac{2}{\left(1 + \frac{m_2}{m_1}\right)}$$

Therefore, T_1/T_2 will depend upon the values of m_1 and m_2 .

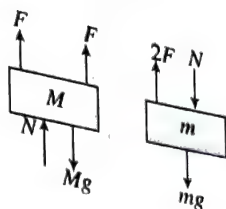
$$N_1 = 2T_1, N_2 = 2T_2$$

So the relation of N_1/N_2 will be same as T_1/T_2 .

39. (2) $a = \frac{3T - mg \sin \theta}{m}$

$$= \frac{3 \times 250 - 100 \times 10 \times \sin 30^\circ}{100} = 2.5 \text{ ms}^{-2}$$

(3) From figure,
 $2F + N - Mg = Ma$
 $2F - mg - N = ma$
 $4F - (M + m)g = (M + m)a$
 $a = \frac{4F - (M + m)g}{M + m}$



(4) Extension in the spring is

$$x = AB - R = 2R \cos 30^\circ - R = (\sqrt{3} - 1)R$$

Spring force:

$$F = kx = \frac{(\sqrt{3} + 1)mg}{R} \times (\sqrt{3} - 1)R = 2mg$$

From the figure, we have

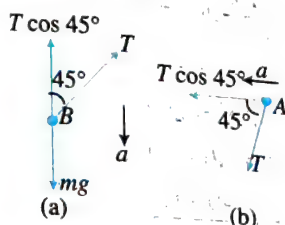
$$N = (F + mg) \cos 30^\circ = \frac{3\sqrt{3}mg}{2}$$

(4) As shown in figure (a and b) from FBD of A,

$$T \cos 45^\circ = ma$$

From FBD of B:

$$Mg - T \cos 45^\circ = ma$$



From (i) and (ii), we get $T = mg/\sqrt{2}$

(3) By virtual work method:

Acceleration of B w.r.t. A will be 10 ms^{-2} downward.

Apart from this, B also has an acceleration 5 ms^{-2} in horizontal direction along with A, so net acceleration of B is

$$\sqrt{10^2 + 5^2} = \sqrt{100 + 25} = \sqrt{125} = 5\sqrt{5} \text{ ms}^{-2}$$

(3) In equilibrium, if θ is the required angle,

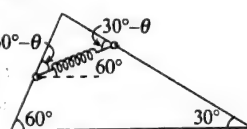
Cylinder A:

$$mg \sin 60^\circ = kx \cos (60^\circ - \theta)$$

Cylinder B:

$$mg \sin 30^\circ = kx \cos (30^\circ + \theta)$$

$$= kx \sin(60^\circ - \theta) \text{ on solving } \theta = 30^\circ$$

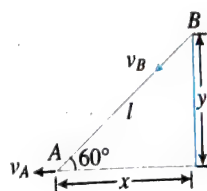


(2) Here y is constant $\frac{dl}{dt} = v_B$

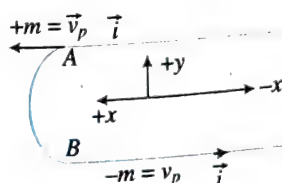
$$l^2 = y^2 + x^2$$

$$\Rightarrow 2l \frac{dl}{dt} = 2x \frac{dx}{dt}$$

$$\Rightarrow v_B = \frac{x}{l} v_A = v_A \cos 60^\circ = 1 \text{ ms}^{-1}$$



(2) Choosing the positive x-y axis as shown in the figure, the momentum of the bead at A is $\vec{p}_i = +m\vec{v}$. The momentum of the bead at B is $\vec{p}_f = -m\vec{v}$



Therefore, the magnitude of the change in momentum between A and B is $\Delta \vec{p} = \vec{p}_f - \vec{p}_i = -2m\vec{v}$, i.e., $\Delta p = 2mv$ along the positive x-axis.

The time taken by the bead to reach from A to B is

$$\Delta t = \frac{\pi d/2}{v} = \frac{\pi d}{2v}$$

Therefore, the average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \left(\frac{2mv}{\frac{\pi d}{2v}} \right) = \frac{4mv^2}{\pi d}$$

47. (2) Equation of motion for M:

Since M is stationary

$$T - Mg = 0 \Rightarrow T = Mg$$

Since the boy moves up with acceleration a,

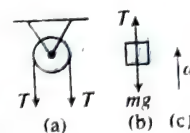
$$\therefore T - mg = ma$$

$$\Rightarrow T = m(g + a)$$

Equating Eqs. (i) and (ii), we obtain

$$Mg = m(g + a)$$

$$\Rightarrow a = \left(\frac{M}{m} - 1 \right) g, \text{ the block M can be lifted}$$

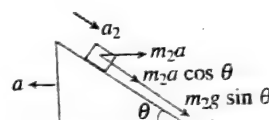


48. (1) In first case, acceleration of m_1 will be $a_1 = g \sin \theta$ down the inclined plane. In second case, acceleration of m_2 w.r.t. incline is

$$a_2 = \frac{m_2 g \sin \theta + m_2 a \cos \theta}{m_2}$$

$$\Rightarrow a_2 = g \sin \theta + a \cos \theta$$

Since $a_2 > a_1$, so $T_2 < T_1$



49. (3) (Check figure in the frame of the car)

Applying Newton's law perpendicular to string,

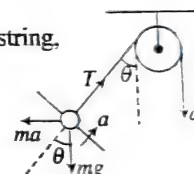
$$mg \sin \theta = ma \cos \theta$$

$$\Rightarrow \tan \theta = \frac{a}{g}$$

Applying Newton's law along string,

$$T - mg \cos \theta - ma \sin \theta = ma$$

$$\Rightarrow T = m\sqrt{g^2 + a^2} + ma$$

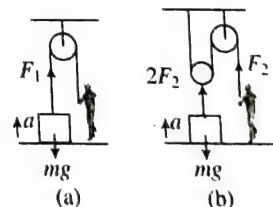


50. (1) Since, $h = \frac{1}{2}at^2$, a should be same in both cases, because h and t are same in both cases as given.

In figure (a), $F_1 - mg = ma \Rightarrow F_1 = mg + ma$

In figure (b), $2F_2 - mg = ma \Rightarrow F_2 = \frac{mg + ma}{2}$

$$\therefore F_1 > F_2$$



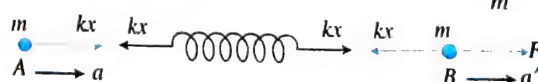
51. (3) Equation of motion for A (figure),

$$kx = ma \Rightarrow a = \frac{kx}{m}$$

For B: $F - T = ma'$

$$\Rightarrow a' = \frac{F - kx}{m}$$

Thus, relative acceleration $= a_r = |a' - a| = \frac{F - 2kx}{m}$



52. (2) As the springs have natural length initially, if one spring is compressed, the other must be expanded. Hence, the compression will be negative.

The free-body diagram of m_2 [Figure]

$$T + F_2 = 80 \text{ N} \quad \text{and} \\ F_2 = 70 \times 0.5 = 35 \text{ N}$$

$$\therefore T = 80 - 35 = 45 \text{ N}$$

FBD of m_1 (figure)

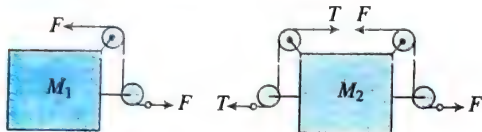
$$T + F_1 = m_1 g \quad \text{or} \quad F_1 = -25 \text{ N}$$

$$\therefore X_1 = \frac{-25}{k_1} = \frac{-25}{50} = -0.5 \text{ m}$$

Therefore, compression in first spring is -0.5 m .
(negative sign indicates that it is extension)

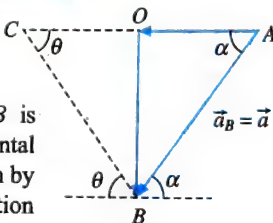
53. (2) From FBD, it is obvious that net force on each block is zero in horizontal direction. So

$$a_1 = a_2 = 0$$



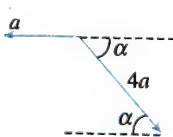
54. (4) Direction of acceleration of B is along the fixed incline, and that of A is along horizontal towards left.

From diagram, acceleration of B is represented by $\vec{AB}$ while its horizontal and vertical components are shown by $\vec{AO}$ and $\vec{OB}$, respectively. Acceleration of A is represented by $\vec{AC}$



$$\vec{AC} = a(\sin \alpha \cot \theta + \cos \alpha)$$

55. (2) If the wedge moves leftward by x , then the block moves down the wedge by $4x$, i.e., w.r.t. wedge the block comes down by $4x$.
So, acceleration of block w.r.t wedge $= 4a$ along the incline plane of wedge.



Acceleration of wedge with respect to ground $= a$, along left.
So acceleration of block w.r.t. ground is the vector sum of the two vectors shown in figure. That is

$$|\vec{a}_{BG}| = \sqrt{a^2 + (4a)^2 + 2 \times a \times 4a \times \cos(\pi - \alpha)} \\ = (\sqrt{17 - 8\cos \alpha}) a \text{ ms}^{-2}$$

56. (4) Using constraint theory,

$$l_1 + l_2 + l_3 = \text{constant}$$

$$\Rightarrow v_1 + 2v_2 + v_3 = 0$$

Take downward as positive and upward as $-ve$.

$$\text{So, } 12 + 2(-4) + v_3 = 0$$

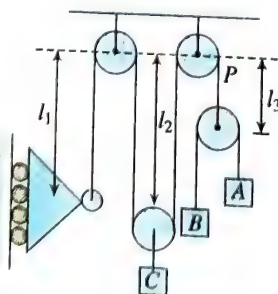
$$v_3 = \text{velocity of pulley } P = -4 \text{ ms}^{-1} \\ = 4 \text{ ms}^{-1} \text{ in upward direction}$$

$$\vec{v}_{AP} = -\vec{v}_{BP}$$

$$\Rightarrow v_{AP} = -(v_B - v_P)$$

$$v_B = v_P - v_{AP} = -4 - (-3) = -7 \text{ ms}^{-1}$$

i.e., block B will move up with speed 7 ms^{-1} .



57. (2) $\vec{u} = 4\hat{i} + 2\hat{j}$, $\vec{a} = \frac{\vec{F}}{m} = \hat{i} - 4\hat{j}$

Let at any time, the coordinates be (x, y) .

$$x - 2 = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x - 2 = 4t + \frac{1}{2} t^2 \quad \text{and} \quad y - 3 = 2t - \frac{1}{2} 4t^2$$

$$\Rightarrow y - 3 = 2t - 2t^2$$

When $y = 3 \text{ m}$, $t = 0, 1 \text{ s}$; when $t = 0$, $x = 2 \text{ m}$

When $t = 1 \text{ s}$, $x = 6.5 \text{ m}$

58. (3) Let the plank move up by x , then pulley 2 will move down by x . Let the end of string C moves down by a distance y .

Let the initial length of string passing over pulley 2,

$$l_1 + l_2 \quad \dots(i)$$

After displacements x and y mentioned above, the lengths become

$$(l_1 - 2x) + (l_2 + y - x) \quad \dots(ii)$$

Equating Eqs. (i) and (ii), we get $y = 3x$

Length of string that slips through A is $y + x = 4x$ and through B is $y = 3x$.

$$\text{Required ratio} = \frac{4x}{3x} = \frac{4}{3}$$

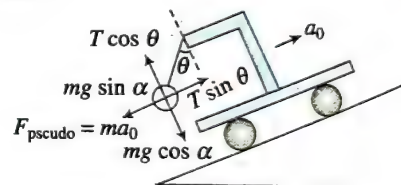


59. (3) $T = 2ma$

$$mg - T = ma$$

$$a = g/3$$

60. (4) As in Figure

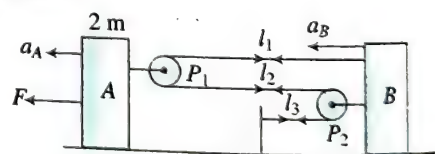


$$T \sin \theta = ma_0 + mg \sin \alpha$$

$$T \cos \theta = mg \cos \alpha$$

$$\tan \theta = \frac{a_0 + g \sin \alpha}{g \cos \alpha}$$

61. (2) As in Figure,



$$l_1 + l_2 + l_3 = C$$

$$l_1' + l_2' + l_3' = 0 \Rightarrow -V_B + V_B - V_B + V_A - V_B = 0$$

$$3V_B = 2V_A \Rightarrow 3a_B = 2a_A$$

Applying Newton's law on A and B,

$$F - 2T = 2ma_A, \quad 3T = 2ma_B$$

$$\text{Solve to get } a_B = \frac{3F}{13m}$$

62. (2) If initial acceleration of M towards right is A, then we can show that acceleration of m w.r.t. M down the incline is

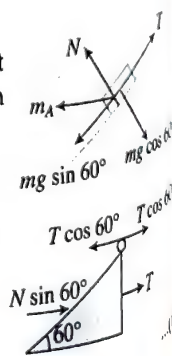
$$a = A(1 + \cos \theta) = \frac{3A}{2} \quad (\because \theta = 60^\circ)$$

FBD of block m (w.r.t. M) is shown below:

FBD of M (Figure)

Equation of motion:

$$\text{For } m: mg \frac{\sqrt{3}}{2} + mA \times \frac{1}{2} - T = m \frac{3}{2} A$$



$$N + mA \frac{\sqrt{3}}{2} = mg \frac{1}{2} \quad \dots(ii)$$

$$\text{For } M: T + N \frac{\sqrt{3}}{2} = MA \quad \dots(iii)$$

$$\text{From Eqs. (i), (ii), and (iii)} \quad A = \frac{3\sqrt{3}g}{23} \text{ m s}^{-1}$$

- (4) Acceleration of two mass systems is $a = \frac{F}{2m}$ leftwards. FBD of block A is shown in below.

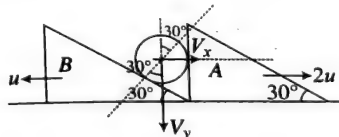
$$N \cos 60^\circ - F = ma = \frac{mF}{2m}$$

Solving, we get $N = 3F$

- (4) **Method 1:** Velocity components perpendicular to the contact surface remain same.

As cylinder will remain in contact with wedge A

$$V_x = 2u$$

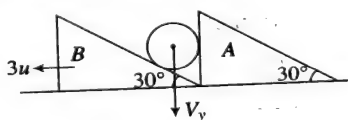


As it also remain in contact with wedge B,
 $u \sin 30^\circ = V_y \cos 30^\circ - V_x \sin 30^\circ$

$$V_y = V_x \frac{\sin 30^\circ}{\cos 30^\circ} + \frac{u \sin 30^\circ}{\cos 30^\circ} = 3u \tan 30^\circ = \sqrt{3}u$$

$$\Rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$$

Method 2: In the frame of A



$$3u \sin 30^\circ = V_y \cos 30^\circ$$

$$V_y = 3u \tan 30^\circ = \sqrt{3}u \text{ and } V_x = 2u$$

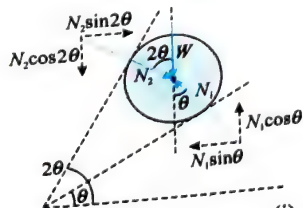
$$\Rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$$

$$5. (1) v_2 \cos \alpha + v_1 \cos \alpha = v_1 \Rightarrow v_2 = v_1 \left[\frac{2 \sin^2(\alpha/2)}{\cos \alpha} \right]$$

$$6. (3) \text{ In horizontal direction: } N_1 \sin \theta = N_2 \sin \theta$$

$$N_1 = N_2 \cos \theta$$

$$\frac{N_1}{N_2} = \cos \theta$$

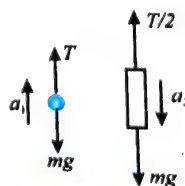


$$67. (1) a_2 = 2a_1 \quad \dots(i)$$

$$T - mg = ma_1 \quad \dots(ii)$$

$$mg - \frac{T}{2} = ma_2 \quad \dots(iii)$$

By solving (i), (ii) & (iii), we get



$$a_1 = \left(\frac{g}{5} \right) \text{ m/s}^2, a_2 = \left(\frac{2g}{5} \right) \text{ m/s}^2$$

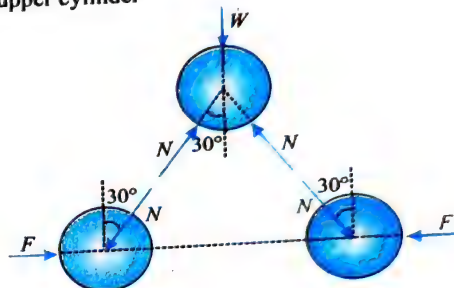
$$\bar{a}_{12} = \bar{a}_1 - \bar{a}_2 = \frac{g}{5} - \left(-\frac{2g}{5} \right) = \frac{3g}{5}$$

$$\bar{s}_{12} = \bar{u}_{12} t + \frac{1}{2} \bar{a}_{12} t^2 \text{ and } \bar{s}_{12} = l$$

$$\therefore l = 0 + \frac{1}{2} \times \frac{3}{5} g t^2$$

$$\Rightarrow t = \sqrt{\frac{10l}{3g}}$$

68. (4) For upper cylinder



$$2N \cos 30^\circ = W$$

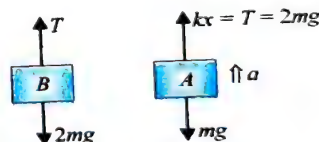
For lower block

$$N \cos 60^\circ = F$$

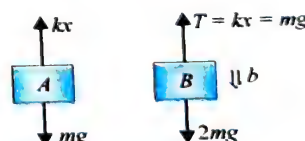
From (i) & (ii)

$$\frac{2 \cos 30^\circ}{\cos 60^\circ} = \frac{W}{F} \Rightarrow F = \frac{W}{2\sqrt{3}}$$

69. (1) In 1st case



In 2nd case

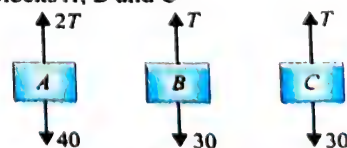


$$T = 2mg \text{ \& } a = \frac{2mg - mg}{m} \Rightarrow a = g \uparrow$$

$$kx = mg \text{ \& } b = \frac{2mg - mg}{2m} \Rightarrow b = \frac{g}{2} \uparrow$$

$$\therefore \frac{a}{b} = 2$$

70. (2) FBD of the blocks A, B and C



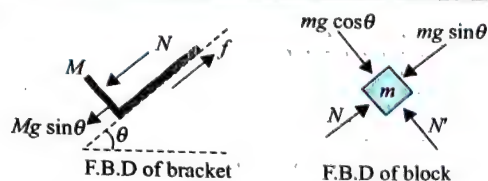
Conditions of forces on B & C are same so they both will move with same downward acceleration 'a'. The block A moves with the same acceleration a upwards.

$$\text{From block A: } 2T - 40 = 4a$$

$$\text{From block B or C: } 30 - T = 3a$$

$$\text{From above two equations } a = 2 \text{ m/s}^2$$

71. (4)



$$N' = mg \cos \theta$$

$$\Rightarrow R = \sqrt{N^2 + N'^2} = \sqrt{(mg \sin \theta)^2 + (mg \cos \theta)^2} = mg$$

72. (4) $N = mg - m(\alpha t)$

$$N = (-m\alpha)t + mg$$

73. (2) Mass of block A: $m_A = M$

$$\text{Mass of block B; } m_B = M - m$$

Acceleration of B after the insect falls,

$$a_B = \left(\frac{m_A - m_B}{m_A + m_B} \right) g (\uparrow) = \frac{mg}{2M - m}$$

Acceleration of the insect = $g(\downarrow)$

The two objects separate with a relative acceleration of

$$a = g + \frac{mg}{2M - m} = \frac{2Mg}{2M - m}$$

$$\therefore \frac{1}{2} at^2 = L \quad \left(\frac{Mg}{2M - m} \right) t^2 = L$$

$$\Rightarrow t = \sqrt{\frac{(2M - m)L}{Mg}}$$

74. (1) Acceleration of the two blocks,

$$a = \frac{mg \sin 60^\circ - mg \sin 30^\circ}{3m} = \left(\frac{\sqrt{3} - 1}{4} \right) g$$

If the two blocks move a distance x along respective inclines in time t , $x = \frac{1}{2} at^2$

The centre of two blocks will be at equal height if

$$x \sin 60^\circ + x \sin 30^\circ = h$$

$$\therefore x \left(\frac{\sqrt{3}}{2} + \frac{1}{2} \right) = h \Rightarrow \frac{1}{2} \left(\frac{\sqrt{3} - 1}{4} \right) g t^2 \left(\frac{\sqrt{3} + 1}{2} \right) = h$$

$$\therefore t^2 = \frac{16h}{(\sqrt{3} - 1)(\sqrt{3} + 1)g} = \frac{8h}{g}$$

$$\Rightarrow t = 2\sqrt{2} \sqrt{\frac{h}{g}}$$

75. (4) For pulley $2F = ma$

$$\text{For particle } F = ma'$$

$$\text{Hence, } a = 2a' = 2F/m$$

Acceleration of the point of application of the force is

$$= 2a + a' = \frac{5a}{2} = \frac{5F}{m}$$

Multiple Correct Answers Type

1. (1), (4)

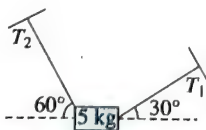
$$T_2 \cos 60^\circ = T_1 \cos 30^\circ \quad \dots(i)$$

$$\text{and } T_2 \sin 60^\circ + T_1 \sin 30^\circ = 5g \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$T_1 = 25 \text{ N and } T_2 = 25\sqrt{3} \text{ N}$$

2. (2), (3), (4)

Acceleration of particle w.r.t. frame S_1 : $\vec{a}_p - \vec{a}_{s_1} = 2\hat{n}$ Acceleration of particle w.r.t. frame S_2 : $\vec{a}_p - \vec{a}_{s_2} = 2\hat{m}$ where $\hat{m}$ and $\hat{n}$ are unit vectors in any directions. Now relative acceleration of frames:

$$\vec{a}_{s_2} - \vec{a}_{s_1} = 2(\hat{n} - \hat{m})$$

Its magnitude can have any value between 0 to 4 ms^{-2} , depending upon the directions of $\hat{m}$ and $\hat{n}$.

3. (1), (3)

$$a_1 = \frac{2mg - mg}{m} = g$$

$$a_2 = \frac{mg + mg - mg}{2m} = g/2$$

$$a_3 = \frac{2mg - mg}{3m} = g/3$$

$$\text{Clearly } a_1 > a_2 > a_3$$

4. (1), (3), (4)

In first case, m will remain at rest. $a_m = F/M$

In second case, both will accelerate

$$a_m = a_M = F/(M + m)$$

In second case, force on $m = ma_m = mF/(M + m)$

5. (1), (3)

$$Mg - T = Ma \quad \dots(i)$$

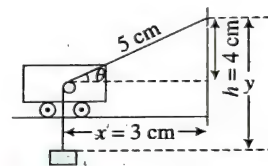
$$T = ma \quad \dots(ii)$$

$$\text{Solving Eqs. (i) and (ii), } a = \frac{Mg}{M + m}$$

FBD of man,

$$Mg - N = Ma \Rightarrow N = \frac{Mmg}{(M + m)}$$

6. (3), (4)



$$(y - h) + \sqrt{x^2 + h^2} = l \quad \text{or} \quad \frac{dy}{dt} + \frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{\sqrt{x^2 + h^2}} \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{3}{5} (-v_A)$$

$$v_B = \frac{3}{5} v_A \quad \dots(i)$$

$$\frac{d^2y}{dt^2} = \frac{v_A^2 h^2}{(x^2 + h^2)^{3/2}} \Rightarrow a_B = v_A^2 \frac{16}{(5)^3} = \frac{16}{125} v_A^2 \quad \dots(ii)$$

7. (1), (3)

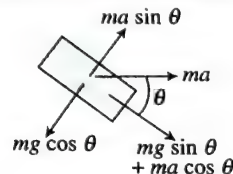
$$M'g - T = M'a \quad \dots(i)$$

$$T = Ma \quad \dots(ii)$$

$$M'g = a(M + M')$$

$$\Rightarrow a = \frac{M'g}{(M + M')}$$

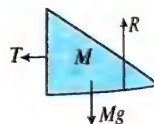
$$ma \sin \theta = mg \cos \theta \rightarrow \text{so that normal force is zero.}$$



$$a = g \cot \theta$$

$$g \cot \theta = \frac{M'g}{(M + M')} \Rightarrow \cot \theta M + \cot \theta M' = M'$$

$$\text{or } M' = \frac{M \cot \theta}{(1 - \cot \theta)}, T = Ma = Mg \cot \theta = Mg/\tan \theta$$



8. (1), (4)

$$T \cos \theta_0 = mg$$

$$T \sin \theta_0 = ma_0$$

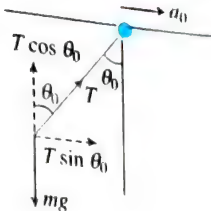
Dividing Eq. (ii) by (i), we get

$$\tan \theta_0 = \frac{a}{g} \Rightarrow \theta_0 = 30^\circ$$

$$T = \frac{mg}{\cos 30^\circ} = \frac{2mg}{\sqrt{3}}$$

... (i)

... (ii)

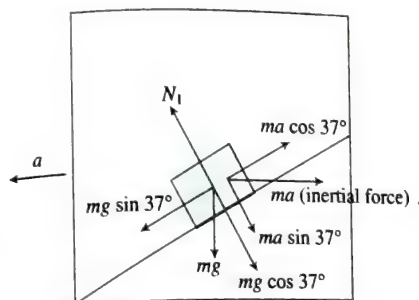


9. (1), (4)

 Acceleration of M, $a = \left(\frac{F}{M} \right)$

$$l = \frac{1}{2} \frac{F}{M} t^2 \Rightarrow t = \sqrt{\frac{2Ml}{F}}$$

10. (1), (3)

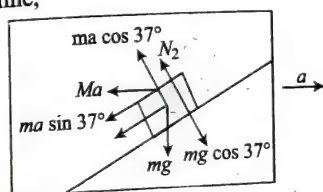


Balancing forces perpendicular to incline

$$N_1 = mg \cos 37^\circ + ma \sin 37^\circ$$

$$N_1 = \frac{4}{5}mg + \frac{3}{5}ma$$

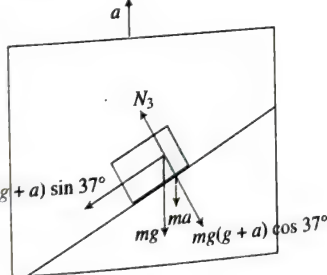
And along incline,



$$mg \sin 37^\circ - ma \cos 37^\circ = mb_1$$

$$b_1 = \frac{3}{5}g - \frac{4}{5}a$$

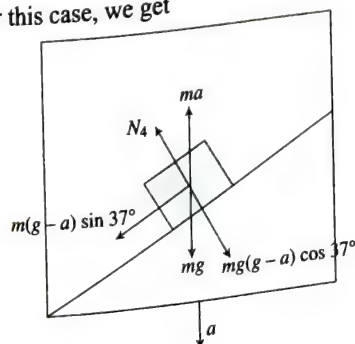
Similarly, for this case, we get



$$N_2 = \frac{4}{5}mg - \frac{3}{5}ma$$

$$\text{And } b_2 = \frac{3}{5}g + \frac{4}{5}a$$

Similarly, for this case, we get



$$N_3 = \frac{4}{5}mg + \frac{4}{5}ma$$

$$b_1 = \frac{3}{5}g + \frac{3}{5}a$$

Similarly, for this case, we get

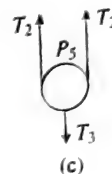
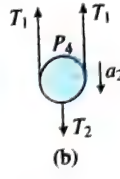
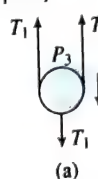
$$N_4 = \frac{4}{5}mg - \frac{4}{5}ma$$

$$\text{and } b_4 = \frac{3}{5}g - \frac{3}{5}a$$

11. (1), (3)

 First of all, draw FBD of P_3 . Let tensions, in three strings be T_1 , T_2 and T_3 , respectively.

$$2T_1 - T_1 = 0 \times a \Rightarrow T_1 = 0$$

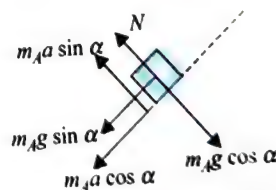
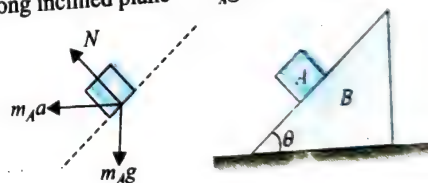

 Now draw FBD of P_4 and P_5 .

$$2T_1 - T_2 = 0 \Rightarrow T_2 = 0$$

$$2T_2 - T_3 = 0 \Rightarrow T_2 = T_3 = 0$$

 So forces acting on P_6 and P_7 will be that of gravity and they will be in free fall. Hence, acceleration of each of them will be g downwards.

12. (1), (4)

 Force along inclined plane = $m_A g \sin \alpha + m_A a \cos \alpha$

 Acceleration along inclined plane = $g \sin \alpha + a \cos \alpha$

 So, $a_A > g \sin \alpha$

$$N = m_A g \cos \alpha - m_A a \sin \alpha$$

$$N < m_A g \cos \alpha$$

13. (1), (2), (4)

Acceleration of 2 kg block

$$(a_2) = \frac{50 - 20}{2} = 15 \text{ m/s}^2 \uparrow$$

Acceleration of 4 kg block

$$(a_4) = \frac{50 - 40}{4} = 2.5 \text{ m/s}^2 \uparrow$$

14. (1), (3), (4)

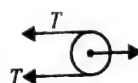
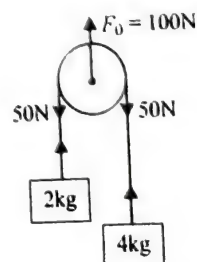
$$15 - T = 10a$$

$$T = 5a$$

$$a = 1 \text{ m/s}^2$$

$$T = 5 \text{ N}$$

$$2T = 10 \text{ N}$$



(1), (3), (4)

$$F = (m_B + m_C)a$$

$$T = 10a$$

$$T = m_A a = m_A \cdot 4$$

$$T + m_A(A - a)$$

$$T = 30(A - a)$$

$$N = m_C a$$

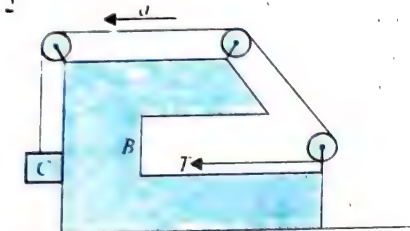
$$m_C g - T = m_C a$$

$$50 - T = 5a$$

$$1 + 2a = 10 \text{ and } 10a = 30(A - a)$$

$$4a = 3A$$

$$1 + \frac{3A}{2} = 10$$



$$A = 4, a = 3, T = 30 \text{ N}, N = 15 \text{ N}$$

Contact force between B and ground is 380 N

$$N_G = 5g + 30g + T = 380 \text{ N}$$

(1), (3), (4)

$$2T = 5g$$

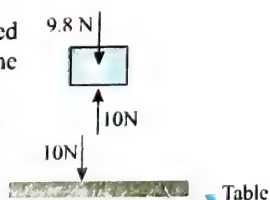
$$4T = 10g$$

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = 2.5g$$

$$\Rightarrow \frac{m_1 m_2}{m_1 + m_2} = \frac{5}{4}g$$

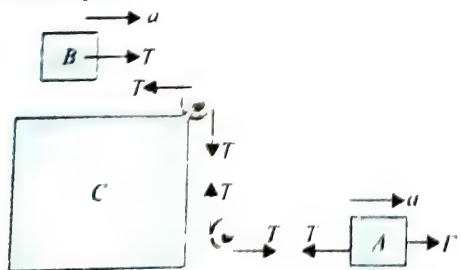
(1), (4)

Newton's third law states that force applied by one object on other is exactly of same magnitude that applied by other on first. As net force on the block is in upward direction hence the block accelerates in upward direction.



(2), (3)

From F.B.D. it is clear that no net force acts on the block C. Hence the acceleration of block C should be zero. The blocks A and B are connected directly by a string they will move with common acceleration (say a).



$$\text{For block B, } T = 10a \quad \dots(i)$$

$$\text{For block A, } F - T = 10a \quad \dots(ii)$$

$$\text{From (i) \& (ii), } a = \frac{F}{20} = \frac{20}{20} = 1 \text{ m/s}^2$$

$$\text{and } T = 10 \times 1 = 10 \text{ N}$$

19. (1), (2), (3), (4)

The acceleration of block A, w.r.t pulley P_1 ;

$$a_{AP_1} = (a_1 - a) \uparrow$$

$$a_{BP_1} = (a_2 - a) \downarrow$$

The acceleration of block B, w.r.t pulley P_1 ;

$$\text{But, } |\vec{a}_{AP_1}| = -|\vec{a}_{BP_1}|$$

$$a_1 - a = a_2 + a$$

$$\text{Hence, } a_2 = 2a - a_1 \uparrow$$

The acceleration of block C w.r.t pulley P_2 ;

$$a_{CP_2} = (a_c + a) \uparrow$$

The acceleration of block D w.r.t pulley P_2 ;

$$a_{DP_2} = (a_d - a) \downarrow$$

$$a_c + a = a_d - a$$

$$a_d = (2a + a_1) \downarrow$$

$$\text{Hence, } \vec{a}_{BC} = \vec{a}_B - \vec{a}_C = 2(a_1 - a) \downarrow$$

$$\text{and } \vec{a}_{BD} = \vec{a}_B - \vec{a}_D = 4a \uparrow$$

20. (1), (3)

For equilibrium $T_1 = Mg$ and $T_2 = mg$

But $2T_2 = 2T_1$ (for equilibrium of pulley P_1)

$$\therefore m = M$$

When $m = 2M$, let acceleration of A be a ($\uparrow$).

Acceleration of pulley P_2 will be $\frac{a}{2}$ ($\downarrow$)

Acceleration of P_1 = acceleration of $P_2 = \frac{a}{2}$ ($\downarrow$)

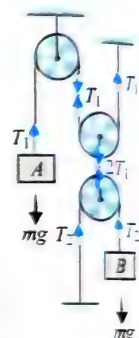
Acceleration of B = $2 \times$ acceleration of $P_2 = a$ ($\downarrow$)

For pulley P_1 ; $2T_1 = 2T_2 \Rightarrow T_1 = T_2 \quad \dots(i)$

For A; $T_1 - Mg = Ma \quad \dots(ii)$

For B; $2Mg - T_2 = 2Ma \quad \dots(iii)$

Solving (i), (ii) and (iii); $a = \frac{g}{3}$



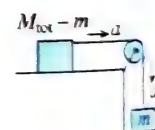
Linked Comprehension Type

For Problems 1 and 2

$$1. (1) \quad a = \left(\frac{m}{M_{\text{tot}}} \right) g$$

$$2. (4) \quad T = \frac{M_1 M_2 g}{M_1 + M_2} = \frac{m(M_{\text{tot}} - m)}{M_{\text{tot}}} g$$

$$= \left(\frac{m(M_{\text{tot}} - m^2)}{M_{\text{tot}}} \right) g$$



For Problems 3-5

3. (1) Let m_1 and m_2 do not accelerate up or down, then

$$F_1 = m_1 g, F_2 = m_2 g. \text{ But } m_1 \neq m_2, \text{ so } F_1 \neq F_2.$$

Hence net horizontal force on M is $F_1 - F_2$. So M cannot be in equilibrium. If M accelerates horizontally, then m_1 and m_2 also accelerate horizontally.

$$4. (3) \quad a = \frac{\text{Net force}}{\text{Net mass}} = \frac{F_1 - F_2}{M + m_1 + m_2} = \frac{m_1 g - m_2 g}{M + m_1 + m_2}$$

5. (4) In horizontal direction, both m_1 and m_2 will have same acceleration for any case. In vertical direction also, they should have same acceleration. Let it be a upwards for both, then

$$F_1 - m_1 g = m_1 a \quad \dots(i)$$

$$F_2 - m_2 g = m_2 a \quad \dots(ii)$$

From (i) and (ii), $\frac{F_1}{m_1} = \frac{F_2}{m_2}$

Problems 6-8

6. (4) Initial velocity: $u = 0$

$$I = \int F dt \Rightarrow m(v - u) = \int_0^t (6t - 2t^2) dt$$

$$\Rightarrow 2v = 3t^2 - \frac{2}{3}t^3$$

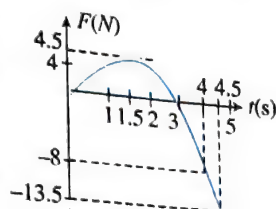
Put $v = 0$, we get $t = 4.5$ s

7. (2) If we draw $F-t$ graph, we can see that upto $t = 3$ s, force is positive, so velocity increase up to 3 s and after that velocity starts decreasing. So at 3 s, velocity is maximum.

8. (2) We can see from the figure above, when force becomes maximum (at $t = 1.5$ s) for the first time, velocity is not maximum at that instant. Hence, (1) is incorrect.

Option (2) is correct as explained in the above solution.

Option (3) is incorrect because when force becomes zero, velocity does not become zero.



Problems 9-11

9. (3) For equilibrium of block A

$$F = N \sin \theta$$

$$N = F / \sin \theta$$

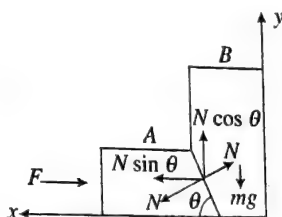
To lift block B from ground

$$N \cos \theta \geq mg$$

$$\Rightarrow \frac{F}{\sin \theta} \cos \theta \geq mg$$

$$F \geq mg \tan \theta = mg \left(\frac{3}{4} \right)$$

$$\text{So, } F_{\min} = \frac{3}{4} mg$$



10. (3) If both the blocks are stationary.

Balancing forces along x-direction

$$F = N \sin \theta$$

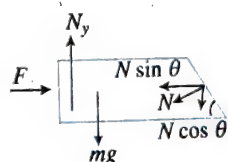
$$\Rightarrow N = F / \sin \theta$$

Balancing forces along y-direction

$$N_y = mg + N \cos \theta$$

$$= mg + \left(\frac{F}{\sin \theta} \right) \cos \theta = mg + F \cot \theta$$

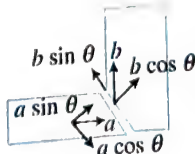
$$N_y = mg + \frac{4F}{3}$$



11. (1) To keep regular contact

$$a \sin \theta = b \cos \theta$$

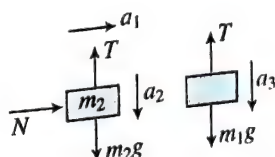
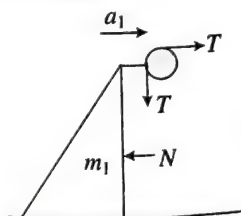
$$b = a \tan \theta = \frac{3}{4} a$$



Problems 12-14

12. (1), 13. (4), 14. (2)

Free-body diagrams



Constraint relations:

$$x_1 = x_2 + x_3$$

$$a_1 = a_2 + a_3$$

Equation of motion:

$$T - N = m_1 a_1$$

$$N = m_2 a_1$$

$$m_2 g - T = m_2 a_2$$

$$m_3 g - T = m_3 a_3$$

...(i)

...(ii)

...(iii)

...(iv)

Using above equations, we can calculate the values.

For Problems 15-18

15. (1), 16. (2), 17. (3), 18. (3)

Acceleration of different objects is shown in figure. All the accelerations are w.r.t. ground

Given: $a_3 = a_4$

$$a_2 + a_1 = 1$$

$$a_3 + a_1 = 5$$

...(i)

...(ii)

...(iii)

From figure, we can write

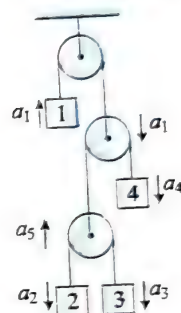
$$-a_5 = \frac{a_2 + a_3}{2} \quad \dots(\text{iv})$$

$$a_1 = \frac{a_4 - a_5}{2} \quad \dots(\text{v})$$

Solving the above equations, we get

$$a_1 = 2 \text{ ms}^{-2}, a_2 = -1 \text{ ms}^{-2},$$

$$a_3 = a_4 = 3 \text{ ms}^{-2}$$



For Problems 19-21

19. (2), 20. (1), 21. (4)

$$T_3 = 20t, T_1 = T_2 = 10t$$

For A to lose contact: $10t = 1g \Rightarrow t = 1$ s

For B to lose contact: $10t = 2g \Rightarrow t = 2$ s

For C to lose contact: $20t = 3g \Rightarrow t = 1.5$ s

$$a_A = \frac{T_1 - 1g}{1} \text{ Velocity of A when B loses contact}$$

$$V_1 = \int_1^2 a_A dt = \int_1^2 (10t - g) dt = 5 \text{ ms}^{-1}$$

$$\text{At } t = 2 \text{ s, } a_B = 0, a_A = \frac{10 \times 2 - 10}{1} = 10 \text{ ms}^{-2}$$

$$a_{A/B} = a_A - a_B = 10 - 0 = 10 \text{ ms}^{-2}$$

For Problems 22-24

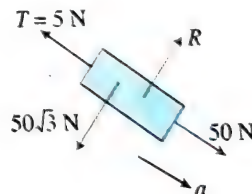
22. (4), 23. (1), 24. (1)

$$mg \sin 30^\circ - T = ma$$

$$50 - 5 = 10a$$

$$a = 4.5 \text{ ms}^{-2}$$

$$a_{P_4} = \frac{0 + 4.5}{2} = 2.25 \text{ ms}^{-1}$$



For Problems 25-27

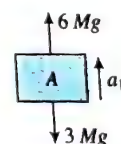
25. (4), 26. (2), 27. (1)

Force in spring can't change abruptly whereas tension in string can change.

25. When PQ is cut, no effect on the forces acting on C, hence its acceleration remains zero.

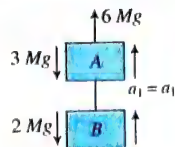
26. Tension in PQ is $6Mg$ before cutting RS.

$$\text{After cutting RS, } a_1 = \frac{6Mg - 3Mg}{3M} = g$$



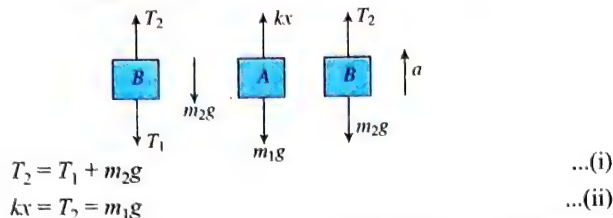
27. After TU is cut

$$a_1 = a_2 = \frac{6Mg - 3Mg - 2Mg}{5M} = g/5$$



For Problems 28–30

28. (2), 29. (1), 30. (4)

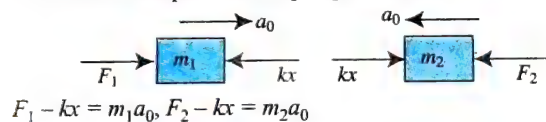
Before burning BC , the free-body diagrams are shown in the figure.where x is extension in the spring. Just after burning, T_1 will become zero, but T_2 will remain same.

$$T_2 - m_2g = m_2a \Rightarrow a = \frac{(m_1 - m_2)g}{m_2}$$

As T_2 remains same, acceleration of block A will still remain zero.

For Problems 31–33

31. (2), 32. (4), 33. (3)

Let x be the compression in spring at any time

$$\text{Solve to get: } a_0 = \frac{F_1 - F_2}{m_1 - m_2} \text{ and } kx = \frac{m_1F_2 - F_1m_2}{m_1 - m_2}$$

Just after m_2 is removed:

$$a_2 = \frac{kx}{m_2} = \frac{F_2 - m_2a_0}{m_2} = \frac{F_2}{m_2} - a_0$$

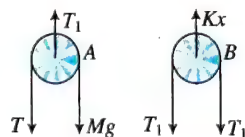
For Problems 34–36

34. (1) $T = Mg$

$$T_1 = T + Mg = 2Mg$$

$$Kx = 2T_1$$

$$\text{or } x = \frac{2T_1}{K} = \frac{4Mg}{K}$$



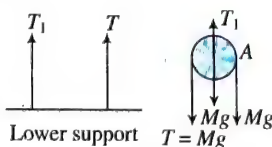
35. (3) Total tension on lower support:

$$T_1 + T = 2Mg + Mg = 3Mg$$

36. (4) $T_1 = 3Mg$

Net tension at lower support

$$T + T_1 = Mg + 3Mg = 4Mg$$



Matrix Match Type

1. i $\rightarrow$ c, d; ii $\rightarrow$ a, c; iii $\rightarrow$ a, c; iv $\rightarrow$ c, d.

For (i), it is not mentioned whether the object is accelerated or moving with constant velocity. So nothing can be predicted with surety.

If no net force is acting along east, then also it can move with constant velocity, and if no force is acting at all, then also it can move with constant velocity.

For (ii) and (iii): As the object is accelerated (whether uniform or non-uniform) a force must act on the object in such a manner that a component or whole of the force would be along east, and also the net force must be towards east.

For (iv): It is moving with constant velocity, so net force must be zero that implies no force may act on the object.

2. i $\rightarrow$ a, c; ii $\rightarrow$ b, c; iii $\rightarrow$ b, c; iv $\rightarrow$ b, d.

Acceleration of the whole system towards right:

$$a = \frac{F}{M + m}$$

$$F - mg \sin \theta = ma \cos \theta$$

$$= m \frac{F}{M + m} \cos \theta = \frac{(M + m)mg \sin \theta}{M + m - m \cos \theta}$$

Pseudo force on m as seen from the frame of M :

$$F_{s1} = ma = \frac{mF}{M + m}$$

$$= mg \sin \theta \left(\frac{m}{M + m(1 - \cos \theta)} \right) < mg \sin \theta$$

Pseudo force on M as seen from the frame of m :

$$F_{s2} = Ma = \frac{MF}{M + m} \left(> \frac{mF}{M + m} \right)$$

$$= mg \sin \theta \left(\frac{M}{M + m(1 - \cos \theta)} \right) < mg \sin \theta$$

Now $mg \cos \theta - N = m \sin \theta \Rightarrow N = mg \cos \theta - m \sin \theta$ Hence N is less than $mg \cos \theta$. Hence, it will also be less than $mg \sin \theta$, because $\theta = 45^\circ$.Applying equation on m in horizontal direction:

$$F \cos \theta - N \sin \theta = ma$$

$$\Rightarrow F \cos \theta - N \sin \theta = m \frac{F}{M + m}$$

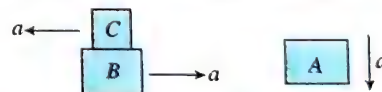
$$\Rightarrow N = \frac{mF}{M + m} \left(\frac{(M + m) \cos \theta - m}{m \sin \theta} \right)$$

Put $\theta = 45^\circ$

$$\Rightarrow N = \frac{mF}{M + m} \left(\frac{M + m - \sqrt{2}m}{m} \right) > \frac{mF}{M + m}$$

Normal force between ground and M will be $(M + m)g$. It is greater than $mg \sin \theta$. It is also greater than $\frac{mF}{M + m}$, because $\frac{mF}{M + m}$ is less than $mg \sin \theta$.3. i $\rightarrow$ b; ii $\rightarrow$ a; iii $\rightarrow$ c, d; iv $\rightarrow$ c, d.

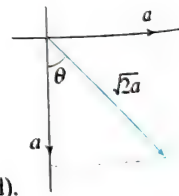
The direction of accelerations of various blocks are as shown.

Acceleration of B is towards right. Hence, (i) $\rightarrow$ b. Acceleration of C w.r.t. B is towards left.Hence, (ii) $\rightarrow$ aAcceleration of A w.r.t. C :

$$= \vec{a}_{A/C} = \vec{a}_A - \vec{a}_C$$

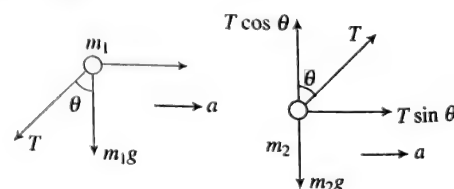
$$= -a\hat{j} - (-a\hat{i}) = a\hat{i} - a\hat{j}$$

as shown below

Hence, (iii) $\rightarrow$ (c, d), Similarly (iv) $\rightarrow$ (c, d).4. i $\rightarrow$ d; ii $\rightarrow$ c; iii $\rightarrow$ b; iv $\rightarrow$ a.

$$F = (m_1 + m_2)a$$

$$T \sin \theta = m_2 a$$



$$T \cos \theta = m_2g \Rightarrow T = m_2g \sec \theta$$

From, (ii) and (iii), $a = g \tan \theta$
 Put in (i), $F = (m_1 + m_2)g \tan \theta$

Net force acting on $m_2 = m_2 a = \frac{m_2 F}{m_1 + m_2}$
 Force acting on m_1 by wire:

$$m_1 g + T \cos \theta = m_1 g + m_2 g$$

5. i $\rightarrow$ b, c; ii $\rightarrow$ a, d; iii $\rightarrow$ d; iv $\rightarrow$ c.

Let the accelerations of various blocks are as shown in figure. Pulley P_2 will have downward acceleration a .

$$\text{Now } a = \frac{a_1 + a_2}{2} \Rightarrow a_2 = 2a - a_1 > 0$$

So acceleration of 2 is upwards

Hence, (i) $\rightarrow$ (b, c)

$$\text{and } a = \frac{-a_1 + a_4}{2} \Rightarrow a_4 = 2a + a_1 > 0$$

So, acceleration of 4 is downwards.

Hence (ii) $\rightarrow$ (a, d)

Acceleration of 2 w.r.t. 3:

$$a_{2/3} = a_2 - a_3 = a_2 - a_1 = 2(a - a_1) < 0$$

This is downwards, Hence (iii) $\rightarrow$ (d)

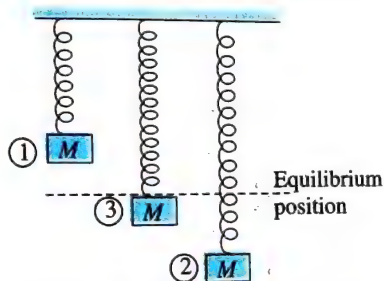
Acceleration of 2 w.r.t. 4:

$$a_{2/4} = a_2 - (-a_4) = 4a > 0$$

This is upwards. Hence, (iv) $\rightarrow$ (c)

6. i $\rightarrow$ a; ii $\rightarrow$ c, d; iii $\rightarrow$ a, d; iv $\rightarrow$ b, d.

In figure is the equilibrium position where velocity is maximum and acceleration is zero. 1 and 2 are the extreme positions where velocity is zero and acceleration is maximum. 1 is the unstretched position.



When the block is at position 3, then $mg = kx$. So net force is zero. Hence, acceleration is zero. But velocity may be either in upward or downward direction.

Hence (ii) $\rightarrow$ (c, d)

When the block is between position 3 and 2, then $kx > mg$. So net force is in upward direction, hence acceleration is in upward direction. But velocity may be either in upward or downward direction.

Hence (iii) $\rightarrow$ (a, d)

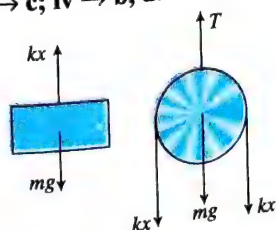
But if the block is at position 2, then velocity is zero and acceleration is in upward direction.

Hence (i) $\rightarrow$ (a)

When the block is between position 3 and 1, $mg > kx$. So net force is in downward direction, hence acceleration is in downward direction. But velocity may be either in upward or downward direction.

Hence (iv) $\rightarrow$ (b, d)

7. i $\rightarrow$ c; ii $\rightarrow$ c; iii $\rightarrow$ c; iv $\rightarrow$ b, d.



(i) Let the maximum downward displacement of m is x_0 , then

$$\frac{1}{2} kx_0^2 = mgx_0 \Rightarrow x_0 = 2mg/k$$

To lift the block (M): $kx_0 = Mg$

$$\Rightarrow 2mg = Mg \Rightarrow mg = Mg/2$$

Hence (i) $\rightarrow$ (c).

(ii) When m is in equilibrium,

$$kx = mg, T = 2kx + mg = 3mg = \frac{3M}{2}g$$

Hence (ii) $\rightarrow$ (a).

(iii) $N + kx = Mg$

$$N = Mg - kx = Mg - \frac{M}{2}g = \frac{M}{2}g$$

Hence (iii) $\rightarrow$ (c)

(iv) Tension $= kx_0 = Mg = 2mg$

Hence (iv) $\rightarrow$ (b, d)

8. i $\rightarrow$ c; ii $\rightarrow$ b; iii $\rightarrow$ b, d; iv $\rightarrow$ b.

(i), (ii)

After spring 2 is cut, tension in string AB will not change.

$$(T_{CD})_i = 4mg$$

$$(T_{CD})_f = m_D g + m_D \cdot \frac{m_A + m_B - m_C - m_D}{m_A + m_B + m_C + m_D} \cdot g$$

$$= 2mg \left(1 + \frac{1}{5} \right) = 2.4mg$$

Hence, T_{CD} decreases.

(iii), (iv)

After string between C and pulley is cut, tension in string AB will become zero.

$$(T_{CD})_i = (m_D + m_E)g = 4mg$$

Acceleration of C and D blocks is

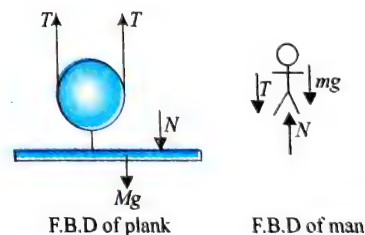
$$(m_C + m_D)g + m_E g = (m_C + m_D) \cdot a$$

$$a = \frac{6mg}{4mg} = \frac{3}{2}g, (T_{CD})_f + m_C g = m_C a$$

$$(T_{CD})_f = 2m \cdot \frac{3}{2}g - 2mg = mg$$

The tension decreases.

9. (2)



F.B.D of plank

F.B.D of man

(i) At equilibrium: $2T = N + mg$... (i)

$$\text{and } N = T + mg$$

... (ii)

$$\text{From (i) and (ii) } T = (M + m)g \text{ and } N = (M + 2m)g$$

Hence (i) $\rightarrow$ a, b

(ii) When the system falls freely, $a = g$, then $N = 0$ and $T = 0$

Hence (ii) $\rightarrow$ c, e

(iii) When accelerating upward

$$\text{For platform: } 2T - N - Mg = Ma$$

... (i)

$$\text{For man: } N - T - mg = ma$$

... (ii)

$$\text{From (i) and (ii) } T = (M + m)(g + a) \text{ and } N = (M + 2m)(g + a)$$

$$\text{It means } N > (2m + M)g$$

Hence (iii) $\rightarrow$ d

- (iv) When the system accelerating downward

Then $T = (M + m)(g - a)$ and $N = (M + 2m)(g - a)$

It means (iv) $\rightarrow c, e$.

10. (4)

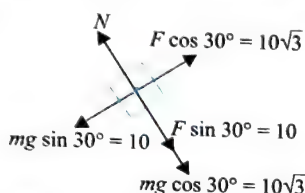
(a)



F exactly balances mg .

So body stays in its position without acceleration and with zero contact. So it does not have any match.

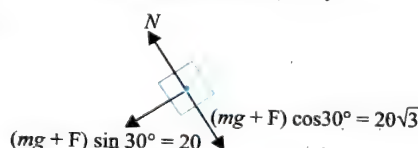
(b)



Slides up the inclined with acceleration $< 10 \text{ m/s}^2$;

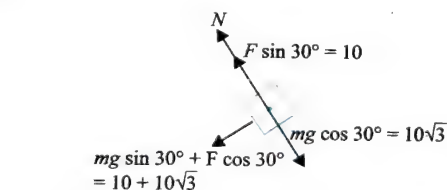
$N \neq 0 \Rightarrow$ matches with (i) only.

(c)



$N \neq 0 \Rightarrow$ Matches with (i); slides down the incline with acceleration $= 10 \text{ m/s}^2$ Matches (ii) also.

(d)



$$N + 10 = 10\sqrt{3} \Rightarrow N = 10\sqrt{3} - 10 \neq 0$$

Slides down with acceleration

$$a = \frac{10 + 10\sqrt{3}}{2} > 10 \text{ m/s}^2$$

Numerical Value Type

1. (6) Consider the forces on the person:

$$\sum F_y = ma_y$$

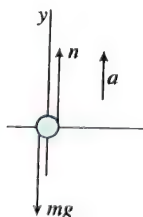
$$n - mg = ma$$

$$n = 1.6mg$$

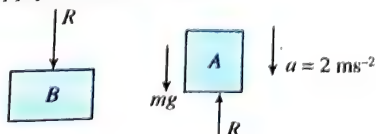
So, $a = 0.60g = 6 \text{ m/s}^2$

$$v^2 = u^2 + 2as = 0^2 + 2 \times 6 \times 3$$

$$\Rightarrow v = 6 \text{ m/s}^{-1}$$



2. (4) Let A apply a force R on B



Then B also applies an opposite force R on A as shown in figure

For A: $mg - R = ma$

$$\Rightarrow R = m(g - a) = 0.5[10 - 2] = 4 \text{ N}$$

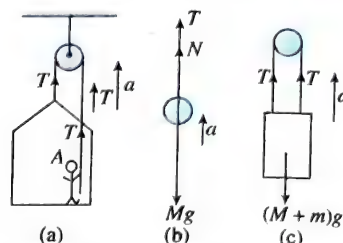
3. (2) Let $M =$ mass of painter $= 10 \text{ kg}$

$$m = \text{mass of crate} = 25 \text{ kg}$$

Let F_A be the action force exerted by painter on crate, reaction force exerted by crate on man

$$N = F_A = 450 \text{ N}$$

The free-body diagram of painter is shown in figure (b)



Therefore, equation of motion of painter is

$$N + T - Mg = Ma \quad \dots(i)$$

The equation of motion of whole system is

$$2T - (M + m)g = (M + m)a \quad \dots(ii)$$

Multiplying (i) by 2, we get

$$2N + 2T - 2Mg = 2Ma \quad \dots(iii)$$

Subtracting (ii) from (iii), we get

$$2N - 2Mg + (M + m)g = (2M - M - m)a$$

$$\text{or } 2N - (M - m)g = (M - m)a$$

$$a = \frac{2N - (M - m)g}{M - m}$$

$$= \frac{2 \times 450 - (100 - 25) \times 10}{100 - 25} = \frac{900 - 750}{75} = 2 \text{ m/s}^2$$

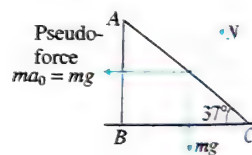
4. (1) Drawing the free-body diagram of block with respect to plane. Acceleration of the block up the plane is

$$a = \frac{mg \cos 37^\circ - mg \sin 37^\circ}{m}$$

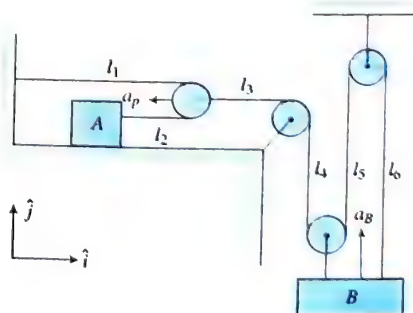
$$= g \left(\frac{4}{5} - \frac{3}{5} \right) = 2 \text{ m/s}^2$$

Applying, $s = \frac{1}{2}at^2$ we get

$$\Rightarrow t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1}{2}} = 1 \text{ s}$$



5. (2) $l_1 + l_2 = C \Rightarrow l_1'' + l_2'' = 0$



$$\Rightarrow -a_p + (12 - a_p) = 0 \Rightarrow a_p = 6 \text{ m/s}^2$$

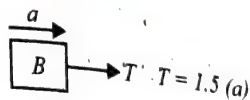
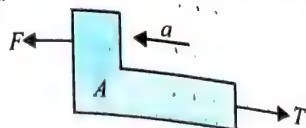
$$l_3 + l_4 + l_5 + l_6 = C$$

$$\Rightarrow l_3'' + l_4'' + l_5'' + l_6'' = 0$$

$$a_p - a_B - a_B - a_B = 0$$

$$\Rightarrow a_p = 3a_B \Rightarrow a_B = 2 \text{ m/s}^{-1}$$

$$(3) F - T = 1 \times a$$



$$\Rightarrow a = 1.5 \text{ ms}^{-2}, a_{\text{rel}} = 1.5 + 4.5 = 3 \text{ ms}^{-2} \text{ here } u_{\text{rel}} = 0$$

$$\text{Hence, } s_{\text{rel}} = 0 + \frac{1}{2}(a_{\text{rel}})t^2$$

$$\Rightarrow t = \sqrt{\frac{2s_{\text{rel}}}{a_{\text{rel}}}} = \sqrt{\frac{2 \times 13.5}{3}} = 3 \text{ sec}$$

$$(5) \text{ At equilibrium } 2T = m_1 g \Rightarrow T = 45 \text{ N} \quad \dots(i)$$

$$T - m_2 g = T_1 \quad \dots(ii)$$

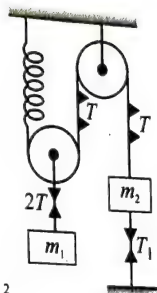
As string is at cut then $T_1 = 0$

$$T - m_2 g = m_1 a$$

$$45 - 30 = 3 \times a$$

$$a = 5 \text{ m/s}^2$$

$$(2) \text{ Acceleration of A and B} = \left(\frac{\frac{11}{3} - 3}{\frac{11}{3} + 3} \right) g = 1 \text{ m/s}^2$$



Relative acceleration of A and B = 2 m/s^2

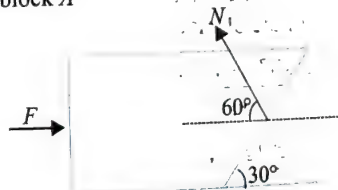
$$\text{Now by using } s = ut + \frac{1}{2}at^2;$$

$$4 = 0 + \frac{1}{2}(2)(t)^2$$

$$\Rightarrow t = 2 \text{ s}$$

$$(30) \text{ Acceleration of two mass system is } a = \frac{F}{2m} \text{ leftward.}$$

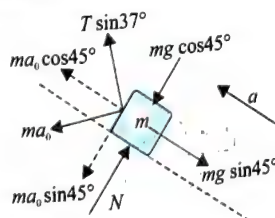
F.B.D of block A



$$N \cos 60^\circ - F = ma = \frac{mF}{2m}$$

$$\text{Solving we get } N = 3F = 30 \text{ N}$$

$$(2) \text{ Let } a_0 \text{ be acceleration of wedge with respect to ground and 'a' be acceleration of block with respect to wedge.}$$



From the frame of wedge

$$ma = ma_0 \cos 45^\circ - mg \sin 45^\circ \quad \dots(1)$$

$$a = \frac{a_0 - g}{\sqrt{2}}$$

$$N = ma_0 \sin 45^\circ + mg \cos 45^\circ \quad \dots(2)$$

$$N = \frac{2g}{\sqrt{2}} + \frac{2a_0}{\sqrt{2}}$$

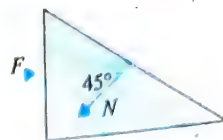
$$\text{For the wedge } Ma_0 = 210 - N \sin 45^\circ$$

$$a_0 = 20 \text{ m/s}^2$$

$$\text{From equation (1), } a = \frac{10}{\sqrt{2}} \text{ m/s}^2$$

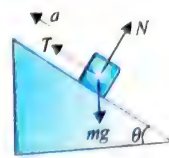
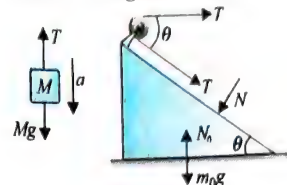
$$\text{Time taken is } 10\sqrt{2} = \frac{1}{2} \left(\frac{10}{\sqrt{2}} \right) t^2$$

$$t = 2 \text{ sec}$$



$$11. (5) \text{ For } M: Mg - T = Ma$$

$$\text{For } m: T - mg \sin \theta = ma$$



$$\text{and } N = mg \cos \theta$$

$$\text{For } m_0: T + T \cos \theta = N \sin \theta$$

[In horizontal direction]

Eliminating a between (i) and (ii),

$$g - \frac{T}{M} = \frac{T}{M} - g \sin \theta$$

$$\Rightarrow T \left(\frac{1}{m} + \frac{1}{M} \right) = g(1 + \sin \theta)$$

$$\Rightarrow T = \frac{mMg}{m+M}(1 + \sin \theta)$$

Put this value of T and value of N from equation (iii) into equation (iv),

$$\frac{mMg(1 + \sin \theta)}{m+M}(1 + \cos \theta) = mg \cos \theta \cdot \sin \theta$$

$$\therefore \frac{M}{m+M} = \frac{\cos \theta \cdot \sin \theta}{(1 + \sin \theta)(1 + \cos \theta)}$$

$$= \frac{\frac{4}{5} \cdot \frac{3}{5}}{\left(1 + \frac{3}{5}\right)\left(1 + \frac{4}{5}\right)} = \frac{1}{6} \Rightarrow \frac{m}{M} = 5$$

$$12. (4) \text{ Let initial extension in } S_1 \text{ be } x_0$$

$$\text{Then, } kx_0 = Mg \quad \dots(i)$$

Let extension in S_2 be x_2 and further extension in S_1 be x_1 as point A moves down by L .

$$2x_1 + x_2 = L$$

For equilibrium of M

$$kx_0 + kx_1 = Mg + 2kx_2$$

$$\text{But } kx_0 = Mg$$

$$\therefore x_1 = 2x_2 \quad \dots(ii)$$

$$\text{Solving (i) and (ii) } x_2 = \frac{L}{5}, x_1 = \frac{2L}{5}$$

$$\text{Hence the displacement of the block } M, x_1 = \frac{2 \times 10}{5} = 4 \text{ cm}$$



$$13. (120)$$

Let T_1 be the tension in rope and T_2 the tension in the tail of monkey A. Let the force exerted by monkey A on the rope be F .

Let M_A and M_B be masses of monkeys A and B, respectively. The free-body diagrams of monkeys A and B are shown in figure (a) and (b). $F = T_1$

The equation of motion of monkeys A and B are

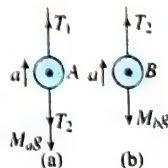
$$T_1 - M_A g - T_2 = M_A a \quad \dots(i)$$

$$T_2 - M_B g = M_B a \quad \dots(ii)$$

For maximum tension in tail

From Eq. (ii), we get

$$a = \frac{T_2 - M_B g}{M_B} = \frac{30 - 2 \times 10}{2} = 5 \text{ m/s}^2$$



Then from Eq. (i), we get

$$T_1 = T_2 + M_A(g + a) = 30 + 6(10 + 5) = 120 \text{ N}$$

14. (1.82)

Since blocks A and B remain stationary relative to block C , it means motion of blocks A and B is identical with block C or acceleration of all the three blocks is same as a horizontally (rightwards). Now considering FBDs w.r.t wedge C ,

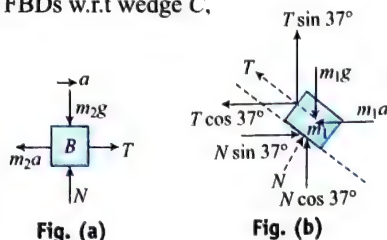


Fig. (a)

Fig. (b)

For blocks B ,

$$N_2 = m_2 g \quad \text{or} \quad N_2 = 55 \text{ N}$$

$$T = m_2 a$$

...(i)

For block A ,

$$N_1 \cos 37^\circ + T \sin 37^\circ = m_1 g$$

...(ii)

$$N_1 \sin 37^\circ - T \cos 37^\circ = m_1 a$$

...(iii)

Solving above equations,

$$T = 10 \text{ N}$$

$$N_1 = 20 \text{ N}$$

$$\text{and } a = 20/11 \text{ m s}^{-2} = 1.82 \text{ m s}^{-2}$$

15. (3) Let acceleration of block B relative to pulley E be b downward, the acceleration of block C relative to pulley E will be b upward and let acceleration of block A be a upward. Then acceleration of pulley E will also be a but downward. Hence, resultant accelerations of blocks B and C become $(a + b)$ and $(a - b)$, both downward, respectively.

Acceleration of B relative to A ,

$$a_{B,A} = (a + b) + a \Rightarrow a_{B,A} = (2a + b) = 5 \text{ m s}^{-2} \quad \text{...(i)}$$

And downward acceleration of C relative to A ,

$$a_{C,A} = (a - b) + a \Rightarrow a_{C,A} = (2a - b) = 3 \text{ m s}^{-2} \quad \text{...(ii)}$$

From Eqs (i) and (ii),

$$a = 2 \text{ m s}^{-2} \quad \text{and} \quad b = 1 \text{ m s}^{-2}$$

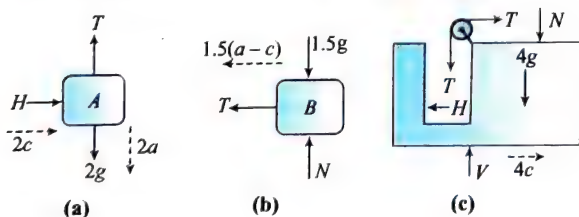
Resultant acceleration of $B = (a + b) = 3 \text{ m s}^{-2}$ (downward)

And that of $C = (a - b) = 1 \text{ m s}^{-2}$ (downward)

16. (5) Let vertically downward component of acceleration of A be a and let acceleration of C be c (rightward).

Then horizontal component of acceleration of A is c (rightward) and acceleration of B , relative to C is a (leftward). Hence, resultant acceleration of B is, $(a - c)$ leftward.

Now considering free body diagrams



(a)

(b)

(c)

For block A ,

$$H = 2c$$

...(i)

$$2g - T = 2a$$

...(ii)

For horizontal forces on block B ,

$$T = 1.5(a - c)$$

...(iii)

For horizontal forces on block C ,

$$T - H = 4c$$

...(iv)

From above equations,

$$a = 6.25 \text{ m s}^{-2} \quad \text{and} \quad c = 1.25 \text{ m s}^{-2}$$

Vertical acceleration of $A = a = 6.25 \text{ m s}^{-2}$ (downward)

Horizontal acceleration of $A = c = 1.25 \text{ m s}^{-2}$ (rightward)

Acceleration of $C = c = 1.25 \text{ m s}^{-2}$ (rightward)

Acceleration of $B = (a - c) = 5.00 \text{ m s}^{-2}$ (leftward)

17. (15)

$$N \cos 37^\circ = 1g$$

...(i)

$$N \sin 37^\circ = 1a$$

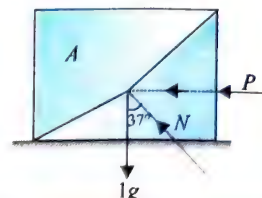
...(ii)

$$\frac{a}{g} = \tan 37^\circ$$

$$a = 10 \times \frac{3}{4} = 7.5 \text{ m s}^{-2}$$

$$P = 2a$$

$$P = 15 \text{ N}$$



Archives

JEE Main

Single Correct Answer Type

1. (2) From the graph, it can be seen that it is a straight line. So the motion is uniform. Because of impulse, the direction of velocity changes as can be seen from the slope of the graph.

$$\text{Initial velocity} = \frac{2}{2} = 1 \text{ m/s}$$

$$\text{Final velocity} = -\frac{2}{2} = -1 \text{ m/s}$$

$$\vec{P}_i = 0.4 \text{ N-s}$$

$$\vec{P}_f = -0.4 \text{ N-s}$$

$$\vec{J} = \vec{P}_f - \vec{P}_i = -0.4 - 0.4 = -0.8 \text{ N-s} \quad (\vec{J} = \text{impulse})$$

$$|\vec{J}| = 0.8 \text{ N-s}$$

2. (4) $mg \sin \theta = ma$

$$\therefore a = g \sin \theta$$

where a is along the inclined plane.

Therefore, the vertical component of acceleration is $g \sin^2 \theta$.

Hence, the relative vertical acceleration of A with respect to B is $g [\sin^2 60^\circ - \sin^2 30^\circ] = \frac{g}{2} = 4.9 \text{ m s}^{-2}$ in vertical direction.

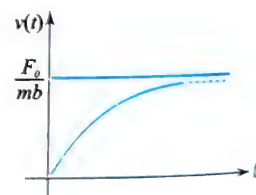
3. (3) $F = ma = F_0 e^{-bt}$

$$\Rightarrow \frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\Rightarrow \int_0^v dv = \frac{F_0}{m} \int_0^t e^{-bt} dt$$

$$\Rightarrow v = \frac{F_0}{m} \left[\frac{e^{-bt}}{-b} \right]_0^t$$

$$\Rightarrow v = \frac{F_0}{mb} (1 - e^{-bt})$$



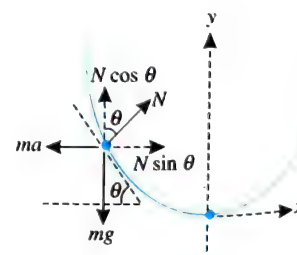
JEE Advanced

Single Correct Answer Type

1. (2) $\tan \theta = \frac{a}{g}$

$$\tan \theta = \frac{dy}{dx} = 2kx$$

$$\Rightarrow x = \frac{a}{2gk}$$

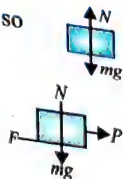


Chapter 7

Concept Application Exercises

Exercise 7.1

1. (a) When $F_{\text{ext}} = 0$, the block has no tendency to slip, so
 $F_{\text{friction}} = 0$



- (b) First, we calculate maximum friction force.
 $F_{\text{max}} = \mu_s N = (0.40 \times 20) = 8 \text{ N}$
 Since in this case, $P < F_{\text{max}}$, the block is in static equilibrium, i.e.,
 $F = P = 5 \text{ N}$

- (c) $P_{\text{min}} = F_{\text{max}} = \mu_s N = 8 \text{ N}$

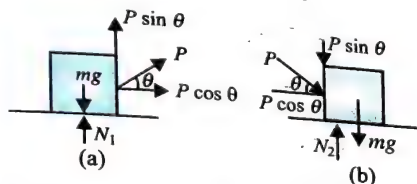
- (d) When the block is in motion,

$$F = \mu_k N = 0.2 \times 20 = 4 \text{ N}$$

$$P_{\text{min}} = F_{\text{kinetic}} = \mu N = 4 \text{ N}$$

- (e) Since in this case $P > \mu_s N$, the block accelerates and the friction force will be kinetic = 4 N

2. (a)



In first case: $N_1 = mg - P \sin \theta$

In second case: $N_2 = mg + P \sin \theta$

Since normal force in second case is greater so the friction will be greater in second case when the block impends the motion. Hence in first case, less effort is required.

- (b) $P \cos \theta = \mu N_1 = \mu (mg - P \sin \theta) \Rightarrow$ for first case

$$\Rightarrow P = \frac{\mu mg}{\cos \theta + \mu \sin \theta} \Rightarrow P_{\text{min}} = \frac{\mu mg}{\sqrt{1 + \mu^2}}$$

- (c) For pushing, $P \cos \theta > \mu N_2$

$$\Rightarrow P \cos \theta > \mu (mg + P \sin \theta)$$

$$\Rightarrow P > \frac{\mu mg}{\cos \theta - \mu \sin \theta}$$

As θ increases, denominator $\cos \theta - \mu \sin \theta$ decreases and hence P increases. Let at $\theta = \theta_0$, denominator becomes zero. So $\cos \theta_0 - \mu \sin \theta_0 = 0 \Rightarrow \theta_0 = \cot^{-1}(\mu)$.

At $\theta = \theta_0$, $P > \infty$ which is not possible.

3. $f_l = \mu_s N = \mu_s mg = 0.3 \times 1 \times 10 = 3 \text{ N}$

$$f_k = \mu_k N = \mu_k mg = 0.25 \times 1 \times 10 = 2.5 \text{ N}$$

- (a) $F = 1 \text{ N}$, $F < f_l$, so the motion will not start and the friction will be static. So friction $f = F = 1 \text{ N}$.

- (b) $F = 2 \text{ N}$, $F < f_l$, so the motion will not start and the friction will be static. So friction $f = F = 2 \text{ N}$.

- (c) $f = 3 \text{ N}$, $F = f_l$. Here the motion is just about to start. Hence the friction force will be limiting. So friction $f = f_l = 3 \text{ N}$.

- (d) $F = 4 \text{ N}$, $F > f_l$, so the motion will start and friction will be kinetic. Hence friction will be $f = f_k = 2.5 \text{ N}$.

- (e) $F = 20 \text{ N}$, $F > f_l$, so motion will start and friction will be kinetic. Hence friction is $f = f_k = 2.5 \text{ N}$.

4. Here $f_l = 10 \text{ N} \Rightarrow \mu_s mg = 10$

$$\Rightarrow \mu_s = \frac{10}{mg} = \frac{10}{5 \times 10} = 0.2$$

$$f_k = 8 \text{ N} \Rightarrow \mu_k mg = 8$$

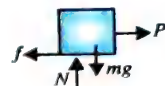
$$\Rightarrow \mu_k = \frac{8}{mg} = \frac{8}{5 \times 10} = 0.16$$

5. Since the body is not moving.

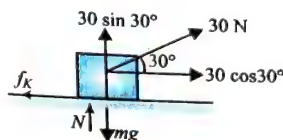
So $f = P$ and $N = mg$

Net force applied by surface on the body:

$$F = \sqrt{f^2 + N^2} = \sqrt{P^2 + m^2 g^2}$$



6.



$$N = mg - 30 \sin 30^\circ$$

As the speed is constant, so the acceleration is zero.

- (a) Hence friction $f_k = 30 \cos 30^\circ = 15\sqrt{3} \text{ N}$

- (b) $f_k = 30 \cos 30^\circ = \mu_k (mg - 30 \sin 30^\circ)$

Solving, we get $\mu_k = 3\sqrt{3}/7$

7. The maximum frictional force f_{max} acting on the block over the surface is given by,

$$f_{\text{max}} = \mu N$$

where $N = mg$ since the block is in equilibrium.

$$\Rightarrow \mu = \frac{f_{\text{max}}}{mg} = \frac{5}{2 \times 10} = 0.25$$

Since the applied horizontal force is $F = 4 \text{ N} \Rightarrow F < f_{\text{max}}$

We known that when $F < f_{\text{max}}$ the block does not slide. Hence friction force will be static nature.

$\therefore$ The frictional force on the block is $f = 4 \text{ Newton}$.



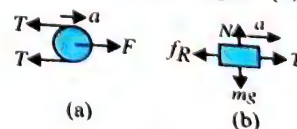
8. $T = Mg$ (T = tension in the rope)

$N = 60 \text{ g} - T \sin 60^\circ$ (N is normal reaction between the man and the ground) and $T \cos 60^\circ = \mu N$

Solving these three equations with proper substitutions, we get

$$M = \frac{120}{\sqrt{3} + 2} \text{ kg}$$

9. From the FBD of pulley shown in figure (a),



$$F - 2T = m_p a_p \therefore T = \frac{F}{2}$$

...(i) (because $m_p \approx 0$)

From FBD of the block in figure. (b),

$$N = mg$$

$$\therefore T - f_k = ma$$

...(ii)

$$\text{i.e. } T - \mu_k N = ma$$

...(iii)

from (i), (ii) and (iii),

$$\frac{F}{2} \mu_k mg = ma \quad \therefore a = \frac{\frac{F}{2} - \mu_k mg}{m} = \frac{F}{2m} - \mu_k g$$

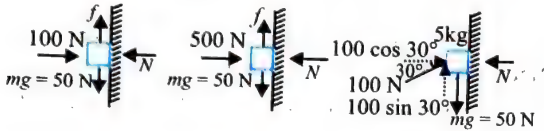
$$\therefore a = \left(\frac{20}{2 \times 2} - 3 \times 10 \right) \text{ms}^{-2} = (5 - 3) \text{ms}^{-2} = 2 \text{ms}^{-2}$$

10. (a) $f_{\max} = \mu N = (0.2)(100) = 20 \text{ N}$

Since $mg > f_{\max}$ therefore, friction force is equal to $f_{\max}$ i.e., $f = f_{\max} = 20 \text{ N}$

(b) $f_{\max} = \mu N = 0.2 \times 500 = 100 \text{ N}$

Since $mg < f_{\max}$, therefore, friction force is equal to mg . This means that $f = mg = 50 \text{ N}$



(c) $f_{\max} = \mu N = (\mu 100 \cos 30^\circ) = (0.2)100 \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ N}$

Since the vertical component of the external force $100 \sin 30^\circ = 50 \text{ N}$ can balance the weight of the block, therefore $f = 0$.

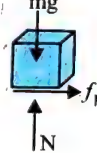
11. (a) $\vec{v}_{\text{block, plank}} = \vec{v}_{\text{block}} - \vec{v}_{\text{plank}}$

$$= v_0 \hat{i} - 2v_0 \hat{i} = -v_0 \hat{i}$$

Friction oppose the relative motion

Hence friction will act in forward direction

$$f_1 = \mu N = \mu mg$$



(b) $\vec{v}_{\text{block, plank}} = \vec{v}_{\text{block}} - \vec{v}_{\text{plank}} = v_0 \hat{i} - (-v_0 \hat{j}) = v_0 \hat{i} + v_0 \hat{j}$

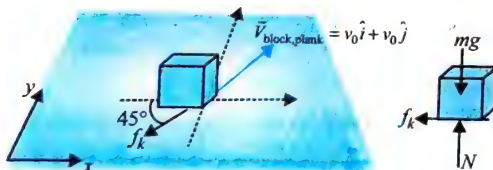
Friction opposes the relative motion.

Hence, friction will act in forward direction.

$$\vec{f}_k = \mu N = \mu mg$$

$$\vec{f}_k = \mu mg \cos 45^\circ (-\hat{i}) + \mu mg \cos 45^\circ (-\hat{j})$$

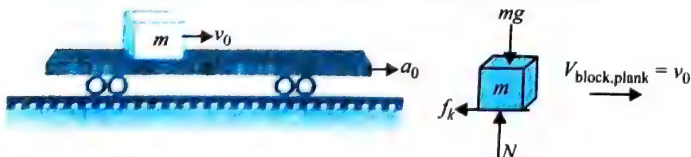
$$\vec{f}_k = -\frac{\mu mg}{\sqrt{2}} (\hat{i} + \hat{j})$$



12. (a) At $t = 0$, the relative motion of the block with respect to plank is in forward direction.

$$\vec{v}_{\text{block, plank}} = \vec{v}_{\text{block}} - \vec{v}_{\text{plank}} = v_0 - 0 = v_0$$

Hence friction on the block acts in backward direction.



$$f_k = \mu N = \mu mg$$

(b) Acceleration of the block. $a = -\frac{\mu mg}{m} = -\mu g$

Analyzing block with respect to plank

Acceleration of the block with respect to plank,

$$\vec{a}_{\text{block, plank}} = \vec{a}_{\text{block}} - \vec{a}_{\text{plank}} = -\mu g - a_0 = -(\mu g + a_0)$$

Initial velocity of block with respect to plank $u = v_0 - 0 = v_0$

Final velocity of block with respect to plank $v = 0$

Using $v = u + at$

$$\Rightarrow 0 = v_0 + -(\mu g + a_0)t_0 \Rightarrow t_0 = \frac{v_0}{(\mu g + a_0)}$$

Exercise 7.2

1. $N = mg \cos \theta$

$$f_k = \mu_k N = \mu_k mg \cos \theta$$

Hence, required force is $\sqrt{f_k^2 + N^2} = 6\sqrt{13} \text{ N}$

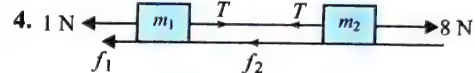
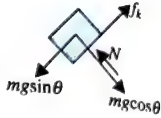
2. Apply $a = g \sin 60^\circ - \mu_k g \cos 60^\circ$ and get $\mu_k = \sqrt{3} - 1$

3. From FBD, friction $f = mg \sin \alpha$

$$N = mg \cos \alpha, \frac{f}{N} = \tan \alpha$$

For the maximum possible value of α , the friction becomes limiting friction, so

$$\mu = \tan \alpha \Rightarrow \cot \alpha = 3$$



$$f_1 = \mu_1 m_1 g = 0.1 \times 2 \times 10 = 2 \text{ N}$$

$$f_2 = \mu_2 m_2 g = 0.2 \times 3 \times 10 = 6 \text{ N}$$

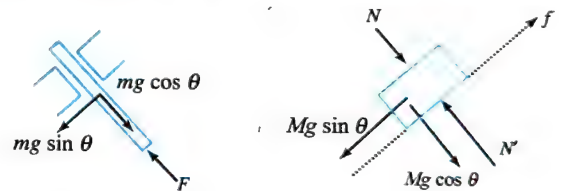
$$T + f_{12} = 8 \Rightarrow T = 2 \text{ N}$$

$$f_1 + 1 = T \Rightarrow f_1 = T - 1 = 2 - 1 = 1 \text{ N}$$

5. Here the weights of the blocks will balance each other so that no friction force is required.

6. Angle of repose, $\phi = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$

Here $\theta > \phi$. Hence, the block has a tendency to slide down.



$$Mg \cos \theta + N = N', N = mg \cos \theta$$

$$N' = Mg \cos \theta + mg \cos \theta$$

For the block to remain stationary,

$$f = Mg \sin \theta$$

If f is static in nature, $f < f_{\max}$

$$Mg \sin \theta < \mu N'$$

$$Mg \sin \theta < \mu (Mg \cos \theta + mg \cos \theta)$$

$$Mg \sin \theta - \mu Mg \cos \theta < \mu mg \cos \theta$$

$$m > M \left(\frac{\tan \theta}{\mu} - 1 \right) \Rightarrow m > 10 \left(\frac{\sqrt{3}}{1/\sqrt{3}} - 1 \right) \Rightarrow m > 20 \text{ kg}$$

7. From force diagram of bead,

$$N = ma_0 = 4 \text{ m}$$

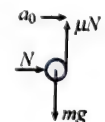
$$mg - \mu N = ma$$

$$\text{or } mg - \frac{1}{2} 4m = ma$$

$$\text{or } 10m - 2m = ma \Rightarrow a = 8 \text{ms}^{-2}$$

$$L = \frac{1}{2} at^2 \text{ or } 1 = \frac{1}{2} \times 8 \times t^2 \text{ or } t^2 = \frac{1}{4}$$

$$\therefore t = \frac{1}{2} \text{ s}$$



$$a = \mu g$$

$$0^2 = v_0^2 - 2\mu gL \quad \text{or} \quad v_0^2 = 2\mu gL$$

$$v_0 = \sqrt{2\mu gL}$$

9. We see that a portion of the chain is lying on the table top. Let the mass of that portion be m_1 . Let the mass of the remaining (hanging) portion of the chain be m_2 . Since the chain is at the point of slipping, the weight of the hanging portion of the chain counterbalances the maximum static frictional force $f_{\max}$ between m_1 and the surface.

$$\Rightarrow m_2 g = f_{\max}$$

$$m_2 g = \mu N_1$$

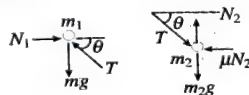
where $N_1 - m_1 g = 0$ for the equilibrium of the portion of chain lying on the table.

$$\Rightarrow m_2 g - \mu m_1 g = 0$$

$$\Rightarrow \mu = \frac{m_2}{m_1} = \frac{\frac{M}{L}x}{\frac{M}{L}(L-x)}$$

$$\Rightarrow \mu = \frac{x/L}{1-x/L} \Rightarrow \mu = \frac{\eta}{1-\eta}$$

10. For the equilibrium of m_1



$$\Sigma F_x = 0 \Rightarrow N_1 - T \cos \theta = 0$$

$$\Rightarrow N_1 = T \cos \theta$$

$$\Sigma F_y = 0 \Rightarrow T \sin \theta - m_1 g = 0$$

$$\Rightarrow T \sin \theta = m_1 g$$

For the equilibrium of m_2

$$\Sigma F_x = 0 \Rightarrow T \cos \theta - \mu N_2 = 0$$

$$\Rightarrow T \cos \theta = \mu N_2$$

$$\Sigma F_y = 0 \Rightarrow N_2 - T \sin \theta - m_2 g = 0$$

$$\Rightarrow N_2 = T \sin \theta + m_2 g$$

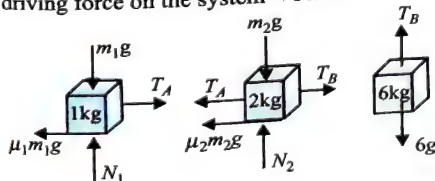
$$(ii) + (iv) \Rightarrow N_2 = (m_1 + m_2)g$$

Substituting N_2 from (v) in (iii), we obtain

$$T \cos \theta = \mu(m_1 + m_2)g$$

$$(ii) \div (vi), \tan \theta = \frac{m_1}{\mu(m_1 + m_2)} \Rightarrow \theta = \cot^{-1} \left[\mu \left(1 + \frac{m_2}{m_1} \right) \right]$$

11. (a) Net driving force on the system = 60 N



Net resisting force on the system

$$= \mu_1 m_1 g + \mu_2 m_2 g = 0.4 \times 1 \times 10 + 0.2 \times 2 \times 10 = 8 \text{ N}$$

As driving force acting on the system is greater than resisting force, hence the system will slide.

Hence, acceleration of the system

$$a = \frac{F_{\text{net}}}{\text{Total mass}} = \frac{(60 - 8)}{1 + 2 + 6} = \frac{52}{9} \text{ m/s}^2$$

For tension in string A,

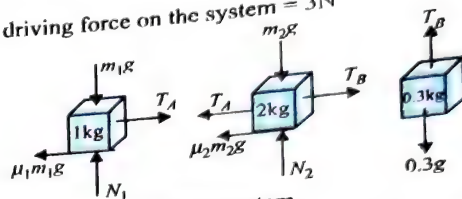
$$T_A - \mu_1 m_1 g = m_1 a \Rightarrow T_A = \mu_1 m_1 g + m_1 a$$

$$T_A = 0.4 \times 1 \times 10 + 1 \times \frac{52}{9} = \frac{88}{9} \text{ N}$$

For tension in string B,

$$6g - T_B = 6a \Rightarrow T_B = 6g - 6a = 60 - 6 \times \frac{52}{9} \Rightarrow T_B = \frac{76}{3} \text{ N}$$

(b) Net driving force on the system = 3 N



Net resisting force on the system

$$= \mu_1 m_1 g + \mu_2 m_2 g = 0.4 \times 1 \times 10 + 0.2 \times 2 \times 10 = 8 \text{ N}$$

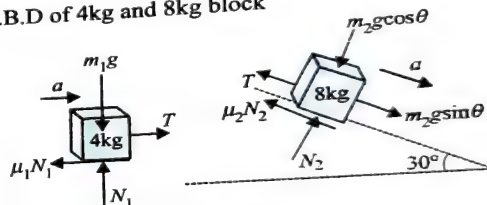
As driving force acting on the system is less than resisting force hence the system will slide.

Hence, acceleration of the system will be zero.

$$T_B = 0.3g = 3 \text{ N}$$

The friction acting on block 2 is of static nature. Hence, tension on string A will be zero.

12. From F.B.D of 4kg and 8kg block



For 4-kg block: $T - \mu_1 N_1 = m_1 a$

$$32 - \mu_1 (40) = 4 \times 0.5$$

$$\Rightarrow 32 - \mu_1 (40) = 2 \Rightarrow \mu_1 = \frac{3}{4}$$

For 8-kg block: $m_2 g \sin \theta - T - \mu_2 N_2 = m_2 a$

$$80 \sin 30^\circ - 32 - \mu_2 80 \cos 30^\circ = 8 \times 0.5$$

$$40 - 32 - \mu_2 40\sqrt{3} = 4$$

$$40 - 32 - \mu_2 40\sqrt{3} = 4 \Rightarrow \mu_2 = \frac{1}{10\sqrt{3}}$$

Exercise 7.3

$$1. a_1 = -\frac{\mu m_1 g}{m_1} = -\mu g \quad \text{and} \quad a_2 = \frac{\mu m_1 g}{m_2}$$

$$\therefore a_{\text{rel}} = a_1 - a_2 = -\mu g - \mu \frac{m_1 g}{m_2} \Rightarrow a_{\text{rel}} = -5 - 0.5 \frac{1}{2} \times 10$$

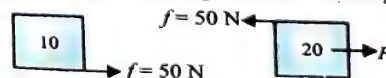
$$a_{\text{rel}} = -5 - \frac{5}{2} = -7.5 \text{ m/s}^2$$

$$v_{\text{rel}} = 0, u_{\text{rel}} = v_0$$

$$\therefore v_{\text{rel}} = u_{\text{rel}} + a_{\text{rel}} t \quad \text{or} \quad 0 = v_0 - 7.5 t$$

$$\therefore t = \frac{v_0}{7.5} = \frac{10}{7.5} = \frac{4}{3} \text{ s}$$

2. At just sliding condition limiting friction is acting.



$$F - 50 = 20a$$

$$f = 10a$$

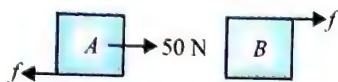
$$50 = 10a$$

$$\therefore a = 5 \text{ m s}^{-2}$$

$$\text{Hence, } F = 50 + 20 \times 5 = 150 \text{ N}$$

$$\therefore F_{\min} = 150 \text{ N}$$

3. $f_{\max} = \mu N = 50 \text{ N}$ (available friction)



If move together,

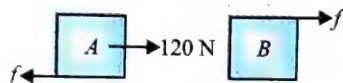
$$a = \frac{50}{10 + 10} = 2.5 \text{ m s}^{-2}$$

Check friction for B:

$$f = 10 \times 2.5 = 25$$

25 N is required which is less than available friction hence they will move together and $a_A = a_B = 2.5 \text{ m s}^{-2}$

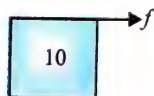
4. $f_{\max} = 50 \text{ N}$



If they move together

$$a = \frac{120}{20} = 6 \text{ m s}^{-2}$$

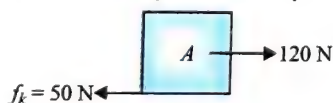
Check friction on B



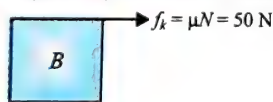
$$f_B = 10 \times 6 = 60 \text{ N}$$

$$60 \text{ N} > 50 \text{ N} \text{ (therefore required} > \text{available)}$$

Hence, they will **not** move together.



Hence, they move separately so kinetic friction is involved.



$$\therefore f \text{ or } a_A = \frac{120 - 50}{10} = 7.0 \text{ m s}^{-2} \Rightarrow a_B = \frac{50}{10} = 5 \text{ m s}^{-2}$$

5. $a = \frac{60}{30} = 2 \text{ m s}^{-2}$ No need to calculate.

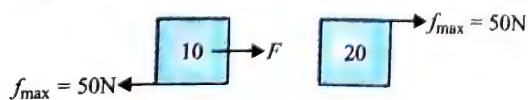
Check friction on 20 kg.

$$f = 20 \times 2 = 40 \text{ (which is required)}$$

$$40 < 50 \text{ (therefore required} < \text{available)}$$

Therefore, will move together.

Observing the critical situation where friction becomes limiting.



$$\therefore F - f_{\max} = 10a \quad \dots(1)$$

$$f_{\max} = 20a \quad \dots(2)$$

$$\therefore F = 75 \text{ N}$$

6. For the block A, $f_{\max} = ma_{\max}$

$a_{\max}$ of A = maximum acceleration of plank in order to have no slipping between A and B.

$$\dots(i)$$

$$\dots(ii)$$

$$\therefore f_{\max} = \mu mg$$

$$\therefore \mu mf = ma_{\max}$$

$$\therefore \mu = \frac{a_{\max}}{g} = \frac{2}{10} = 0.2$$

7. Let a_1 and a_2 be the retardation of m and M , respectively.

$$\Rightarrow a_{\text{rel.}} = |a_1 - a_2|,$$

$$\text{where } a_1 = \frac{\mu/2 mg}{m} = \frac{\mu}{2} g$$

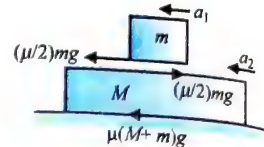
$$\text{and } a_2 = \frac{\mu(M+m)g - (\mu/2)mg}{M}$$

The relative acceleration between the blocks

$$= a_{\text{rel.}} = |a_2 - a_1| = \frac{\mu(M+m)g}{2M}$$

Now, $l = \frac{1}{2} a_{\text{rel.}} t^2$, where t = time of sliding of m over M .

$$\Rightarrow t = \sqrt{\frac{2l}{a_{\text{rel.}}}} = \sqrt{\frac{4Ml}{(M+m)\mu g}}$$



8. The force diagram and the FBD are shown in figure, taking the ground as reference frame.

$$N_2 = mg \quad \dots(i)$$

$$F - T - f = 0$$

From figure,

$$N_1 = N_2 + Mg \quad \dots(iii)$$

$$T - f = 0 \quad \dots(iv)$$

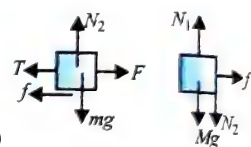
$$\text{Again } f_{\max} = \mu N_2$$

$$\text{i.e. } f_{\max} = \mu mg \quad \dots(v)$$

For m not to slide relatively on M ,

$$f \leq f_{\max} \quad \dots(vi)$$

Using Eq. (i) to (v) and the condition (vi), $F_{\max}$ can be found out as, $F_{\max} = 2 \mu mg$.



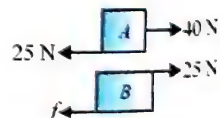
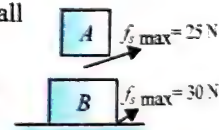
9. First of all find values of limiting friction at all contact surfaces ($f_{s \max}$).

Maximum force upper surface of 10 kg can experience is 25 N so it will not move relative to ground.

Hence, only 5 kg will move.

$$a_A = \frac{40 - 25}{5} = 3 \text{ m s}^{-2}$$

and $a_B = 0$



10. Let both blocks move together the friction between the blocks should be static.

$$\text{For block (2), } P - f = 1a \quad \dots(i)$$

$$\text{For block (1), } f - P/2 = 1a \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } P/2 = 2a$$

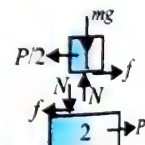
$$\text{or } a = \frac{P}{4} \text{ and } f = \frac{3P}{4}$$

$$\text{As } f \text{ is static } f = \frac{3P}{4} \leq \mu_s mg$$

$$\mu_s mg = 0.75 \times 10 = 7.5$$

$$\text{Thus, } P \leq \frac{4}{3} \mu_s mg = \frac{4}{3} \times 0.75 \times 10$$

$$P = 10 \text{ N}$$



11. Weighing machine measure normal reaction. Draw FBD of man. System of man and platform have acceleration at an angle of 37° . a_x and a_y are x and y components of acceleration. What is cause of vertical acceleration and horizontal acceleration.

$$(80 - 72)g = ma_y$$

$$a_y = 1 \text{ m/s}^2$$

$$\text{Now } a_y = a \sin 37^\circ = 1 \text{ m/s}^2$$

$$\text{or } a = 5/3 \text{ m/s}^2$$

Now apply Newton's second law on man in direction of acceleration. Note that x component of acceleration of man is due to friction.

$$mg \sin 37^\circ - \mu mg \cos 37^\circ = m \times \left(\frac{5}{3}\right)$$

$$6 - 8\mu = \frac{5}{3}$$

$$6 - \left(\frac{5}{3}\right) = 8\mu \Rightarrow \mu = \frac{13}{24}$$

12. Acceleration of block $a_1 = \frac{\mu m_1 g}{m_1} = \mu g = 0.5 \times 10 = 5 \text{ m/s}^2$

(towards left)

$$\text{Acceleration of block } a_2 = \frac{\mu m_1 g}{m_2} = \frac{0.5 \times 1 \times 10}{2} = \frac{5}{2} \text{ m/s}^2$$

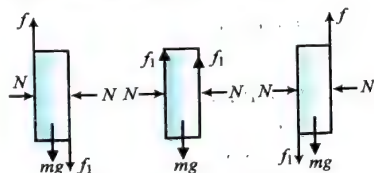
(towards right)

The block comes to rest with respect to block when the velocities of both block and plank becomes equal.

$$v_1 = v_2 \Rightarrow v - a_1 t = a_2 t \quad \text{or} \quad 10 - 5 \times t = \frac{5}{2} t$$

$$\text{Which gives } t = \frac{4}{3} \text{ sec}$$

13. From FBD, for center book, $2f_1 = mg$



$$f \leq \mu N \Rightarrow \mu N > \frac{mg}{2}$$

$$N > \frac{mg}{2\mu} \quad \dots(i)$$

For side book, $f - f_1 = mg$

$$f = \frac{3mg}{2} \leq \mu N$$

$$N \geq \frac{3mg}{2\mu} \quad \dots(ii)$$

By Eqs. (i) and (ii),

$$\text{Thus, minimum } N = \frac{3mg}{2\mu}$$

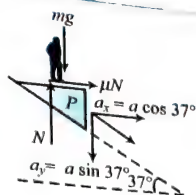
14. FBD of plank:



(only horizontal forces are shown)

$$\therefore a_1 = \frac{f}{m}$$

(i) (acceleration of plank w.r.t. ground)



For man w.r.t. plank.

$$ma_1 + f = ma$$

$$\text{From (i) } 2ma_1 = ma \Rightarrow a_1 = a/2$$

Acceleration of man w.r.t. ground

$$\vec{a}_m = \vec{a}_{m,p} + \vec{a}_p = a + \left(-\frac{a}{2}\right) = \frac{a}{2}$$

$$f = \frac{ma}{2}$$

(pseudo force)



Exercises

Single Correct Answer Type

1. (4) Force of friction is independent of the area of contact.

2. (3) When free fall of lift normal reaction on block by lift floor will be zero. Hence no friction.

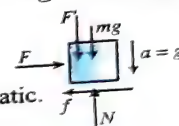
3. (4) Here normal reaction $N = F' = 100 \text{ N}$ ($F' + mg - N = mg$)

$$\text{Hence } f_{\text{lim}} = \mu_s N = 0.1 \times 100 = 10 \text{ N}$$

$$\text{Here } F \text{ (driving force)} = 5 \text{ N}$$

Hence, block will not move friction will be static.

$$\text{Hence, } f = 5 \text{ N.}$$



4. (3) Given horizontal force $F = 25 \text{ N}$ and coefficient of friction between block and wall (μ) = 0.4.

We know that at equilibrium horizontal force provides the normal reaction to the block against the wall. Therefore, normal reaction to the block (R) = $F = 25 \text{ N}$.

We also know that weight of the block (W) = Frictional force = $\mu R = 0.4 \times 25 = 10 \text{ N}$.

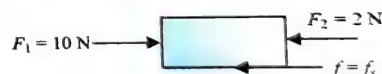
5. (1) $N = mg \cos \theta$, $f = mg \sin \theta$

Net force applied by M on m (or m on M):

$$\begin{aligned} F &= \sqrt{N^2 + f^2} \\ &= \sqrt{(mg \cos \theta)^2 + (mg \sin \theta)^2} \\ &= mg \end{aligned}$$



6. (3) The driving force on the block



$$F_1 - F_2 = 10 - 2 = 8 \text{ N}$$

As the block is at rest the friction will be static and towards left.

$$f = f_s = 8 \text{ N}$$

If F_1 is removed only $F_2 = 2 \text{ N}$ is acting on the block.



If the block is at rest $f = F_2 = 2 \text{ N}$ this value of friction is within static range as in previous case even $f = 8 \text{ N}$ is static case.

7. (1) Zero. No driving force. So friction is zero.

8. (1) During downward motion:

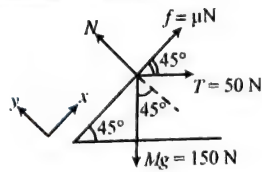
$$F = mg \sin \theta - \mu mg \cos \theta$$

During upward motion:

$$2F = mg \sin \theta + \mu mg \cos \theta$$

Solving above two equations, we get $m = (\tan \theta)/3$

9. (1) The string is under tension. Hence, there is limiting friction between the block and the plane (figure)



$$\sum F_x = 0 \Rightarrow \mu N + 50 \cos 45^\circ = 150 \sin 45^\circ \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N = 50 \sin 45^\circ + 150 \cos 45^\circ \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get $m = 1/2$

10. (3) The minimum force required to just move a body will be $f_1 = \mu_s mg$. After the motion is started, the friction will become kinetic. So the force which is responsible for the increase in velocity of the block is

$$F = (\mu_s - \mu_k) mg = (0.8 - 0.6) \times 4 \times 10 = 8 \text{ N}$$

$$\text{So, } a = \frac{F}{m} = \frac{8}{4} = 2 \text{ m s}^{-2}$$

11. (1) For first half acceleration = $g \sin \phi$
Therefore, velocity after travelling half distance

$$v^2 = 2(g \sin \phi)l$$

For second half, acceleration
 $= g(\sin \phi - \mu_k \cos \phi)$

$$\text{So } 0^2 = v^2 + 2g(\sin \phi - \mu_k \cos \phi)l \quad \dots(ii)$$

Solving (i) and (ii), we get $\mu_k = 2 \tan \phi$

$$12. (2) t_A = \sqrt{\frac{2s}{a_A}} \Rightarrow t_D = \sqrt{\frac{2s}{a_D}}, t_A = \frac{1}{2} t_D$$

$$\sqrt{\frac{2s}{g \sin \theta + \mu g \cos \theta}} = \frac{1}{2} \sqrt{\frac{2s}{g \sin \theta - \mu g \cos \theta}}$$

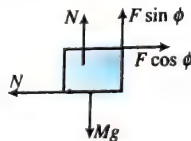
$$4g \sin \theta - 4\mu g \cos \theta = g \sin \theta + \mu g \cos \theta$$

$$3g \sin \theta = 5\mu g \cos \theta \quad \text{or} \quad \mu = \frac{3}{5} \tan \theta = \frac{3}{5} \quad (\because \theta = 45^\circ)$$

13. (3) $N = Mg - F \sin \phi$

From figure

$$a = \frac{F \cos \phi - \mu (Mg - F \sin \phi)}{M}$$



14. (3) From $s = ut + \frac{1}{2} at^2 = 0 + \frac{1}{2} at^2, t = \sqrt{\frac{2s}{a}}$

For smooth plane $a = g \sin \theta$

For rough plane, $a' = g(\sin \theta - \mu \cos \theta)$

$$\therefore t' = nt \Rightarrow \sqrt{\frac{2s}{g(\sin \theta - \mu \cos \theta)}} = n \sqrt{\frac{2s}{g \sin \theta}}$$

$$\therefore n^2 g(\sin \theta - \mu \cos \theta) = g \sin \theta$$

$$\text{When } \theta = 45^\circ, \sin \theta = \cos \theta = 1/\sqrt{2}$$

$$\text{Solving, we get } \mu = \left(1 - \frac{1}{n^2}\right)$$

15. (3) $a_1 = \frac{F - f_1}{m_1} = \frac{F - \mu m_1 g}{m_1} = 10 \text{ m s}^{-2}$

$$a_2 = \frac{F - \mu m_2 g}{m_2} = 1 \text{ m s}^{-2}$$

$$\therefore s = \frac{1}{2} a_{\text{real}} t^2 = \frac{1}{2} [10 + 1] t^2 \Rightarrow t = 2 \text{ s}$$

16. (1) Coefficient of friction is independent of the normal reaction. Hence, it will remain same.

17. (1) $f_1 = \mu Mg$. If motion does not start, then

$$f = F = F_0 t$$

Motion will start when $f = f_1$

$$\Rightarrow F_0 T = \mu Mg \Rightarrow T = \frac{\mu Mg}{F_0}$$



18. (4) If the blocks move together,

$$a = \frac{F}{m_A + m_B} = \frac{10}{6} = \frac{5}{3} \text{ m s}^{-1}$$

$$f_B \text{ (frictional force on B)} = m_B a = \frac{m_B F}{m_A + m_B} = \frac{20}{3} \text{ N}$$

$$f_{B \text{ max}} = \mu m_A g = 0.4 \times 2 \times 10 = 8 \text{ N}$$

As $f_{B \text{ max}} > f_B$, the blocks will not be separated and move together with common acceleration $5/3 \text{ m s}^{-2}$.

19. (1) Minimum effort is required by pulling a block at the angle of friction.

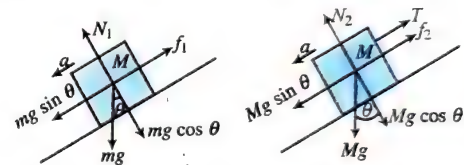
20. (2) Retardation of train = $20/4 = 5 \text{ m s}^{-2}$

It acts in the backward direction. Fictitious force on suitcase = $5m$ Newton, where m is the mass of suitcase.

In acts in the forward direction. Due to this force, the suitcase has a tendency to slide forward. If suitcase is not to slide, then $5m = \text{force } f \text{ of friction}$

$$\text{or } 5m = m mg \quad \text{or} \quad m = \frac{5}{10} = 0.5$$

21. (3) $N_1 = mg \cos \theta$ and $f_1 = \mu mg \cos \theta$



$$N_2 = Mg \cos \theta \quad \text{and} \quad f_2 = \mu Mg \cos \theta$$

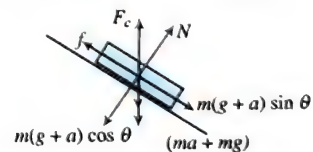
Equations of motion are

$$T - f_1 + mg \sin \theta = ma \quad \dots(i)$$

$$Mg \sin \theta - T - f_2 = Ma \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get $T = 0$

22. (2) From figure, $F_c = \sqrt{f^2 + N^2} = m(g + a)$



$$f = m(g + a) \sin \theta, \quad N = m(g + a) \cos \theta$$

$$\text{Net contact force: } F_c = \sqrt{f^2 + N^2} = m(g + a)$$

23. (2) Frictional force = $\mu R = \mu (mg + Q \cos \theta)$ and horizontal push = $P - Q \sin \theta$

For equilibrium, we have

$$\mu (mg + Q \cos \theta) = P - Q \sin \theta \Rightarrow \mu = \frac{P - Q \sin \theta}{mg + Q \cos \theta}$$

24. (4) As there is no tendency of relative slipping between the block and cube, the friction force is zero.

25. (2) For the equilibrium of block of mass M_1 :

Frictional force, $f = \text{tension in the string, } T$

$$\text{where } T = f = \mu(m + M_1)g \quad \dots(i)$$

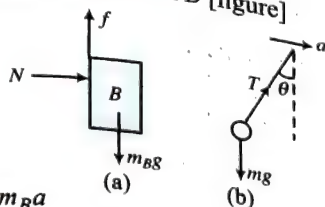
For the equilibrium of block of mass M_2 :

$$T = M_2 g$$

From Eqs. (i) and (ii), we get

$$\mu(m + M_1)g = M_2 g \quad m = \frac{M_2}{\mu} - M_1$$

16. (4) In the free-body diagram of B [figure]



$$N = m_B a$$

$$f = m_B g$$

$$\mu N = m_B g$$

From Eqs. (i) and (ii),

$$a = \frac{g}{\mu} = 20 \text{ ms}^{-2}$$

FBD of bob:

$$T \sin \theta = ma$$

$$T \cos \theta = mg$$

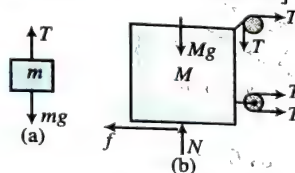
From, Eq. (iii) and (iv),

$$\tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}(2)$$

17. (3) In the free-body diagram of m [figure (a)],

$$T = mg$$

[No friction will act between M and m]



In the free-body diagram of M [figure (b)]

$$f = \mu_2 N = 3T$$

$$\text{and } T + Mg = N$$

From Eq. (iii), $N = (m + M)g$

From Eq. (ii), $\mu_2(m + M)g = 3T$

$$m = \frac{\mu_2 M}{(3 - \mu_2)} = \frac{1/3 \times 8}{(3 - 1/3)} = 1 \text{ kg}$$

28. (2) FBD of m in frame of wedge,

$$N = mg \cos \alpha - ma \sin \alpha$$

$$\text{Now } f = \mu N = ma \cos \alpha + mg \sin \alpha$$

$$\mu = \frac{a \cos \alpha + g \sin \alpha}{g \cos \alpha - a \sin \alpha}$$

$$= \frac{a + g \tan \alpha}{g - a \tan \alpha} = \frac{5}{12}$$

29. (4) Maximum acceleration of B or C can be mg so that they do not slip with each other or on A.

For the system of (A + B + C):

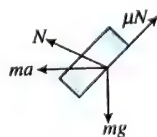
$$T = 3ma = 3\mu mg$$

For D:

$$Mg - T = Ma$$

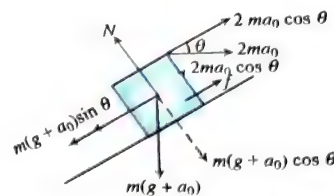
$$\Rightarrow Mg - 3\mu mg = M\mu g$$

$$\Rightarrow M = \frac{3\mu m}{1 - \mu}$$



30. (1) For chain to move with constant speed, P needs to be equal to frictional force on the chain. As the length of chain on the rough surface increases. Hence, the friction force $f_k = \mu_k N$ increases.

31. (4) The FBD of block from lift frame is as shown in figure. From given data, as $m(g + a_0) \sin \theta > 2ma_0 \cos \theta$



So friction force acts upwards.

$$f = m(g + a_0) \sin \theta - 2ma_0 \cos \theta$$

$$= \frac{9mg}{10} - \frac{4mg}{5} = \frac{mg}{10}$$

$$N = m(g + a_0) \cos \theta + 2ma_0 \sin \theta = \frac{9mg}{5}$$

$$\text{As } f_L = \mu_s N = \frac{18mg}{50} = \frac{9mg}{25} > f \text{ so static friction.}$$

Reaction force,

$$R = \sqrt{f^2 + N^2} = \frac{mg}{5} \sqrt{\frac{1}{4} + 9^2} = \frac{mg\sqrt{13}}{2}$$

Alternative solution:

$$\text{Net force, } \vec{F} - m\vec{g} = m(a_0\hat{j} - 2a_0\hat{i})$$

$$\Rightarrow \vec{F} = m(-2a_0\hat{i} + (a_0 + g)\hat{j}) = m(-g\hat{i} + \frac{3g}{2}\hat{j})$$

$$\Rightarrow F = m\sqrt{g^2 + \left(\frac{3g}{2}\right)^2} = \frac{\sqrt{13}mg}{2}$$

$$32. (1) f_1 = \mu_1 m_A g = 0.3 \times 300 = 90 \text{ N}$$

$$f_2 = \mu_2 (m_A + m_B)g$$

$$= 0.2 (300 + 100) = 80 \text{ N}$$

$$f_3 = \mu_3 (m_A + m_B + m_C)g$$

$$= 0.1(300 + 100 + 200) = 60 \text{ N}$$

33. (1) Let m start moving down and extension produced in spring be x at any time. Value of x required to move the block m is

$$kx = \mu mg \cos \theta + mg \sin \theta$$

$$\Rightarrow kx = 0.5 mg \frac{4}{5} + mg \frac{3}{5} = mg$$

For minimum M , it will stop after producing extension in the spring x .

$$Mgx = \frac{1}{2} kx^2 \Rightarrow Mg = \frac{1}{2} kx$$

$$\Rightarrow Mg = \frac{1}{2} mg \Rightarrow M = \frac{m}{2}$$

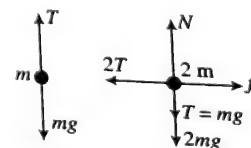
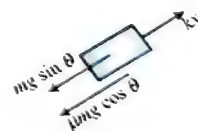
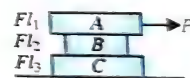
34. (3) In equilibrium (figure)

$$T = mg, N = 3mg \text{ and } f = 2T = 2mg$$

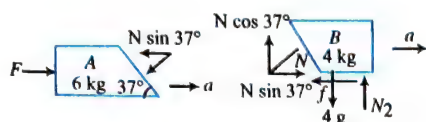
In limiting case, $f < f_{\max}$.

$$2mg < \mu N$$

$$\Rightarrow 2mg \leq 3\mu mg \Rightarrow \mu \geq \frac{2}{3}$$



35. (3) $F - N \sin 37^\circ = 6a \Rightarrow F - \frac{3N}{5} = 6a \quad \dots(i)$



$N_2 = 4g - N \cos 37^\circ = 40 - \frac{4N}{5} \quad \dots(ii)$

$N \sin 37^\circ - f = 4a \quad \dots(iii)$

From Eqs. (i) and (iii): $F - f = 10a$

$\Rightarrow F - \mu N_2 = 10a \quad \dots(iv)$

$\Rightarrow F - \mu \left[40 - \frac{4N}{5} \right] = 10a \quad \dots(v)$

Put the value of N from Eq. (i) in (v) and also put the value of μ to get $a = \frac{5F - 60}{42}$

Now to start motion: $a > 0 \Rightarrow F > 12 \text{ N}$.

So the minimum force F to just start the motion is 12 N.

Now maximum F will be when N_2 just becomes zero. Then from Eqs. (ii): $N = 50 \text{ N}$.

From Eqs. (i) and (iv), we get $F = 75 \text{ N}$ (by putting $N_2 = 0$)

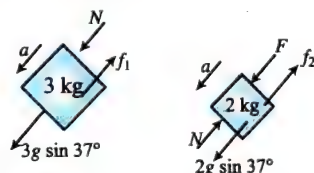
If we apply $F > 75 \text{ N}$, then B will start sliding up on A , but we do not want this.

36. (3) The free-body diagrams of two blocks are shown in figure. Under the assumption that both blocks are moving together,

$F + 2g \sin 37^\circ + 3g \sin 37^\circ - f_1 - f_2 = 5a$

where $f_1 = \mu \times 3g \cos 37^\circ$

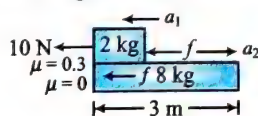
and $f_2 = \mu \times 2g \cos 37^\circ$



$\Rightarrow a = \frac{46}{5} \text{ m s}^{-2}$

For 3-kg block, $N + 3g \sin 37^\circ - f_1 = 3a \Rightarrow N = 12 \text{ N}$

37. (3) Friction between 2-kg and 8-kg blocks is kinetic in nature, so



$F = m \times 2g = 0.3 \times 2 \times 10 = 6 \text{ N}$

For 2 kg block, $10 - 6 = 2a_1$

For 8-kg block, $6 = 8a_2 \Rightarrow a_1 = 2 \text{ m s}^{-2}, a_2 = \frac{3}{4} \text{ m s}^{-2}$

Acceleration of 2-kg block relative to 8-kg block is

$a_{\text{rel}} = a_1 - a_2 = \frac{5}{4} \text{ m s}^{-2}$

Using the equation of motion, $3 = \frac{1}{2} \times \frac{5}{4} t^2 \Rightarrow t = 2.19 \text{ s}$

38. (2) For equilibrium,

$\int \mu dm g \cos \theta \geq \int dm g \sin \theta$

or $\int \mu \lambda dl g \cos \theta \geq \int \lambda dl g \sin \theta$

$\mu \int dl \cos \theta \geq \int dl \sin \theta \left(\because \sin \theta = \frac{dy}{dl}, \cos \theta = \frac{dx}{dl} \right)$

or $\mu \int dx \geq \int dy \text{ or } \mu l \geq h$

39. (4) As the block does not slip on prism, the combined acceleration of the prism is

$a = g \sin \theta$

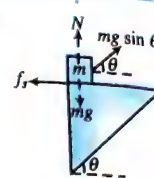
$mg \sin \theta$ is the pseudo force on m .

$N + mg \sin \theta \sin \theta = mg$

or $N = mg \cos^2 \theta$

And for no slipping, $mg \sin \theta \cos \theta \leq \mu N$

$mg \sin \theta \cos \theta \leq \mu mg \cos^2 \theta \text{ or } \mu \geq \tan \theta$



40. (1) As sphere is at rest

$T = mg \quad \dots(i)$

$f = T \quad \dots(ii)$

and $f \leq \mu N$

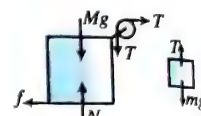
Here $N = Mg + T = (M + m)g$

$T \leq \mu(M + m)g$

$mg \leq \mu(M + m)g$

$\frac{m}{M} \leq \frac{\mu}{(1 - \mu)}$

$\frac{m}{M} \leq \frac{0.25}{(1 - 0.25)} \Rightarrow \frac{m}{M} \leq \frac{1}{3}$



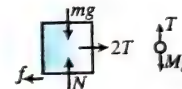
41. (1) $f = 2T \quad \dots(i)$

and $T = Mg \quad \dots(ii)$

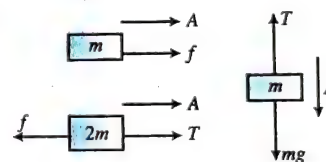
If m be at rest friction will be static

$f \leq \mu N$

$2Mg \leq \mu mg \Rightarrow M = \frac{\mu g}{2}$



42. (3)



$f = mA$

$mg - T = mA$

$T - f = 2mA$

From Eqs. (ii) and (iii),

$f = mg/4$

If friction is static, $f \leq \mu mg$

$\Rightarrow \mu = \frac{1}{4}$

43. (2) Horizontal acceleration of the system is

$a = \frac{F}{2m + m + 2m} = \frac{F}{5m}$

Let N be the normal reaction between B and C .

Free body diagram of C gives

$N = 2ma = \frac{2}{5}F$

Now B will not slide downward if

$\mu N \geq m_B g$

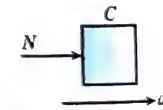
or $\mu \left(\frac{2}{5}F \right) \geq mg \text{ or } F \geq \frac{5}{2\mu}mg$

So $F_{\text{min}} = \frac{5}{2\mu}mg$

44. (1) Common acceleration, $a = \frac{F}{5m}$

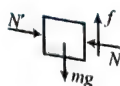
Taking A and B together as system,

$N = (2m + m) \frac{F}{5m} = \frac{3}{5}F$



If block B is not sliding, $f = mg$ and $f \leq \mu N$

$$mg \leq \mu \cdot \frac{3}{5} F \Rightarrow F \geq \frac{5mg}{3\mu}$$



46. (2) The friction force on block A is $\mu_k w_A = (0.25)(2 \text{ N}) = 0.5 \text{ N}$. This is the magnitude of the friction force that block A exerts on block B, as well as the tension in the string. The force F must then have magnitude

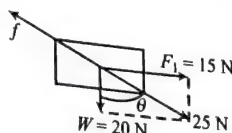
$$F = \mu_k(w_B + w_A) + \mu_k w_A + T$$

$$= \mu_k(w_B + 3w_A) = (0.25)(6 \text{ N} + 3(2 \text{ N})) = 3 \text{ N}.$$

Note that the normal force exerted on block B by the table is the sum of the weights of the blocks.

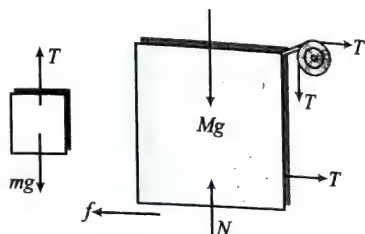
47. (1) $25 - f = ma$
 $\Rightarrow 25 - 20 = 2a \Rightarrow 2.5 \text{ m/s}^2$

$$\tan \theta = \frac{15}{20} \Rightarrow \theta = 37^\circ$$



47. (3) Wedge will start slipping when
 $F \geq \mu(2mg)$

48. (4) F.B.D. of m and M



From F.B.D.: $T = mg$

$$2T = f \Rightarrow 2mg = f \quad \dots(i)$$

and $N = Mg + T = Mg + mg \quad \dots(ii)$

If M is not moving

$$f \leq \mu N$$

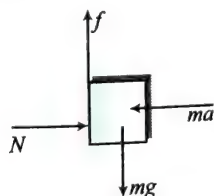
$$2mg \leq \mu(M + m)g \quad \dots(iii)$$

$$(2m - \mu m) \leq \mu M$$

$$m \leq \frac{\mu M}{(2 - \mu)}$$

$$m \leq \frac{0.5 \times 20}{(2 - 0.5)} \Rightarrow m \leq \frac{20}{3} \text{ kg}$$

49. (1) $a = \frac{m_A g - \mu_1(M + m)g}{(m_A + m + M)} \quad \dots(i)$



If the block m is not sliding acceleration of the system

F.B.D. of m w.r.t. M

If m is not sliding

$$f = mg \quad \dots(ii)$$

and $N = ma \quad \dots(iii)$

and $f \leq \mu_2 N$

$$mg \leq \mu_2 ma$$

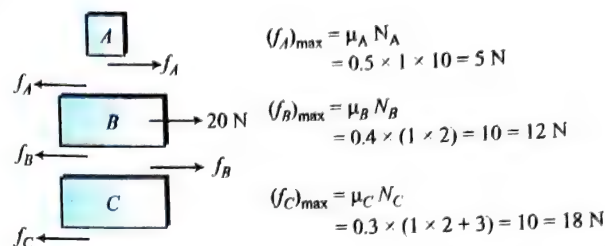
$$g \leq \mu_2 \left[\frac{m_A g - \mu_1(M + m)g}{m_A + m + M} \right]$$

$$(m_A + m + M) \leq \mu_2(m_A - \mu_1(M + m))$$

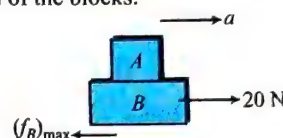
$$(\mu_2 - 1)m_A \geq (M + m) + \mu_1 \mu_2(M + m)$$

$$m_A \geq \frac{(M + m)[1 + \mu_1 \mu_2]}{(\mu_2 - 1)}$$

50. (2)



Let us assume blocks A and B move together with common acceleration of the blocks.



$$a = \frac{20 - 12}{(1 + 2)} = \frac{8}{3} \text{ m/s}^2$$

Now considering block 'A' alone from F.B.D. of 'A'



$$f_A = 1 \times \frac{8}{3} = \frac{8}{3} \text{ N} < (f_A)_{\max}$$

Hence blocks 'A' and 'B' will move together with acceleration $\frac{8}{3} \text{ m/s}^2$.

51. (2) For uppermost block $a_{\max} = 1 \text{ m/s}^2$



$$\text{For lowermost block } a_{\max} = \frac{(24 - 16)}{4} = 2 \text{ m/s}^2$$

Hence, sliding between middle and lower block will start only after sliding between middle and upper block has already started.

For middle block $F - 18 = 6 \times 2 \Rightarrow F = 30 \text{ N}$.

52. (3) The tension in the string initially is zero. If $\mu_1 > \tan \alpha$ and $\mu_2 > \tan \beta$, both the blocks will not move down the incline and the tension in the string shall continue to remain zero.

53. (4) Let the value of 'a' be increased from zero. As long as $a \leq \mu g$, there shall be no relative motion between m_1 or m_2 and platform, that is, m_1 and m_2 shall move with acceleration a.

As $a > \mu g$ the acceleration of m_1 and m_2 shall become μg each.

Hence, at all instants the velocity of m_1 and m_2 shall be same

$\therefore$ The spring shall always remain in natural length.

54. (2) For man + board system

$$f = 2T$$

$$T = N = (M + m)g \quad \dots(i)$$

$$N = (M + m)g - T \quad \dots(ii)$$

If board is not sliding $f \leq \mu N$

$$2T \leq \mu[(M + m)g - T] \quad \dots(iii)$$

$$[2 + \mu]T \leq \mu(M + m)g; T \leq \frac{\mu(M + m)g}{(2 + \mu)}$$

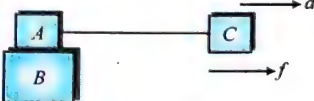
$$\text{Hence, } T_{\max} = \frac{\mu(M+m)g}{(2+\mu)}$$

After substituting the values,

$$T_{\max} = \frac{0.25(70+11)}{(2+0.25)} \times 100 = \frac{0.25 \times 81 \times 10}{2.25} = 90 \text{ N}$$

55. (3) Let all the four blocks move with common acceleration a , then
- $$a = \frac{F}{2(m+M)}$$

Friction between C and D will be more than friction between A and B. Let this friction is f , then



$$f = (m+m+M)a = \frac{(2m+M)F}{2(m+M)}$$

For no slipping: $f \leq f_1$

$$\frac{(2m+M)F}{2(m+M)} \leq \mu_s mg \Rightarrow F \leq \frac{2\mu_s mg(m+M)}{2m+M}$$

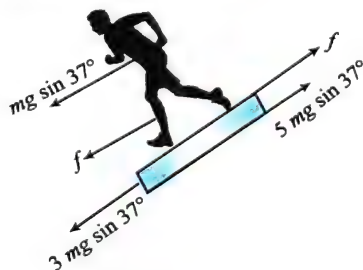
56. (2) Taking (A + B) as the system, net external force on system is less than the limiting value of static friction. Hence, the blocks are at rest.

Further, if F_1 is more than μMg , then there will be tension in the string. If F_1 is less than μMg , then there will be no tension in the string and friction on B acts towards right.

57. (2) Since the blocks cannot accelerate in horizontal direction therefore the normal interaction force between the blocks as well as between 5 kg block and the wall is $F = 1000 \text{ N}$. Again both the blocks accelerate downward with acceleration $\bar{g} \text{ m/s}^2$ and therefore the relative acceleration between the blocks is zero. Hence the friction force between the blocks is zero.

58. (1) For equilibrium of

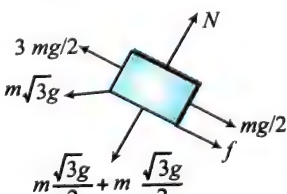
$$3mg \sin 37^\circ = f + 2mg \cos 37^\circ \Rightarrow f = 2m$$



For man, $mg \sin 37^\circ + f = ma$

$$6m + 2m = ma \Rightarrow a = 8 \text{ m/s}^2$$

59. (1) $N = \sqrt{3} mg$



$$f_{\max} = \sqrt{3} \times \frac{\sqrt{3}}{2} mg = \frac{3}{2} mg$$

$\therefore$ Block will not slide

$$\text{Since } f = \frac{3mg}{2} - \frac{mg}{2} = mg < f_{\max}$$

Multiple Correct Answers Type

1. (1, 2, 3)

For (1): Consider a block at rest on a rough surface and no force (horizontal) is acting on it. Now friction force on it would be zero.



For (2): Consider a heavy block, under the application of small force F which is not sufficient to cause its motion, so friction force is static in nature and block does not move.

For (3): Refer theory.

For (4): Friction force and normal force always act perpendicular to each other.



2. (1, 2, 3)

If the block is at rest, then force applied has to be greater than limiting friction force for its motion to begin.

$$f_L = \mu_s mg = 0.25 \times 3g = 7.5 \text{ N} > F_{\text{applied}}$$

So friction is static in nature and its value would be equal to applied force, i.e., 7 N. If the body is initially moving, then kinetic friction is present ($f_k = \mu_k mg = 6 \text{ N}$), acting opposite to direction of motion.

As $F > f_k$ the block is accelerated with an acceleration of

$$a = \left[\frac{F - f_k}{m} \right] = \frac{1}{3} \text{ ms}^{-2} \text{ and hence its speed is continuously increasing.}$$

If applied force is opposite to direction of motion, then block is

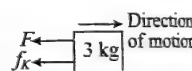
$$\text{under deceleration of } a = -\left[\frac{F + f_k}{m} \right] = -\frac{13}{3} \text{ ms}^{-2} \text{ and hence after}$$

some time block stops and kinetic friction vanishes but applied force continues to act.

But as $F < f_L$, the block remains at rest and friction force acquires the value equal to applied force, i.e., friction is static in nature.

3. (2, 3, 4)

For some time, the block would not move due to friction force. When $F > f_L$, the motion of block starts.



$$a = \frac{F - f_k}{m} = \frac{kt - f_k}{m}$$

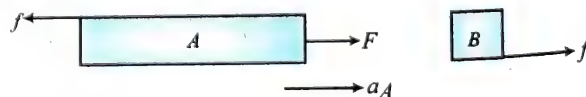
$$\Rightarrow \frac{dv}{dt} = a = \frac{kt - f_k}{m}$$

$$v = \frac{\frac{kt^2}{2} - f_k t}{m}$$

$$\Rightarrow \frac{ds}{dt} = \frac{\frac{kt^2}{2} - f_k t}{m} \Rightarrow s = \frac{\frac{kt^3}{6} - \frac{f_k t^2}{2}}{m}$$

4. (1, 2, 3)

As the acceleration of A and B are different, it means there is relative motion between A and B. The free-body diagram of A and B can be drawn as below.



$$\text{For A, } F - f = Ma_A = 50 \times 3 = 150 \text{ N}$$

$$\text{For B, } f = ma_B = 20 \times 2 \Rightarrow f = 40 \text{ N, } F = 190 \text{ N}$$

5. (1, 2, 3)

Friction on A and B acts as shown in figure.



From the figure it is clear that friction on A supports its motion and on B opposes its motion. And friction always opposes relative motion.

6. (1, 2, 3, 4)

When friction between the blocks becomes zero, the relative sliding between the block will be stopped hence; $v_A = v_B$ and $a_A = a_B$. Also when the friction becomes zero, only forces to move the blocks are F_A and F_B .

$$a_A = a_B \Rightarrow \frac{F_A}{m_A} = \frac{F_B}{m_B}$$

7. (1, 2, 3)

Here $m_1 g \sin 30^\circ = m_2 g = 20 \text{ N}$, so there is no tendency of motion in any direction.

Hence, there is no friction on m_1 . Contact force will be only normal force.

8. (2, 3)

Here $F > \mu, mg \left(1 + \frac{m}{M}\right)$, so slipping will occur between the blocks.

$$\text{For } m F - \mu_k mg = m \cdot a \Rightarrow a = 1.2 \text{ ms}^{-2}$$

$$\text{For } M \mu_k mg = MA \Rightarrow A = 0.4 \text{ ms}^{-2}$$

9. (1, 4)

If initially acceleration of A is greater than that of B, then there will be extension and if that of B is greater than A, then there will be compression in the spring. Otherwise the length of spring will remain same.

10. (1, 2, 3)

(1) and (3) Let the acceleration of each block be a .

$$10g - T_2 = 10a, \quad T_2 - T_1 - f = 3a$$

$$T_1 - f = 2a, \quad \text{where } f = 0.3 \times 2g = 6 \text{ N}$$

From above equations

$$10g - 2f = 15a \Rightarrow 10 \times 10 - 2 \times 6 = 15a$$

$$\Rightarrow a = 88/15 \text{ ms}^{-2}$$

$$T_2 = 10g - 10a = 10 \times 10 - 10 \times \frac{88}{15} = 41.3 \text{ N}$$

$$T_1 = f + 2a = 6 + 2 \times \frac{88}{15} = 17.7 \text{ N}$$

Clearly $T_2 > T_1$

(2) This is correct because of greater mass of 3 kg since acceleration is same for both.

(4) This is incorrect, because net force acting on 10 kg mass is greater due to its larger mass, not due to its acceleration downward.

11. (2, 3)

$$F = 10a_1 + 40a_2$$

$$\Rightarrow 100 = 10a_1 + 40 \times 2$$

$$\Rightarrow a_1 = 2 \text{ ms}^{-2}$$

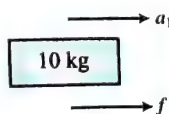
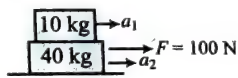
So acceleration of A must be 2 ms^{-2} for given conditions to be satisfied.

$$f = 10a_1 = 10 \times 2 = 20 \text{ N}$$

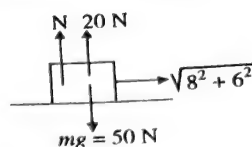
$$f \leq f_l \Rightarrow 20 \leq \mu m_A g$$

$$\Rightarrow 20 \leq \mu \times 10g \Rightarrow \mu \geq 0.2$$

Hence, μ can be greater or equal to 0.2.



12. (2, 3, 4)



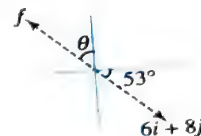
$$N = 50 - 20 = 30 \text{ N}$$

Limiting friction force $= \mu N = 12 \text{ N}$ and applied force in horizontal direction is less than the limiting friction force, therefore the block will not slide.

For equilibrium in horizontal direction, friction force must be equal to 10 N.

From the top view, it is clear that $\theta = 37^\circ$ i.e. 127° from the x-axis that is the direction of the friction force. It is opposite to the applied force.

$$\text{Contact force} = \sqrt{N^2 + f^2} = 10\sqrt{10} \text{ N}$$



13. (2, 4)

(2) The blocks m_1 will just begin to move up the plane if the downward force $m_2 g$ due to mass m_2 trying to pull the mass m_1 up the plane just equals the force

$$m_2 g = (m_1 g \sin \theta + \mu m_1 g \cos \theta)$$

$$mg = mg \sin \theta + \mu mg \cos \theta$$

$$mg(1 - \sin \theta) = \mu mg \cos \theta$$

$$\mu = \sec \theta - \tan \theta$$

So, choice (2) is correct.

(4) The block m_1 will just begin to move down the plane, if the downward force $(m_1 g \sin \theta - \mu m_1 g \cos \theta)$ on m_1 just equals the upward force m_2 of acting on m_1 due to m_2 if

$$m_2 g = m_1 g (\sin \theta - \mu \cos \theta)$$

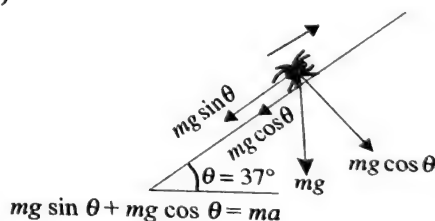
$$m_2 g = 2m_1 g (\sin \theta - \mu \cos \theta)$$

$$\frac{1}{2} = \sin \theta - \mu \cos \theta$$

$$\frac{1}{2 \cos \theta} = \tan \theta - \mu$$

$$\mu = \tan \theta - \frac{1}{2} \sec \theta$$

14. (1, 3)



$$mg \sin \theta + mg \cos \theta = ma$$

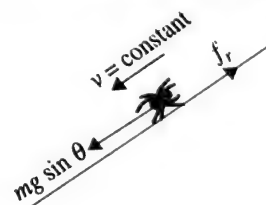
$$= 10 \left[\frac{3}{5} + \frac{4}{5} \right] = 14 \text{ m/sec}^2$$

$$\text{If } f_r = mg \sin \theta$$

$$= mg \times \frac{3}{5} = \frac{3mg}{5} = f_{r \max}$$

$$f_r < f_{r \max}$$

$$\frac{3mg}{5} < \frac{4mg}{5} \text{ hence insect can move with constant velocity.}$$



15. (2, 4)

Case I: Since, no relative motion:

$$a = \frac{F_1 - F_f}{5} = \frac{F_f}{3} \Rightarrow F_{1(\max)} = \frac{8}{3} F_f$$

$$\text{Case II: } a = \frac{F_f}{5} = \frac{F_2 - F_f}{3} \Rightarrow F_{2(\max)} = \frac{8}{5} F_f$$

$$\text{Clearly, } F_{1(\max)} > F_{2(\max)} \text{ and } \frac{F_{1(\max)}}{F_{2(\max)}} = \frac{5}{3}$$

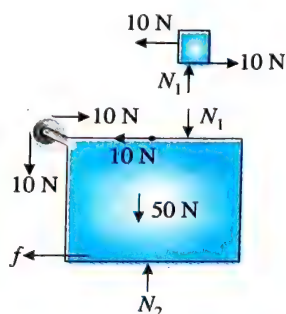
16. (1, 4)

The frictional force on block A is

$$\Rightarrow \mu N_1 = 10 \Rightarrow N_1 = \frac{10}{0.2} = 50 \text{ N}$$

The net force on block B in vertical direction is zero

$$\therefore N_1 = 50 + N_1 + 10 = 110 \text{ N}$$



$\Rightarrow$ Normal reaction exerted by ground on block B is 110 N.

The net force on block B in horizontal direction is zero.

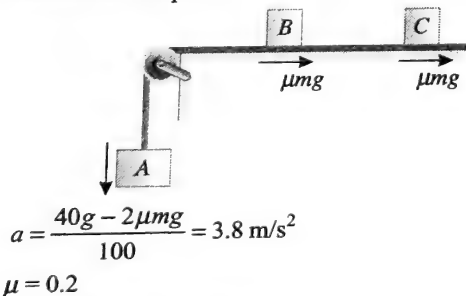
$$\therefore f + 10 - 10 = 0$$

$\Rightarrow$ Frictional force exerted by ground on block B is zero

17. (1, 3)

Maximum possible acceleration of block will be $= \mu g$.

- (i) When $\mu = 0.2$ maximum acceleration possible so that block does not slip is 2 m/s^2 . Now acceleration of system assuming blocks don't slip is



As $3.8 > 2$ hence the assumption is wrong and blocks slip on the belt.

$\therefore$ Maximum force applicable on block B is μmg

$\therefore$ Acceleration of block $= 2 \text{ m/s}^2$

Hence, velocity of B when it strikes pulley is $v^2 = 0^2 + 2 \times 2 \times 2$
 $v^2 = 8$

$$\therefore v = 2\sqrt{2} \text{ m/s}$$

Hence, (1).

- (ii) When $\mu = 0.5$ maximum acceleration possible so that block does not slip is 5 m/s^2

Now acceleration of system assuming blocks don't slip is

$$a = \frac{40g - 2\mu mg}{100} = 1 \text{ m/s}^2$$

$$\mu = 0.5$$

as $1 < 5$ hence, blocks do not slip and acceleration of block B is 1 m/s^2

$\therefore$ Hence velocity of B when it strikes pulley is

$$v^2 = 0^2 + 2 \times 1.2$$

$$v^2 = 4$$

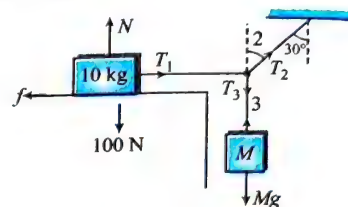
$$\therefore v = 2 \text{ m/s}$$

Hence, (3).

18. (1, 2, 3)

If the block of 10 kg is at rest

$$f = T_1 \text{ and } f \leq \mu N$$



$$T_1 \leq 0.7 \times 100 \Rightarrow T_1 \leq 70 \text{ N}$$

$$T_{1,\max} = 70 \text{ N}$$

At junction of strings

$$T_2 \sin 30^\circ = T_1 \quad \dots(i)$$

$$\text{and } T_2 \cos 30^\circ = Mg = T_3 \quad \dots(ii)$$

From (i)/(ii)

$$\tan 30^\circ = \frac{T_1}{Mg} = \frac{T_1}{T_3}$$

$$\Rightarrow \tan 30^\circ = \frac{T_1}{T_3}$$

$$M = \frac{T_1}{(\tan 30^\circ)g}$$

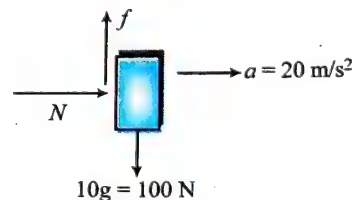
$$M_{\max} = \frac{70}{(1/\sqrt{3}) \times 10} = 7\sqrt{3} \text{ N} \Rightarrow M_{\max} = 12 \text{ N}$$

19. (1, 2, 3, 4)

$$N = ma = 10 \times 20 = 200 \text{ N}$$

$$f_t = \mu N = 0.6 \times 200 = 120 \text{ N}$$

$$f = 100 \text{ N}$$



Since $f < f_t$, so block will not slip down and acceleration of block will also be 20 m/s^2 in horizontal direction.

Net contact force:

$$F = \sqrt{N^2 + f^2} = \sqrt{200^2 + 100^2} = 100\sqrt{5} \text{ N}$$

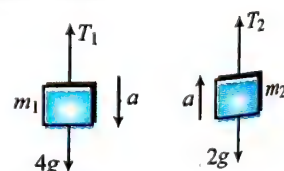
All contact forces are electromagnetic in nature.

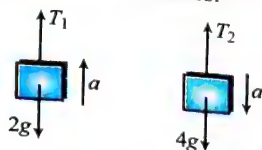
20. (1, 3)

$$T_1 = T_2 + 10, 4g - T_1 = 4a, T_2 - 2g = 2a$$

$$\text{Solve to get } a = \frac{5}{3} \text{ m/s}^2,$$

$$T_2 = 2g + 2a = \frac{70}{3} \text{ N}$$





$$T_2 = T_1 + 10, T_1 - 2g = 2a$$

$$4g - T_2 = 4a, \text{ solve to get } a = \frac{5}{3} \text{ m/s}^2$$

11. (1, 2, 3, 4)

$$\text{Acceleration, } a = \frac{2g}{(2+3)} = 4 \text{ m/s}^2$$

Initial velocity of A is towards left and acceleration is towards right.

$$\begin{aligned} \text{At } t = 1 \text{ sec, } v &= -u + at \\ &= -10 + 4 \times 1 = -6 \text{ m/s} \\ &= 6 \text{ m/s (towards left)} \end{aligned}$$

Block A will stop at $t = 2.5$ second, as

$$v = -10 + 4 \times 2.5 = 0$$

Block A will be at a distance given by (at $t = 4$ seconds)

$$\begin{aligned} S &= -ut + \frac{1}{2}at^2 \\ &= -10 \times 4 + \frac{1}{2} \times 4 \times 16 = -8 \text{ m towards left} \end{aligned}$$

So, distance from pulley = 108 m

Now we can calculate time at which displacement will be zero.

$$\begin{aligned} S &= 0 = -ut + \frac{1}{2}at^2 \\ t &= \frac{2u}{a} = \frac{2 \times 10}{4} = 5 \text{ sec} \end{aligned}$$

Block A will be again at 100 m at $t = 5$ sec.

12. (2, 4)

As the block does not slip on prism, the combined acceleration of the prism is $a = g \sin \theta$.

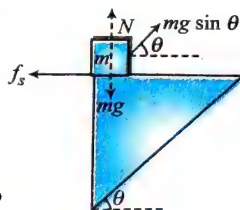
$mg \sin \theta$ is the pseudo force on m .

$$N + mg \sin \theta \sin \theta = mg$$

or $N = mg \cos^2 \theta$ and for no slipping, so

$$mg \sin \theta \cos \theta \leq \mu N$$

$$mg \sin \theta \cos \theta \leq \mu mg \cos^2 \theta \text{ or } \mu \geq \tan \theta$$



Linked Comprehension Type

For Problems 1 and 2

1. (4), 2. (3)

As the monkey moves downwards with respect to rope with an acceleration b , its absolute acceleration is $a + b$, where a is the acceleration of rope. Therefore, equations of motion are

$$mg - T = m(a + b) \quad \dots(i)$$

$$T - \mu Mg = Ma \quad \dots(ii)$$

Putting the value of T from Eq. (ii) into Eq. (i), we get

$$(m - \mu M)g = (M + m)a + mb$$

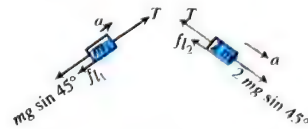
$$\Rightarrow \frac{m(g - b) - \mu Mg}{(M + m)} = a$$

$$T = \mu Mg + \frac{Mmg - Mmb - \mu M^2 g}{M + m}$$

$$\begin{aligned} &= \frac{\mu M^2 g + \mu Mmg + Mmg - Mmb - \mu M^2 g}{M + m} \\ &= \frac{Mm(\mu g + g - b)}{M + m} \end{aligned}$$

For Problems 3-5

$$3. (2) \quad T - mg \sin 45^\circ - f_{l1} = ma, \quad 2mg \sin 45^\circ - T - f_{l2} = 2ma$$



$$\text{where } f_{l1} = \mu_1 mg \cos 45^\circ = 2mg \cos 45^\circ/3$$

$$\text{and } f_{l2} = \mu_2 2mg \cos 45^\circ = 2mg \cos 45^\circ/3$$

$$\text{Now we get } a = -\frac{g}{9\sqrt{2}}$$

This is negative, which is not possible. Hence $a = 0$.

4. (3) $f_{l2} < 2mg \sin 45^\circ$, hence friction only will not be able to prevent slipping of $2m$ mass. So on $2m$ mass, friction will be maximum i.e. f_{l2} .

$$T + f_{l2} = 2mg \sin 45^\circ \Rightarrow T = \frac{2\sqrt{2}}{3} mg$$

$$5. (4) \quad T = mg \sin 45^\circ + f_1 \Rightarrow f_1 = \frac{mg}{3\sqrt{2}}$$

For Problems 6 and 7

$$6. (3) \quad \text{Net force, } F = \sqrt{F_1^2 + F_2^2} = \sqrt{41} \text{ N}$$

$$f_{l1} = 0.4 \times 10 \text{ g} = 40 \text{ N}, f_{l2} = 0.3 \times 10 \text{ g} = 30 \text{ N}$$

Net force is less than f_b , hence

$$\text{Required friction force} = \text{applied force} = \sqrt{41} \text{ N}$$

7. (1) Now applied force will be equal to maximum friction force, i.e., $\sqrt{5^2 + a^2} = f_l = 40$

$$\Rightarrow a = \sqrt{1575} \text{ N}$$

For Problems 8-10

$$8. (3) \quad \text{For } t < t_0: f = \mu mg, \quad a_1 = \frac{f}{m} = \mu g$$

Velocity of block at any time:

$$v_b = v_1 + a_1 t$$

$$\Rightarrow v_b = v_1 + \mu g t$$

Velocity of plank at any time: $v_p = v_2 + at$

At $t = t_0$, both velocities are same.

$$v_1 + \mu g t_0 = v_2 + at_0 \Rightarrow t_0 = \frac{v_2 - v_1}{\mu g - a}$$

9. (1) For $t < t_0$:

Velocity of block at any time: $v_b = v_1 + \mu g t$. Slope of graph is μg .

Velocity of block at $t = t_0$: $v_{b0} = v_1 + \mu g t_0$

For $t > t_0$:

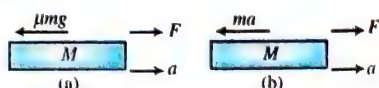
Velocity of block at any time $v_b = v_{b0} + a(t - t_0)$

$$v_b = v_1 + (mg - a)t_0 + at$$

Slope of graph is a . So slope will decrease.

Hence correct graph is (a)



10. (2) For $t < t_0$:Let F be the forward force, then

$$F - \mu mg = Ma \Rightarrow F = \mu mg + Ma$$

For $t > t_0$:Let F be the forward force, then

$$F - ma = Ma \Rightarrow F = (m + M)a$$

For Problems 11–13

11. (2) As acceleration is coming negative for both the possible directions of motion, it means the net applied force is not enough to cause the motion of system or to overcome the limiting friction force.

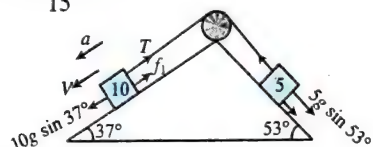
12. (1) As it is given that system is moving initially in a specific direction, it means direction of friction is known and it is kinetic in nature.

$$a = \frac{10g \sin 37^\circ - 5g \sin 53^\circ - f_1 - f_2}{15}$$

$$f_1 = \mu_{k1} \times 10g \cos 37^\circ = 20.0$$

$$f_2 = \mu_{k2} \times 5g \cos 53^\circ = 2.25$$

$$a = \frac{2.25}{15} = \text{ms}^{-2}$$



So the required time,

$$t_0 = \frac{2}{2.25/15} = \frac{30}{2.25} = 13.33 \text{ s}$$

13. (3) Once the block comes to rest, kinetic friction disappears and static friction comes into the existence.

For Problems 14 and 15

14. (2), 15. (1)

Assuming systems move together there is no sliding, acceleration of the system

$$a = \frac{F}{(5+10)} = \frac{F}{15} \quad \dots(i)$$

FBD of m

$$\text{FBD of } M: f - F = ma = 10 \left(\frac{F}{15} \right)$$

$$\Rightarrow f = F \left(1 + \frac{10}{15} \right) = \frac{5}{3} F \quad \dots(ii)$$

If there is no sliding, $F \leq \mu_s N$

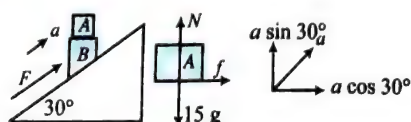
$$F \left[\frac{5}{3} \right] \leq 0.4 \times 10 \times 10 \Rightarrow F \leq 24 \text{ N}$$

$$\text{From (i)} \quad a = \frac{F}{15} = \frac{24}{15} = 1.6 \text{ ms}^{-2}$$

For Problems 16 and 17

16. (1), 17. (2)

$$500 - 55g \sin 30^\circ = 55a \Rightarrow a = \frac{45}{11} \text{ ms}^{-2}$$



$$N - 15g = 15a \sin 30^\circ$$

$$N = 15 \left[10 + \frac{45}{11} \times \frac{1}{2} \right] = \frac{265 \times 15}{22}$$

$$f = 15a \cos 30^\circ = 15 \times \frac{45 \sqrt{3}}{11 \times 2}$$

For A not to slide on B : $f \leq f_l$

$$\Rightarrow \frac{15 \times 45}{11} \times \frac{\sqrt{3}}{2} \leq \mu \left(\frac{265 \times 15}{22} \right) \Rightarrow \mu \geq \frac{9\sqrt{3}}{53} = 0.294$$

For Problems 18–20

$$18. (3) \quad 0.5t = \mu mg \left(1 + \frac{m}{M} \right), t = 12 \text{ s}$$

$$\Rightarrow \mu = 0.2$$

19. (1) Use concept of function force for drawing the graph.

$$20. (4) \quad \frac{t}{2} = (M+m) \frac{dv}{dt}$$

$$\int_0^v dv = \int_0^{12} \frac{t}{2(M+m)} dt \Rightarrow v = 6 \text{ ms}^{-1}$$

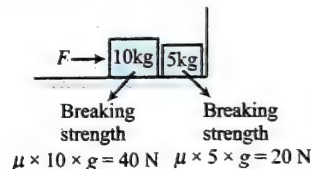
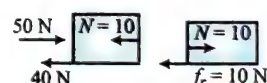
After 12 s up to 15 s for 3 s the acceleration of plank will be

$$a = \frac{0.2 \times 2 \times 10}{4} = 1 \text{ ms}^{-2}$$

$$v' = 6 + 3 \times 1 = 9 \text{ ms}^{-1}, a_{av} = \frac{v' - 0}{15} = \frac{9}{15} = 0.6 \text{ ms}^{-2}$$

For Problems 21–23

21. (4), 22. (1), 23. (2)

21. (4) If $F = 20 \text{ N}$, 10 kg block will not move and it will not press 5 kg block. So $N = 0$.22. (1) If $F = 50 \text{ N}$, force on 5 kg block = 10 N

So friction force = 10 N

23. (2) Until the 10-kg block is stuck with ground ($F = 40 \text{ N}$), No force will be felt by 5-kg block. After $F = 40 \text{ N}$, the friction force on 5-kg increases, till $F = 60 \text{ N}$, and after that, the kinetic friction start acting on 5-kg block, which will be constant (20 N).**For Problems 24 and 25**

24. (4), 25. (2)

Free-body diagrams:



Let both the blocks move together.

$$\text{Acceleration of blocks, } a = \frac{F}{(m+M)}$$

$$f = m \left(\frac{F}{m+M} \right)$$

If both the blocks moves together, $f \leq \mu mg$

$$\frac{mF}{(m+M)} \leq \mu mg$$

$$F \leq \mu(m+M)g$$

If the cube begins to slide then, $F = \mu(m + M)g$

$$a_m = \frac{f}{m} = \mu g \quad (\text{towards } +x \text{ direction})$$

$$a_M = \frac{F - \mu mg}{M} \quad (\text{towards } +x \text{ direction})$$

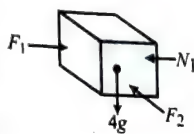
$$\begin{aligned} \bar{a}_{m,M} &= \bar{a}_m - \bar{a}_M = \mu g - \left(\frac{F - \mu mg}{M} \right) \\ &= \frac{\mu(m + M)g - F}{M} \end{aligned}$$

If the cube falls from the plank, it will cover a distance l

$$-l = \frac{1}{2} a_{m,M} t^2 \Rightarrow t = \sqrt{\frac{2l}{a_{m,M}}} = \sqrt{\frac{2lM}{F - \mu(m + M)g}}$$

For Problems 26–29

26. (3) The forces acting on the block are F_1 , F_2 , mg , normal contact force and frictional force. Here frictional force won't act along vertical direction as the component of resultant force along the surface acting on body is not along vertical direction and direction of the friction force is either opposite to the motion of block (direction of acceleration of block) if it is moving or opposite to component of resultant force along the surface if it is not moving.



$$N_1 = 300 \text{ N}$$

$$\text{So, } f_L = \mu N_1 = 0.6 \times 300 = 180 \text{ N}$$

Resultant of $4g$ and F_2 is 107.7 N making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal. As force applied $F_2 = 100 \text{ N}$ is less than f_L , the block doesn't move and friction is static in nature.

$f = 107.7 \text{ N}$ making an angle of $\tan^{-1}\left(\frac{2}{5}\right)$ with the horizontal in upward direction.

27. (1) As $F_{\text{applied}} < f_L$, the block remains at rest.

28. (2) For $F_1 = 150 \text{ N}$, $f_L = 0.6 \times 150 = 90 \text{ N}$

As component of resultant force along the surface is 107.7 N and it is greater than f_L , so kinetic friction comes into existence, i.e., friction force acquires the value $f = \mu_k N_1 = 0.5 \times 150 = 75 \text{ N}$. Its direction is opposite to the component of resultant force along the surface.

29. (4) Acceleration of block $= \frac{107.7 - 75}{4} = 8.175 \text{ ms}^{-2}$

For Problems 30–32

30. (3) $f_i = f_k = 0.5 \times 2g = 10 \text{ N}$

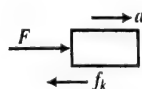
Initially $F = 20 \text{ N} > f_i$, so the block will start accelerating immediately.

At any time t : $F = 20 - 2t$

$$\text{Acceleration: } a = \frac{20 - 2t - f_k}{m} = \frac{10 - 2t}{m} \quad \dots(i)$$

$$\text{For } a = 0, \frac{10 - 2t}{2} = 0 \Rightarrow t = 5 \text{ s} \quad \dots(ii)$$

$$\text{From Eq. (i), } \frac{dv}{dt} = \frac{10 - 2t}{2} = 5 - t$$



$$\int_0^v dv = \int_0^t (5 - t) dt \Rightarrow v = 5t - \frac{t^2}{2}$$

Let us see when the velocity becomes zero. For this:

$$5t - \frac{t^2}{2} = 0 \Rightarrow t = 10 \text{ s}$$

We see that at $t = 10 \text{ s}$, also $F = 0$. So the block has no tendency to move. Hence, acceleration is zero at this time. Now the block will not move from $t = 10 \text{ s}$ to 15 s , because for this magnitude of $F < 10 \text{ N}$. So block will remain at rest from $t = 10 \text{ s}$ to 15 s or acceleration is zero from 10 to 15 s .

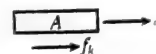
31. (1) From Eq. (ii), first time acceleration, becomes zero at $t = 5 \text{ s}$
Velocity at this time

$$v = 5t - \frac{t^2}{2} = 5 \times 5 - \frac{5^2}{2} = 12.5 \text{ ms}^{-1}$$

32. (4) From the above discussion, clearly, velocity is zero at $t = 12 \text{ s}$

For Problems 33–35

33. (3) Since there is no friction between A and B , B will remain at rest.



$$f_k = \mu(2 + 2)g = 0.1(2 + 2)g = 4 \text{ N}$$

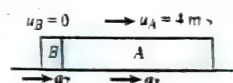
$$\text{Acceleration of A: } a = \frac{f_k}{m} = \frac{4}{2} = 2 \text{ ms}^{-2}$$

$$\text{For B to fall off A: } S = ut + \frac{1}{2} at^2$$

$$\Rightarrow 4 = 0 \times t + \frac{1}{2} 2t^2 \Rightarrow t = 2 \text{ s}$$

34. (2) $v_A = u + at = 0 + 2 \times 2 = 4 \text{ ms}^{-1}$

35. (3) Just when B falls off A , take this instant to be $t = 0$



At $t = 0$:

Velocity of A : $u_A = 4 \text{ ms}^{-1}$,

velocity of B : $u_B = 0$

acceleration of A : $a_1 = m_1 g = 0.1 \times 10 = 1 \text{ ms}^{-2}$

acceleration of B : $a_2 = m_2 g = 0.4 \times 10 = 4 \text{ ms}^{-2}$

Let us see when A comes to rest (w.r.t. belt):

For this: $v_A = u_A + a_1 t$

$$\Rightarrow 8 = 4 + 1 \times t$$

$$\Rightarrow t = 4 \text{ s}$$

Let us see when B comes to rest (w.r.t. belt):

For this: $v_B = u_B + a_2 t$

$$\Rightarrow 8 = 0 + 4t$$

$$\Rightarrow t = 2 \text{ s}$$

So B comes to rest earlier, and till that A continues to move with acceleration 1 ms^{-2} .

So we have to find separation between the blocks at $t = 2 \text{ s}$.
At $t = 2 \text{ s}$:

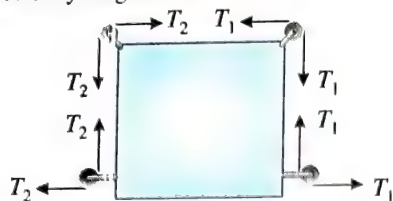
$$S_A = 4 \times 2 + \frac{1}{2} \times 1 \times (2)^2 = 10 \text{ m}$$

$$S_B = 0 \times 2 + \frac{1}{2} \times 4 \times (2)^2 = 8 \text{ m}$$

$$\text{Separation} = S_A - S_B = 10 - 8 = 2 \text{ m}$$

For Problems 36–38

36. (4) The free body diagram of cart is



Hence, net horizontal force on cart is zero.

∴ Acceleration of cart is zero.

37. (3) The acceleration of each block is equal and equal to $\frac{F}{3m}$.

∴ Tension in required string can be found by applying Newton's second law to block C.

$$T_2 = ma = \frac{F}{3}$$

38. (2) For block B to just slip on the cart, the friction force on cart is μmg . The net force on cart is thus μmg . Hence acceleration of cart is

$$a = \frac{\mu mg}{3m} = \frac{\mu g}{3}$$

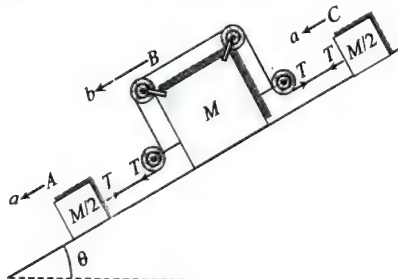
Hence, maximum force, $F_{\max} = (m + m + 3m + m)a$

$$\Rightarrow F_{\max} = 6m \times \frac{\mu g}{3} = 2\mu mg$$

For Problems 39–41

39. (3), 40. (1), 41. (4)

Here acceleration of A and C will be same, let it be a . Acceleration of B will be independent of A and C, let it be b .



$$\text{For B: } T + Mg \sin \theta - T - Mb \Rightarrow b = g \sin \theta$$

$$\text{For A: } \frac{M}{2} g \sin \theta - T = \frac{M}{2} a$$

$$\text{For C: } T + \frac{M}{2} g \sin \theta - \mu \frac{M}{2} g \cos \theta = \frac{M}{2} a$$

Solve to get

$$T = Mg \sin \theta / 8$$

For Problems 42–44

42. (3), 43. (2), 44. (1)

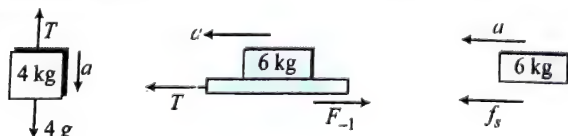
Limiting friction between belt and table

$$f_{l1} = \mu_1 \times 6g = 0.2 \times 6 \times g = 12 \text{ N}$$

Limiting friction between belt and block

$$f_{l2} = \mu_2 \times 6g = 0.2 \times 6 \times g = 12 \text{ N}$$

Let us first assume that the whole system moves together with acceleration a .



$$4g - T = 4a$$

$$\Rightarrow T - 12 = 6a$$

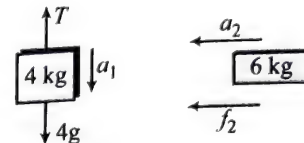
$$\text{from (i) and (ii): } a = 2.8 \text{ m/s}^2$$

$$\text{and } f_s = 6a = 6 \times 2.8 = 16.8 \text{ N}$$

$$\Rightarrow \text{(not possible)}$$

Hence, they will not move together and accelerations of both blocks will be different and 6 kg block will slide on belt.

Now let acceleration of 4 kg block is a_1 and that of 6 kg block is a_2 .



$$T_1 = f_{l1} + f_{l2} = 24 \text{ N}$$

$$4g - T_1 = 4a_1 \Rightarrow 40 - 24 = 4a_1 \Rightarrow a_1 = 4 \text{ m/s}^2$$

$$\Rightarrow 12 = 6a_2 \Rightarrow a_2 = 2 \text{ m/s}^2$$

Acceleration of belt is also a_1 .

Time taken by block to reach pulley:

$$s = ut + \frac{1}{2}at^2 \Rightarrow 2 = \frac{1}{2}2t^2 \Rightarrow t = \sqrt{2} \text{ s}$$

Displacement of belt in this time:

$$s_1 = ut + \frac{1}{2}at^2 = \frac{1}{2}4(\sqrt{2})^2 = 4 \text{ m}$$

Displacement of block w.r.t. belt in this time:

$$s - s_1 = 2 - 4 = -2 \text{ m or } 2 \text{ m towards right.}$$

For Problems 45–48

45. (3) Wedge will start slipping when

$$F \geq \mu(2mg)$$

46. (3) If wedge start slipping, common acceleration

$$a_C = \frac{F - 2\mu mg}{2m}$$

If $mg \sin \theta = ma \cos \theta$ then no force along the plane will be felt by the block and hence friction will be zero.

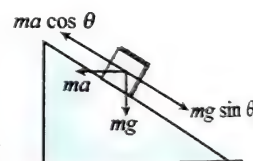
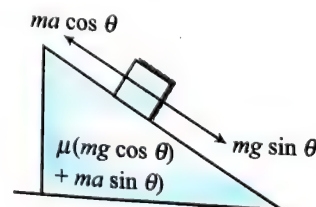
$$\Rightarrow mg \sin \theta = m \left(\frac{F - 2\mu mg}{2m} \right) \cos \theta$$

$$F = 2mg \tan \theta + 2\mu mg$$

47. (2) Block will start sliding if

$$mg \cos \theta \geq mg \sin \theta + \mu(mg \cos \theta + ma \sin \theta)$$

$$\text{get } a > g \left(\frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right)$$



1. i → a, c; ii → b, d; iii → a, b, c, d; iv → c, d

- (i) Force of friction is zero in (a) and (c) because block has no tendency to move.
 (ii) Force of friction is 2.5 N in (b) and (d) because applied force in horizontal direction in both is 2.5 N.
 (iii) Acceleration is zero in all cases.
 (iv) Normal force is not equal to 2g in (c) and (d) because some extra vertical force is also acting.

2. i → a, d; ii → a, d; iii → b, c, d; iv → b, c, d

Maximum possible acceleration of m: $a_0 = \mu g = 0.5g$

So, (d) matches with all (i), (ii), (iii) and (iv).

Let us assume that m and 2m move together with acceleration a:

$$a = \frac{m_1 g}{3m + m_1}$$

$$\text{If } a = a_0 \Rightarrow \frac{m_1 g}{3m + m_1} = 0.5g \Rightarrow m_1 = 3m$$

So 3m is the maximum value of m_1 such that both move together.

- (i) $m_1 = 2m < 3m$, hence (i) → (a, d)
 (ii) $m_1 = 3m$, hence (ii) → (a, d)
 (iii) $m_1 = 4m > 3m$, hence (iii) → (b, c, d)
 (iv) $m_1 = 6m > 3m$, hence (iv) → (b, c, d)

3. i → c; ii → c; iii → b, d; iv → a, d

- (i) If $m_1 = m_2 = 0$, then there is no force on M in horizontal direction. So M does not accelerate.



Hence, (i) – (c)

- (ii) If $m_1 = m_2 \neq 0$

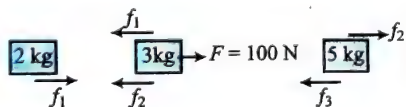
$$f_1 < \mu_1 mg, f_2 < \mu_2 mg$$

f_1 and f_2 will be of same magnitude at any time whether the blocks slip on larger block or not, so net force on M is zero.

Hence, M does not accelerate. So (ii) – (c)

- (iii) $m_1 > m_2$, here $f_1 > f_2$, hence (iii) – (b, d)
 (iv) $m_1 < m_2$, here $f_1 < f_2$, hence (iv) – (a, d)

4. i → b, d; ii → c; iii → a, d; iv → b, d



$$f_{11} = 0.2 \times 2g = 4 \text{ N}$$

$$f_{12} = 0.1 \times 5g = 5 \text{ N}$$

$$f_{13} = 0.1 \times 10g = 10 \text{ N}$$

Friction on 3-kg block is towards left and non-zero. Hence (i) → b, d
 $f_{12} < f_{13}$ hence 5 kg block will not move. So net friction on 5 kg will be zero.

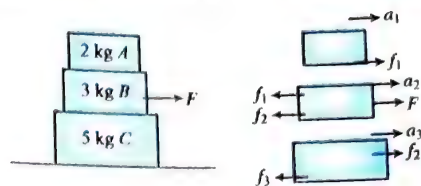
Hence, (ii) → (c)

5. i → c; ii → c; iii → b, c; iv → a, b, c

Let f_1, f_2, f_3 represent the friction forces between three contact surfaces A-B, B-C and C-ground, respectively. Limiting values of friction forces at three surfaces are 8 N, 15 N, and 10 N respectively.

For relative motion between C and ground, the minimum force needed is $F = 10 \text{ N}$.

For $F = 12 \text{ N}$, all the three blocks move together with same acceleration i.e., $a_1 = a_2 = a_3 = a$.



$$F - f_3 = (2 + 3 + 5)a \Rightarrow a = \frac{12 - 10}{10} = \frac{1}{10} \text{ ms}^{-2}$$

$$f_1 = 2a = \frac{2}{5} \text{ N} \Rightarrow f_2 = 12 - f_1 - 3a = 11 \text{ N}$$

$$f_3 = 10 \text{ N}$$

For $F = 15 \text{ N}$, the situation is similar.

For relative motion to start between B and C, $f_2 \geq f_{l2}$

$$F - f_3 = 10a \quad \text{and} \quad F - f_2 = 5a$$

$$f_2 = F - 5a = F - 5 \left[\frac{F - f_3}{10} \right] = \frac{F + f_3}{2}$$

$$\frac{F + 10}{2} > 15$$

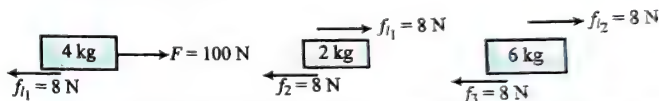
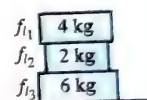
$$\Rightarrow F > 20 \text{ N} \text{ (Condition for relative motion to start)}$$

6. i → b, d; ii → c; iii → a, d; iv → c

$$f_{11} = 0.2 \times 4g = 8 \text{ N}$$

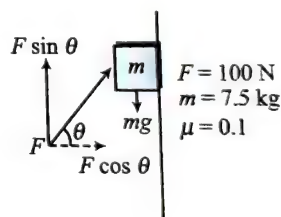
$$f_{12} = 0.4 \times 6g = 24 \text{ N}$$

$$f_{13} = 0.5 \times 12g = 60 \text{ N}$$



Here only 4 kg will accelerate, 2 kg and 6 kg will remain at rest.

7. (2)



- (i) $\theta = 37^\circ, F \sin \theta = 60 \text{ N}$

$$N = F \cos \theta = 80 \text{ N}$$

$$f_l = \mu N = 0.1 \times 80 = 8 \text{ N}$$

$$\text{Force required} = 75 - 60 = 15 \text{ N} > f_l$$

Hence, friction is kinetic in upward direction.

- (ii) $\theta = 45^\circ, N = 50\sqrt{2}$

$$f_l = \mu N = 0.1 \times 50\sqrt{2} = 5\sqrt{2} \text{ N}$$

$$F \sin \theta = 50\sqrt{2} \text{ N}$$

$$\text{Force required} = 75 - 50\sqrt{2} = 25(3 - \sqrt{2}) = 4.29 \text{ N} < f_l$$

$$\text{so, } f = 4.29 \uparrow \text{ (static)}$$

- (iii) $\theta = 53^\circ, F \sin \theta = 80 \text{ N}, N = F \cos \theta = 60 \text{ N}$

$$f_l = \mu N = 0.1 \times 60 = 6 \text{ N}$$

$$\text{Force required } 80 - 75 = 5 \text{ N} < f_l$$

$$\text{so, } f = 5 \downarrow \text{ (static)}$$

8. (4)

$$\text{Angle of repose: } \tan \phi = \mu \Rightarrow \phi = \tan^{-1}(0.75) = 37^\circ$$

- (i) $\theta = 30^\circ < \phi$, so the block will not move and friction force will be $f = mg \sin 30^\circ = 0.5mg$

(ii) $\theta = 30^\circ = \phi$, so the block is on the verge of slipping $= mg \sin \phi$
 $= mg (3/5) = 0.6mg$

(iii) $\theta = 60^\circ > \phi = 3$ $mg/8$

(iv) $\theta = 90^\circ \rightarrow N = 0$, so friction is zero

9. (1)

(i) Till $\theta = \tan^{-1} \mu$, $T = 0$

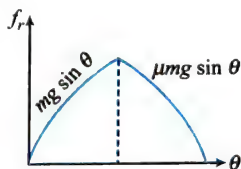


After $\theta = \tan^{-1} \mu$, $T = mg \sin \theta - \mu mg \cos \theta$

(ii) $N = mg \cos \theta$



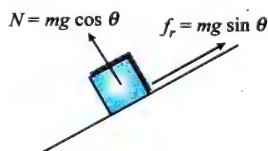
(iii) Till $\theta = \tan^{-1} \mu$



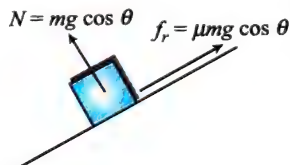
f_r will be static $= mg \sin \theta$, after $\theta = \tan^{-1} \mu$

f_r will be limiting $= \mu mg \cos \theta$

(iv) Net interaction force between the block and incline's
for $\theta \leq \tan^{-1} \mu$



$$\text{Net reaction} = \sqrt{(mg \cos \theta)^2 + (mg \sin \theta)^2} = mg$$



for $\theta > \tan^{-1} \mu$

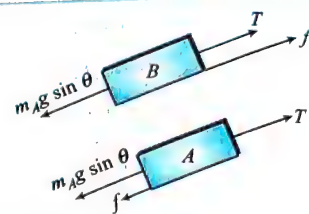
$$\begin{aligned} \text{Net reaction} &= \sqrt{(mg \cos \theta)^2 + (\mu mg \cos \theta)^2} \\ &= \sqrt{1 + \mu^2} mg \cos \theta \end{aligned}$$



10. (1) Limiting value of friction is 2 N. So, block does not move till $t = 2$ sec. At $t = 8$ sec, speed of block is found by energy method. Area under $F-t$ graph is equal to change in momentum. Hence, at $t = 8$ sec, $v = 18$ m/s. After that, a net force 1 N acts on the block opposite to motion. Suppose particle takes further t sec to come to rest. Solving, $t = 16$ i.e., particle comes to rest at 24 sec. Hence, the answers follow.

11. (3)

(i) When $m_B > m_A$, assume f to be the frictional force required to keep the blocks stationary.



$$m_B g \sin \theta = T + f$$

$$m_A g \sin \theta = T - f$$

$$f = \frac{(m_B - m_A) g \sin \theta}{2}$$

...(i)

If $f > f_{\max}$, A and B will move together as $\mu = 0$ between A and surface.

So, for A and B to be stationary

$$m_B > m_A \text{ and } f \leq f_{\max}$$

$$\leq \mu m_B g \cos \theta$$

$$\frac{(m_B - m_A) g \sin \theta}{2}$$

$$\leq \mu m_B g \cos \theta$$

$$\mu \geq \frac{(m_B - m_A) \tan \theta}{2 m_B}$$

(ii) From equation (i), $f = 0$ if $m_A = m_B$

If $m_B < m_A$, f is negative, i.e., its direction will change, but $f \neq 0$

(iii) Block B will move upwards if $m_B < m_A$ and then direction of f will change.

$$m_B \sin \theta = T - f$$

$$m_B \sin \theta = T + f$$

$$f = \frac{(m_A - m_B) g \sin \theta}{2}$$

Blocks will move if $f \geq f_{\max}$

$$\frac{(m_A - m_B) g \sin \theta}{2} \geq \mu m g \cos \theta$$

Numerical Value Type

1. (5) Denote the common magnitude of the maximum acceleration as a . For block A to remain at rest with respect to block B, $a \leq \mu_s g$. Let us assume $a = \mu_s g$ for mass of C to be largest. The tension in the cord is then

$$T = (m_A + m_B)a + \mu_k g(m_A + m_B)$$

$$= (m_A + m_B)(a + \mu_k g)$$

This tension is related to the mass m_C (largest) by

$$T = m_C(g - a).$$

Solving for m_C yields

$$m_C = \frac{(m_A + m_B)(\mu_s + \mu_k)}{1 - \mu_s}$$

$$= \frac{(1.5 + 0.5)(0.6 + 0.4)}{1 - 0.6} = 5 \text{ kg}$$

2. (8) Since $mg \sin 37^\circ > \mu mg \cos 37^\circ$, the block has a tendency to slip downwards.

Let F be the minimum force applied on it, so that it does not slip. Then,

$$N = F + mg \cos 37^\circ$$

$$\therefore mg \sin 37^\circ = \mu N = \mu(F + mg \cos 37^\circ)$$

$$\text{or } F = \frac{mg \sin 37^\circ}{\mu} - mg \cos 37^\circ$$

$$= \frac{(2)(10)(3/5)}{0.5} - (2)(10)\left(\frac{4}{5}\right) = 8 \text{ N}$$

3. (6) Force of friction between the two will be maximum i.e., μmg .
Retardation of A is $a_A = \frac{\mu mg}{m} = \mu g$

and acceleration of B is $a_B = \frac{\mu mg}{2m} = \frac{\mu g}{2}$

Acceleration of B relative to A is $a_{BA} = a_A + a_B = \frac{3\mu g}{2}$

Substituting, $\mu = \frac{1}{2}$; $a_{BA} = \frac{3g}{4}$

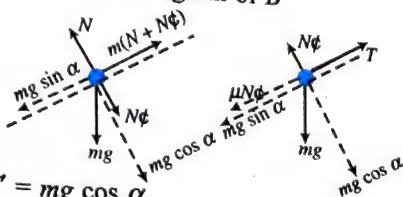
4. (4) Since A tends to slip down, frictional forces act on it from both sides up the plane.

Let N be the reaction of the plank on A and N' be the mutual normal action-reaction between A and B.

From the free-body diagram of A,

$$N' + mg \cos \alpha = N \quad \text{and} \quad mg \sin \alpha = \mu(N + N')$$

From the free-body diagram of B



$$N' = mg \cos \alpha$$

$$mg \sin \alpha + \mu N' = T$$

$$\therefore 2mg \cos \alpha = N$$

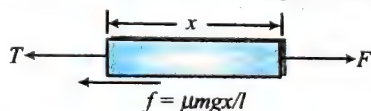
$$\text{and } mg \sin \alpha = \mu(2mg \cos \alpha + mg \cos \alpha)$$

$$\text{or } \mu = \frac{1}{3} \tan \alpha = \frac{1}{3} \times \frac{3}{4} = 0.25 \text{ or } \frac{1}{m} = 4$$

5. (2) $f_l = \mu mg = 0.2 \times 10 \times 10 = 20 \text{ N}$

$F < f_l$ so the chain will not move.

Consider a length x of the chain. If there is some tension T then limiting friction will be acting on this part of chain.



$$F = T + \mu mg x / l \Rightarrow 15 = T + 20x/8$$

Let $T = 0$, then $x = 6 \text{ m}$. Now for $T = 0$, the remaining part of length $l - x = 8 - 6 = 2 \text{ m}$ will not tend to move, hence, friction force on it will be zero.

6. (5) Limiting friction force

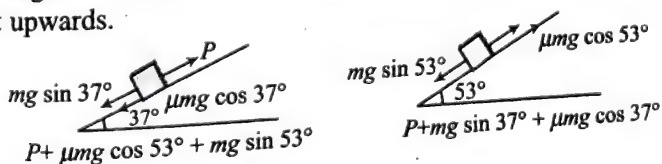
$$f = \mu mg = 0.5 \times 10 \times 10 = 50 \text{ N}$$

Therefore body starts moving at $t = 2 \text{ sec}$

$$Mv - 0 = \text{Area under } F-t \text{ curve } (t = 2 \text{ to } t = 4)$$

$$\Rightarrow Mv = 50 \Rightarrow v = 5 \text{ m/s}$$

7. (7) At angle 37° , friction will act downwards. At 53° , friction will act upwards.

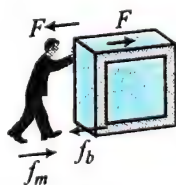


$$\text{Solve to get } \mu = \frac{1}{7} \Rightarrow \frac{1}{\mu} = 7$$

8. (3) From FBD of box, the box will move if

$$F \geq f_b$$

$$\Rightarrow F \geq 0.2 mg$$



If man is moving the box, then there is no slipping of shoes of man on floor. So friction on man f_m will be static.

From FBD of man

$$f_m = F$$

$$\text{but } f_m \leq \mu_s 75g$$

$$\Rightarrow F \leq \mu_s 75g$$

From (i) and (ii)

$$0.2mg \leq F \leq \mu_s 75g \Rightarrow 0.2mg \leq \mu_s 75g$$

$$\Rightarrow m \leq \frac{0.8}{0.2} 75 \Rightarrow m \leq 300 \text{ kg}$$

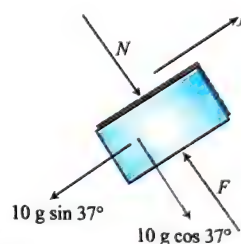
9. (2) Along the belt: $f - mg \sin 37^\circ = ma$

maximum frictional force may be $f = \mu mg \cos 37^\circ$

$$a_{\max} = g(\mu \cos 37^\circ - \sin 37^\circ) = 2 \text{ m/s}^2$$

$$4 = 1/2 \times 2 \times t^2, t = 2 \text{ s}$$

10. (2)



$$f_t = \mu N = 0.5 [F - 10g \cos 37^\circ]$$

$$f = 10g \sin 37^\circ$$

For the block not to slide, $f_t > f$

$$\Rightarrow 0.5 [F - 10g \cos 37^\circ] > 10g \sin 37^\circ$$

$$\Rightarrow F > 200 \text{ N}$$

11. (1) The block begins to slide if

$$F \cos 37^\circ = \mu (mg - F \sin 37^\circ)$$

$$5t [\cos 37^\circ + \mu \sin 37^\circ] = \mu mg$$

$$5t \left[\frac{4}{5} + \frac{3}{5} \right] = 70 \text{ or } t = 10 \text{ second}$$

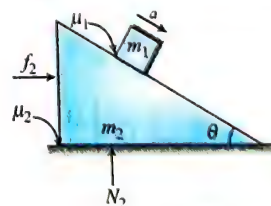
12. (8) Taking block + wedge as system and applying NLM in horizontal direction

$$f_2 = m_1 a \cos \theta$$

$$m_1 [g(\sin \theta - \mu_1 \cos \theta)] \cos \theta$$

...(1)

Again applying NLM in vertical direction



$$(m_1 + m_2)g - N_2 = m_1 a \sin \theta$$

$$N_2 = (m_1 + m_2)g - m_1 \sin \theta (g \sin \theta - \mu_1 g \cos \theta)$$

$$\text{For limiting condition, } f_2 = \mu_2 N_2$$

...(2)

From (1) and (2)

$$\mu_2 = \frac{m_1 \cos \theta (g \sin \theta - \mu_1 g \cos \theta)}{(m_2 + m_2)g - m_1 \sin \theta (g \sin \theta - \mu_1 g \cos \theta)}$$

$$\text{Using values } \mu_2 = \frac{1}{8}$$

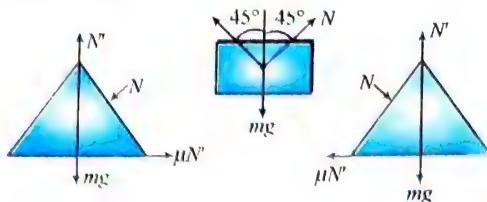
13. (1) The free-body diagrams of all bodies are as shown.

From FBD of block

$$2N \cos 45^\circ = Mg$$

...(i)

For wedge to remain at rest



$$N \sin 45^\circ < \mu N' \quad \dots(ii)$$

$$\text{and } N' = mg + N \cos 45^\circ \quad \dots(iii)$$

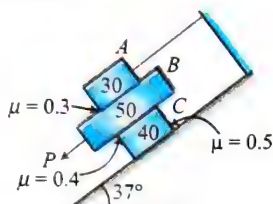
From (i), (ii) and (iii) we get

$$M \leq \frac{2\mu m}{1-\mu} \Rightarrow \frac{M}{m} \leq \frac{2\mu}{1-\mu} \quad \dots(iv)$$

Substituting $\mu = \frac{1}{3}$ in equation (iv) we get

$$\frac{M}{m} \leq \frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{3}} \Rightarrow \frac{M}{m} \leq 1$$

$$14. (3) f_{AB \max} = 0.3 \times 30 g \cos 37^\circ$$



$$f_{AB \max} = 72 \text{ N}$$

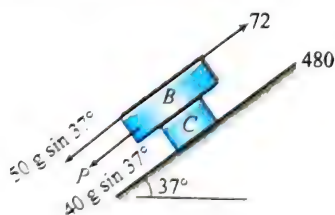
$$f_{BC \max} = 0.4 \times 80 g \cos 37^\circ = 256 \text{ N}$$

$$f_{C \max} = 0.5 \times 120 g \cos 37^\circ = 480 \text{ N}$$

when B block is pulled two cases are possible

- (1) A and C remain stationary, only B tends to move downwards.
- (2) B and C both move together and there is just slipping and A and B contact and C and wedge contact

Taking case (2), let B and C move together and there is just slipping between A and B and between wedge and C

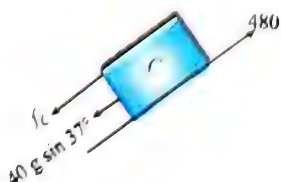


$$\Rightarrow P + 40 g \sin 37^\circ + 50 g \sin 37^\circ - 72 - 480 = 0$$

$$\Rightarrow P = 12 \text{ N}$$

Checking friction between B and C

$$f_{BC} + 40 g \sin 37^\circ - 480 = 0$$

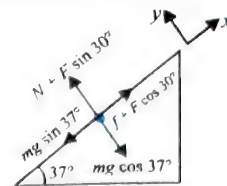


$f_{BC} = 240 \text{ N}$ which is within limit so case is correct.

So maximum force for which there will be no slipping $P_{\max} = 12 \text{ N}$ at this force both B and C tends to move together. Hence, the value of $x = 3$.

15. (30.2)

Drawing free body diagram of block



$$\Rightarrow N + F \sin 30^\circ = mg \cos 37^\circ$$

$$\text{or, } N = mg \cos 37^\circ - F \sin 30^\circ$$

$$= (4)(10)\left(\frac{4}{5}\right) - (10)\left(\frac{1}{2}\right) = 27 \text{ N} \quad \dots(i)$$

$$f_{\max} = \mu N = 0.5 \times 27 = 13.5 \text{ N}$$

$$mg \sin 37^\circ = (4)(10)\left(\frac{3}{5}\right) = 24 \text{ N}$$

$$\text{and } F \cos 30^\circ = (10)\left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3} \text{ N}$$

Now since $mg \sin 37^\circ > f_{\max} + F \cos 30^\circ$

Therefore block will slide down and friction will be maximum. Therefore, net contact force is

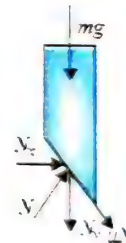
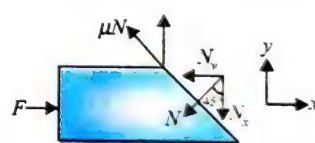
$$F_C = \sqrt{N^2 + (f_{\max})^2} \\ = \sqrt{(27)^2 + (13.5)^2} = 13.5\sqrt{5} \text{ N} = 30.2 \text{ N}$$

16. (14)

For equilibrium of block B

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

$$mg + \frac{\mu N}{\sqrt{2}} = \frac{N}{\sqrt{2}} \Rightarrow N = \frac{\sqrt{2} mg}{1-\mu}$$



For equilibrium of block A

$$\sum F_x = F - \frac{N}{\sqrt{2}} - \frac{\mu N}{\sqrt{2}} = 0$$

$$F = \frac{N}{\sqrt{2}} [1 + \mu] = mg \frac{(1 + \mu)}{(1 - \mu)}$$

$$= 0.6 \times 10 \frac{(1 + 0.4)}{(1 - 0.4)} = \frac{6 \times 1.4}{0.6} = 14 \text{ N}$$

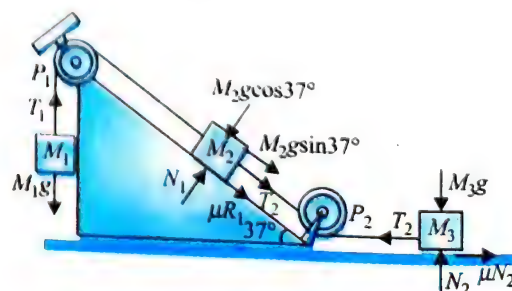
17. (10.5)

Consider mass M_1 . Since it moves down with uniform speed,

$$\therefore T_1 = M_1 g \quad \dots(i)$$

$$\text{For mass } M_2, T_1 + T_2 + \mu M_2 g \cos 37^\circ + M_2 g \sin 37^\circ \quad \dots(ii)$$

$$\text{For mass } M_3, T_2 = \mu M_3 g \quad \dots(iii)$$



Substituting for T_1 and T_2 from (i) and (iii), respectively, in (ii), we get

$$M_1 g = \mu M_3 g + \mu M_2 g \cos 37^\circ + M_2 g \sin 37^\circ$$

$$M_1 = \mu M_3 + \mu M_2 \cos 37^\circ + M_2 \sin 37^\circ$$

$$\therefore M_1 = \frac{1}{4} \times 4 + \frac{1}{4} \times 4 \times \frac{4}{5} + 4 \times \frac{3}{5} = 4.2 \text{ kg}$$

$$T_2 = \mu M_3 g = \frac{1}{4} \times 4.2 \times 10 = 10.5 \text{ N}$$

(162)

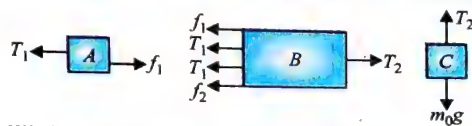
Maximum friction that can be obtained from A and B is

$$f_1 = \mu m_1 g = (0.3)(100)(10) = 300 \text{ N}$$

And maximum friction between B and ground is

$$f_2 = \mu(m_A + m_B)g = (0.3)(100 + 140)(10) = 720 \text{ N}$$

Drawing free body diagram of A, B and C in limiting case



Equilibrium of A gives $T_1 = f_1 = 300 \text{ N}$

...(i)

Equilibrium of B gives $2T_1 + f_1 + f_2 = 0$

$$\text{or } T_2 = 2(300) + 300 + 720 = 1620 \text{ N}$$

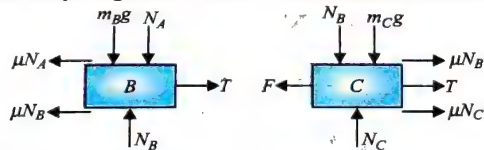
...(ii)

and equilibrium of C given $m_C g = T_2$ or $10m_C = 1620$

$$\text{or } m_C = 162 \text{ kg}$$

(80)

The free body diagram of B and C are separately shown in figures.

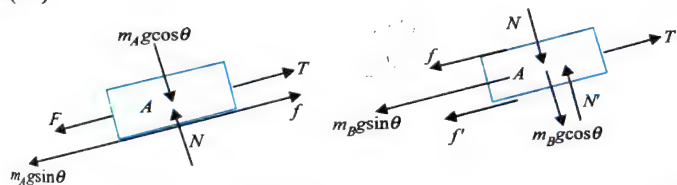


$$T = f_1 + f_2 = 3\mu g + 7\mu g = 10\mu g$$

$$\text{Now, } F = f_2 + f_3 + T = 7\mu g + 15\mu g + 10\mu g$$

$$= 0.25 \times 32 \times 10 = 80 \text{ N}$$

(5.2)



$$m_A g \sin \theta + F - T - f = m_A a$$

...(i)

$$f = \mu_k m_A g \cos \theta$$

...(ii)

$$T - m_B g \sin \theta - f - f' = m_B a$$

...(iii)

$$f' = \mu_k N'$$

...(iv)

$$= \mu_k (m_A g \cos \theta + m_B g \cos \theta)$$

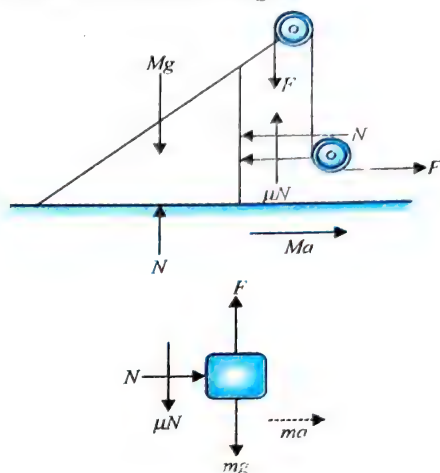
Solving above equations we get $a_A = 5.2 \text{ m/s}^2$

(10)

Since thread is pulled to the right by force F , therefore tension in it is also equal to F . This tension acts on the block vertically upward. Hence, it has tendency to slip up along the vertical surface of the wedge. Hence the friction on the block will act vertically downward. Since the only horizontal force acting on the system on the wedge and block is F , therefore, the system will accelerate horizontally rightward and block remains stationary relative to wedge. Therefore, the acceleration of block and wedge is identical.

$$\text{This acceleration, } a = \frac{F}{m + M}$$

Maximum value of force F will correspond to limiting friction. Hence, FBD will be as shown in figure.



For horizontal forces on the block, $N = ma$

$$N = \frac{mF}{m + M} \text{ or } N = \frac{F}{5}$$

For vertical equilibrium of the block,

$$F = \mu N + mg \text{ or } F = 10 \text{ N}$$

22. (65)

For equilibrium in vertical direction.

$$2f \sin \theta - 2N \cos \theta - mg = 0 \quad \dots(1)$$

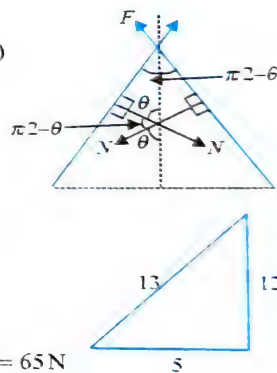
$$2\mu N \sin \theta - 2N \cos \theta = mg$$

$$\tan \theta = \left(\frac{\pi}{2} - \theta \right) = \frac{5}{12}$$

$$\cot \theta = \frac{5}{12}, \sin \theta = \frac{12}{13} \text{ \& } \cos \theta = \frac{5}{13}$$

$$2(0.5) \left(\frac{12}{13} \right) N - 2N \cdot \frac{5}{13} = (1)(10)$$

$$N \left(\frac{12}{13} - \frac{10}{13} \right) = (10) \Rightarrow N = \frac{10 \times 13}{2} = 65 \text{ N}$$



Archives

JEE Main

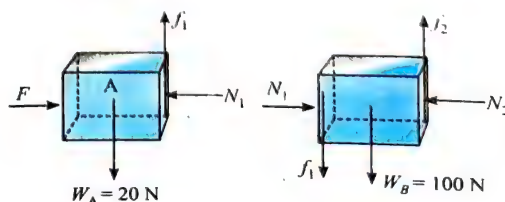
Single Correct Answer Type

1. (3) F.B.D of block A and block B

For block A:

If block A is at equilibrium, then

$$f_1 = W_A = 20 \text{ N}$$



If friction is static, $f_1 \leq \mu_1 N_1$

$$\text{or } 20 \leq 0.1 \times F \Rightarrow F \geq \frac{20}{0.1} = 200 \text{ N}$$

Hence, limiting value of friction between the blocks,

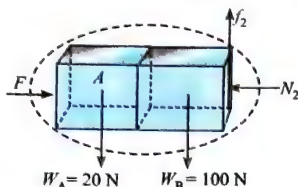
$$f_1 = 0.1 \times 200 = 20 \text{ N}$$

From F.B.D of block B:

$$f_2 = f_1 + W_B = 20 + 100 = 120 \text{ N}$$

Alternative method:

If we take both blocks together as system, then



For equilibrium of the system in vertical direction,

$$f_2 = W_A + W_B = 20 + 100 = 120 \text{ N}$$

2. (3) From F.B.D of the block ' m_1 '

$$T = m_1 g = 5 \times 10 = 50 \text{ N}$$

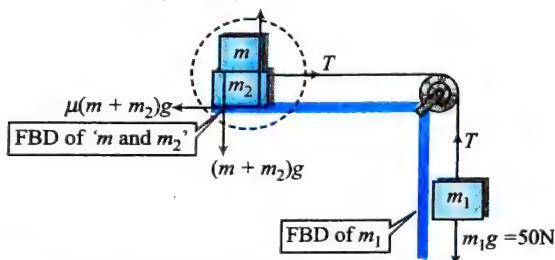
...(i)

From F.B.D. of the block ' $m + m_2$ '

$$T = \mu(m + m_1)g$$

$$= 0.15(m + 10) \times 10$$

...(ii)



From (i) and (ii) $50 = 0.15(m + 10)10$

$$5 = \frac{3}{10}(m + 10) \Rightarrow m + 10 = \frac{100}{3} \Rightarrow m = 23.3 \text{ kg}$$

JEE Advanced

Single Correct Answer Type

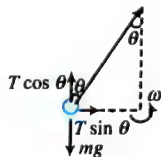
1. (1) Initially the frictional force is upwards as $P < mg \sin \theta$. As P increases frictional force decreases and at value of $P = mg \sin \theta$ friction will be zero. Afterwards friction will change the direction and start increasing.

2. (4)

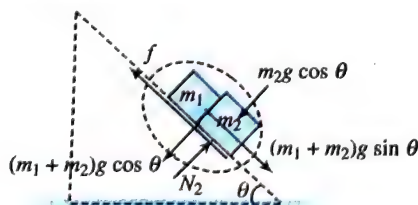
$$T \sin \theta = mL \sin \theta \omega^2$$

$$324 = 0.5 \times 0.5 \times \omega^2$$

$$\omega^2 = \frac{324}{0.5 \times 0.5} = 36 \text{ rad s}^{-1}$$



3. (4) Condition for not sliding,



$$f_{\max} > (m_1 + m_2) g \sin \theta$$

$$\mu N > (m_1 + m_2) g \sin \theta$$

$$0.3 m_2 g \cos \theta \geq (1 + 2)10 \sin \theta$$

$$0.3 \times 2 \times 10 \cos \theta \geq (1 + 2)10 \sin \theta$$

$$6 \geq 30 \tan \theta$$

$$1/5 \geq \tan \theta \Rightarrow 0.2 \geq \tan \theta$$

Now it is clear that in cases (P) and (Q), the blocks will not slip. Friction is $(m_1 + m_2)g \sin \theta$

$$\Rightarrow f = (m_1 + m_2) g \sin \theta$$

In the cases (R) and (S), they must slip.

So friction should be kinetic,

$$\text{i.e. } \mu m_2 g \cos \theta$$

Multiple Correct Answers Type

1. (1, 3)

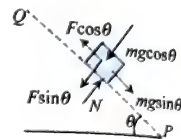
$$f = 0, \text{ If } mg \sin \theta = F \cos \theta$$

$$\Rightarrow \theta = 45^\circ$$

$$f \text{ towards } Q, mg \sin \theta > F \cos \theta$$

$$\Rightarrow \theta > 45^\circ$$

$$f \text{ towards } P, mg \sin \theta < F \cos \theta$$



Linked Comprehension Type

1. (4) $mr\omega^2 = ma$

$$a = r\omega^2$$

$$\frac{v dv}{dr} = r\omega^2$$

$$\int_0^v v dv = \omega^2 \int_{R/2}^r r dr$$

$$v = \omega \sqrt{r^2 - \frac{R^2}{4}}$$

$$\int_{R/2}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

...(1)

$$\text{Assume } r = \frac{R}{2} \sec \theta$$

$$dr = \frac{R}{2} \sec \theta \tan \theta d\theta$$

$$\int \frac{\frac{R}{2} \sec \theta \tan \theta d\theta}{\sqrt{\frac{R^2}{4} \tan^2 \theta}} = \int_0^t \omega dt$$

$$\omega t = \ln \left[\frac{2r}{R} + \frac{\sqrt{4r^2 - R^2}}{R} \right]$$

$$r = \frac{R}{4} [e^{\omega t} + e^{-\omega t}]$$

2. (3) $\vec{F}_{rot} = \vec{F}_{in} + 2m(v_{rot} \hat{i}) \times \omega \hat{k} + m(\omega \hat{k} \times r \hat{i}) \times \omega \hat{k}$

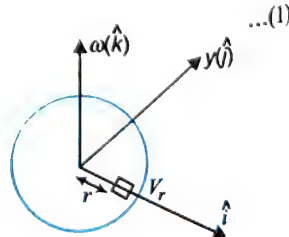
$$m r \omega^2 \hat{i} = \vec{F}_{in} + 2m v_{rot} \omega (-\hat{j}) + m \omega^2 r \hat{i}$$

$$\vec{F}_{in} = 2m v_{rot} \omega \hat{j}$$

$$r = \frac{R}{4} [e^{\omega t} + e^{-\omega t}]$$

$$\frac{dr}{dt} = v_r = \frac{R}{4} [\omega e^{\omega t} - \omega e^{-\omega t}]$$

$$\vec{F}_{in} = 2m \frac{R \omega}{4} [e^{\omega t} - e^{-\omega t}] \omega \hat{j}$$



$$\vec{F}_{in} = \frac{mR\omega^2}{2} [e^{\omega t} - e^{-\omega t}] \hat{j}$$

Also reaction is due to disc surface then

$$\vec{F}_{reaction} = \frac{mR\omega^2}{2} [e^{\omega t} - e^{-\omega t}] \hat{j} + mg \hat{k}$$

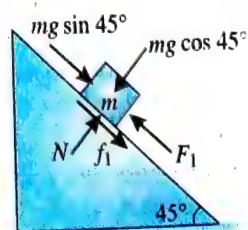
Numerical Value Type

1. (5)

Case I: For pushing the block up

$$F_1 = mg \sin 45^\circ + \mu mg \cos 45^\circ$$

$$\Rightarrow F_1 = \frac{mg}{\sqrt{2}} + \frac{\mu mg}{\sqrt{2}}$$



Case II: Force required to just prevent the block from sliding down

$$F_2 = mg \sin 45^\circ - \mu mg \cos 45^\circ$$

$$\Rightarrow F_2 = \frac{mg}{\sqrt{2}} - \frac{\mu mg}{\sqrt{2}}$$

$$F_1 = 3F_2$$

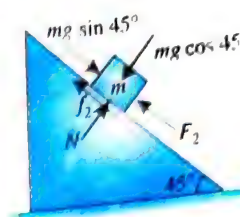
$$\Rightarrow 1 + \mu = 3 - 3\mu$$

$$4\mu = 2$$

$$\mu = \frac{1}{2}$$

$$\Rightarrow N = 10\mu$$

$$\Rightarrow N = 5$$



Concept Application Exercises

Exercise 8.1

1. Since the body is displaced 4 m along the z-axis only,

$$S = 0\hat{i} + 0\hat{j} + 4\hat{k}$$

Also, $F = -i + 2\hat{j} + 3\hat{k}$

Work done,

$$W = \vec{F} \cdot \vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= 12(\hat{k} \cdot \hat{k}) = 12 \text{ J}$$

2. Here force is variable so

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} (3x^2\hat{i} + 2y\hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{\vec{r}_i}^{\vec{r}_f} (3x^2 dx + 2y dy) = \left[x^3 + y^2 \right]_{(2,3)}^{(4,6)} = 83 \text{ J}$$

3. Given

$$\vec{F} = (3x \text{ N})\hat{i} + (4 \text{ N})\hat{j}$$

$$W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy = \int_2^3 3x dx + \int_3^0 4 dy$$

$$= 3 \left[\frac{x^2}{2} \right]_2^3 + 4[y]_3^0 = -4.5 \text{ J}$$

As work done is negative, K.E. of the particle will decrease and hence speed will decrease.

4. $W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy$

$$= \int_0^1 y dx + \int_0^1 x dy = \int_{(0,0)}^{(1,1)} d(xy) = [xy]_{(0,0)}^{(1,1)} = 1 \text{ J}$$

5. $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$

$$= \int_{(2m, 3m, 4m)}^{(1m, 2m, 3m)} (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= [2x + 3y + 4z]_{(2m, 3m, 4m)}^{(1m, 2m, 3m)} = -9 \text{ J}$$

6. Work done = Area under F - s graph

$$W_{AB} = W_{AP} + W_{PQ} + W_{QR} + W_{RB}$$

$$= 10 \times 1 + \frac{1}{2}(10 + 15) \times 1 + \frac{1}{2} \times 1 \times 15 - \frac{1}{2} \times 1 \times 15$$

$$= 22.5 \text{ J}$$

7. The area under the curve with respect to x-axis gives the total work done by the force as the block moves from $x = 0$ to $x = 8 \text{ m}$.

Area = Work done

$$= 20 + \frac{1}{2}(2)(10) + 0 - \frac{1}{2}(5)(2) = 25 \text{ J}$$

8. (a) The work done on the cycle by the road is the work done by the frictional force exerted by the road on the cycle.

Now, $W = \vec{F} \cdot \vec{S} = F \cdot S \cos 180^\circ$

$$= -FS$$

$$= -200 \text{ N} \times 10 \text{ m}$$

$$= -2000 \text{ J}$$

It is this negative work which brings the cycle to rest. This is clearly in accordance with work-energy theorem.

- (b) The displacement of the road is zero. So, work done by the cycle on the road is zero.

(According to Newton's third law of motion, an equal and opposite force acts on the road due to the cycle. The magnitude of this force is 200 N).

9. As the force is acting in the direction opposite to the motion of the box, work done by this force must be negative. As force is not constant, we use

$$W = - \int_0^s f dx = - \int_0^s (ax + b) \cdot dx = \left[\frac{ax^2}{2} + bx \right]_0^s = -\frac{1}{2}as^2 - bs$$

10. $W = -\frac{k}{2}(x_2^2 - x_1^2) = -\frac{k}{2}(x_2 + x_1)(x_2 - x_1)$

$$= -\left(\frac{kx_2 + kx_1}{2} \right)(x) = -\left(\frac{F_1 + F_2}{2} \right)x = -F_{av}x$$

$$= -10 \times \frac{5}{100} = -0.5 \text{ J}$$

11. $W_1 = \frac{1}{2}k(x_2^2 - x_1^2)$ (where $x_2 = 15$ and $x_1 = 10 \text{ cm}$)

$$= \frac{1}{2}k[(15)^2 - (10)^2] = \frac{125}{2}k \quad \dots(i)$$

Similarly, $W_2 = \frac{1}{2}k[(20)^2 - (15)^2] = \frac{175}{2}k \quad \dots(ii)$

From Eqs. (i) and (ii), $\frac{W_2}{W_1} = \frac{(175/2)k}{(125/2)k} = \frac{7}{5}$

12. (a) $W_{f_1} = f_s s$

where $f_s = ma$

and $s = \frac{1}{2}at^2$

Using the above equations,

$$W_{f_1} = \frac{1}{2}ma^2t^2$$

(b) $W_{f_2} = -f_s s = -(ma) \left(\frac{1}{2}at^2 \right) = -\frac{1}{2}ma^2t^2$

(c) $W_f = W_{f_1} + W_{f_2} = (f_s s) + (-f_s s) = 0$

13. When $a > \mu_s g$, the block will slide relative to the plank. Hence, kinetic friction comes into play.



The work done by f_k is

$$W_{f_1} = f_k s_1 \quad \dots(i)$$

$$s_1 = \frac{1}{2}a_1 t^2 \quad \dots(ii)$$

$$f_k = \mu_k mg \quad \dots(iii)$$

$$a_1 = \mu_k g \quad \dots(iv)$$

Then, $W_f = \frac{m}{2} \mu_k^2 g^2 t^2$

14. The work done by the force F is

$$W = \int \vec{F}_{\min} \cdot d\vec{s} = F_{\min} x \cos \theta \quad \dots(i)$$

For slowly pulling the block,

$$F \cos \theta = \mu N$$

In vertical direction,

$$F \sin \theta + N - mg = 0$$

$$N = F \sin \theta - mg$$

Using Eqs. (ii) and (iii),

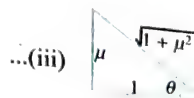
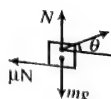
$$F = \frac{\mu mg}{\sin \theta + \mu \cos \theta}$$

F is minimum when $\tan \theta = \mu$

$$F_{\min} = \frac{\mu mg}{\sqrt{1 + \mu^2}} \quad \dots(iv)$$

Using Eqs. (i) and (iv),

$$W = \frac{\mu mgx}{\sqrt{1 + \mu^2}} \cdot \frac{1}{\sqrt{1 + \mu^2}} = \frac{\mu mgx}{1 + \mu^2}$$



Exercise 8.2

1. The ball starts falling from position 1, where its speed is zero. Hence, kinetic energy is also zero.

$$K_{\text{initial}} = 0 \text{ J} \quad \dots(i)$$

During downwards motion of the ball, constant gravitational force mg acts downwards and air resistance R of unknown magnitude acts upwards as shown in the free body diagram. The ball reaches at position 20 m below position 1 with a speed $v = 10 \text{ m/s}$. So the kinetic energy of the ball at position 2 is

$$K_{\text{final}} = \frac{1}{2} mv^2 = 250 \text{ J} \quad \dots(ii)$$

The work done by gravity: $W_{g,1 \rightarrow 2} = mgh = 1000 \text{ J} \quad \dots(iii)$

Denoting the work done by the air resistance $W_{R,1 \rightarrow 2}$ and making use of eq. (i), (ii), and (iii) in work-kinetic energy theorem, we have

$$(W_{1 \rightarrow 2})_{\text{external}} = K_{\text{final}} - K_{\text{initial}} \rightarrow W_{g,1 \rightarrow 2} + W_{R,1 \rightarrow 2}$$

$$\Rightarrow W_{\text{gravity},1 \rightarrow 2} + W_{\text{Resistance},1 \rightarrow 2} = K_{\text{final}} - K_{\text{initial}}$$

$$\Rightarrow 1000 + W_{\text{Resistance},1 \rightarrow 2} = 250 - 0$$

$$\text{or } W_{\text{Resistance},1 \rightarrow 2} = -750 \text{ J}$$

2. According to work-kinetic energy theorem, the net work done on the system equals the change in kinetic energy of the system, as expressed by equation: $W_{\text{external}} = \Delta K$.

$$W_{\text{external}} = \Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - 0 \Rightarrow v_f = \sqrt{\frac{2W_{\text{external}}}{m}}$$

But W_{external} = area under force-displacement curve. It means

$$v_f = \sqrt{\frac{2(\text{area})}{m}}$$

- (a) Work done for the region $0 \leq x \leq 5 \text{ m}$ = Area under the graph

$$(W_{\text{external}})_{0 \rightarrow 5} = \frac{(3.0 \text{ N})(5.0 \text{ m})}{2} = 7.5 \text{ J}$$

$$\text{Hence, } v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.5 \text{ J})}{4 \text{ kg}}} = 1.94 \text{ m/s}$$

- (b) The speed at $x = 10.0 \text{ m}$,

Work done for the region $5 \text{ m} \leq x \leq 10 \text{ m}$ = Area under the graph;

$$(W_{\text{external}})_{5 \rightarrow 10} = (3.0 \text{ N})(5.0 \text{ m}) = 15 \text{ J}$$

Total work done for the region, $0 \leq x \leq 10 \text{ m}$,

$$(W_{\text{external}})_{0 \rightarrow 10} = 7.5 + 15 = 22.5 \text{ J}$$

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5 \text{ J})}{4.0 \text{ kg}}} = 3.35 \text{ m/s}$$

- (c) The speed at $x = 15.0 \text{ m}$,

Work done for the region $10 \text{ m} \leq x \leq 15 \text{ m}$ = Area under the graph;

$$(W_{\text{external}})_{10 \rightarrow 15} = \frac{(3.0 \text{ N})(5.0 \text{ m})}{2} = 7.5 \text{ J}$$

Total work done for the region $0 \leq x \leq 15.0$,

$$W = (7.50 + 7.50 + 15.0) \text{ J} = 30.0 \text{ J}$$

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = 3.87 \text{ m/s}$$

3. **W-E theorem:** Applying work - energy theorem for wedge bead system,

$$W_{\text{ext}} + W_{\text{gravity}} = \Delta K$$

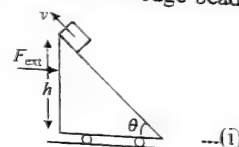
$$\text{We have } W_{\text{ext}} - mgh = \Delta K$$

$$\text{This gives } W_{\text{ext}} = mgh + \Delta K$$

ΔK : The KE of the bead changes by

$$\Delta K = \frac{1}{2} mv^2 \quad \dots(ii)$$

Using Eqs. (i) and (ii), we have $W_{\text{ext}} = \frac{1}{2} mv^2 + mgh$



4. Let the particle move up through a height y between points 1 and 2. The work done by gravity in shifting the particle from point 1 to 2 is

$$W_{\text{gr}} = -mgy$$

The change in kinetic energy is

$$\Delta K = \frac{1}{2} m(v^2 - v_0^2)$$

Substituting $W = W_{\text{gr}} = -mgy$ and $\Delta K = \frac{1}{2} m(v^2 - v_0^2)$ in W-E theorem, $W = \Delta K$, we have

$$\frac{m}{2} (v^2 - v_0^2) = -mgy$$

This gives $v = \sqrt{v_0^2 - 2gy}$



5. The forces acting on the block are N , gravity and friction.

$$W_N + W_f + W_{\text{gr}} = \Delta K$$

where $W_N = 0$, $W_f = -\mu mgl$

and $W_{\text{gr}} = mg(h - h')$ and $\Delta K = 0$

Thus, $-\mu mgl + mg(h - h') = 0$

$$h' = h - \mu l = 5 - \frac{1}{2} \times 8 = 1 \text{ m}$$

6. As the block moves, spring force $F_s (= kx)$ increases till it nullifies $f_k (= \mu mg)$. In consequence, the block will stop.

ΔK : When the block will stop,

$$\Delta K = -\frac{1}{2}mv_0^2$$

W_{sp} : When the block stops finally, let us assume that the spring is deformed by an amount x .

Then the work done by the spring is

$$W_{sp} = -\frac{kx^2}{2}$$

The work done by friction W_f is given as $W_f + W_{sp} = \Delta K$.

Substituting $\Delta K = -\frac{1}{2}mv_0^2$ and $W_{sp} = -\frac{kx^2}{2}$, we have

$$W_f = \frac{kx^2}{2} - \frac{1}{2}mv_0^2$$

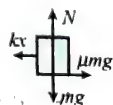
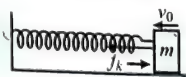
where x can be calculated by equating the forces acting on the stationary block.

Since the block is at the verge of sliding, we have $kx = \mu mg$

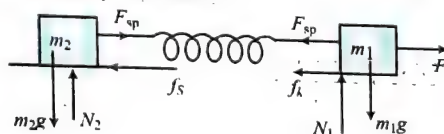
This gives $x = \frac{\mu mg}{k}$

Now substituting $x = \frac{\mu mg}{k}$ in the above expression of work, we

$$\text{have } W_f = \frac{\mu^2 m^2 g^2}{2k} - \frac{1}{2}mv_0^2$$



7. Since the block m_2 does not move force acting on m_2 do not perform work. As blocks do not move in vertical direction, hence work done by normal reaction and weight will be zero. Then, the forces F_{sp} , F and f_k do work on the block



Let v = speed of m_1

Applying W-E theorem,

$$W_{sp} + W_N + W_{gr} + W_{f_k} + W_{ext} = \Delta K$$

$$\left(-\frac{1}{2}kx^2\right) + (0) + (0) - \mu Nx + Fx = \frac{m_1 v^2}{2}$$

where $N = m_1 g$

$$v = \sqrt{2\left(\frac{F}{m_1} - \mu g\right) - \frac{k}{m_1}x^2}$$

8. Work done on the ball by gravity is

$$W_g = -mg(h_2 - h_1)$$

Let work done on the ball by air resistance be W_{air}

$$\therefore W_g + W_{air} = \Delta KE$$

$$\Rightarrow -mg(h_2 - h_1) + W_{air} = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$\Rightarrow W_{air} = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$$

9. (a) From Newton's second law, we get

$$ma = 3\mu mg - \mu mg \Rightarrow a = 2\mu g$$

- (b) From work energy principle

$$W_{spring} + W_{friction} = 0$$

$$\frac{1}{2}k(x_0^2 - x_1^2) - \mu mg(x_1 + x_0) = 0$$

$$x_0 - x_1 = \frac{2\mu mg}{k} \Rightarrow x_1 = \frac{\mu mg}{k}$$

- (c) Speed will be maximum where net force is zero

$$\mu mg = kx$$

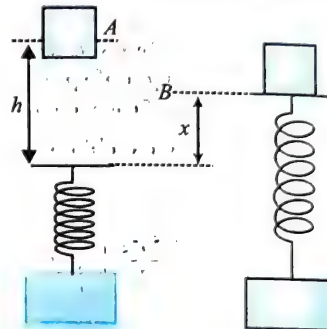
$$\Rightarrow x = \frac{\mu mg}{k} \text{ (extension)}$$

Now from work-energy principle, we have

$$\frac{1}{2}k\left(\frac{\mu mg}{k}\right)^2 (3^2 - 1^2) - \frac{2\mu mg}{k}(\mu mg) = \frac{1}{2}mv^2$$

$$\text{Solving we get, } v = 2\mu g \sqrt{\frac{m}{k}}$$

10. The block m drops a height h from A and compresses the spring. It then goes up x up to B above the initial level of spring.



Applying work-energy theorem between A and B,

$$W_{gravity} + W_{spring} = \Delta K \Rightarrow mg(h - x) - \frac{1}{2}kx^2 = 0 \quad \dots(i)$$

Minimum value of x required so that the block bounces is when the N will become zero

$$kx = Mg \Rightarrow x = \frac{Mg}{k}$$

$$mg\left(h - \frac{Mg}{k}\right) = \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 \Rightarrow mgh = \frac{M^2 g^2}{2k} + \frac{mMg^2}{k}$$

$$\text{Hence, } h = \frac{(M^2 + 2mM)g}{2km}$$

This is the required minimum value of h .

11. (a) At the extreme position blocks stops.

Applying work-energy theorem, we get $W_{gravity} + W_{spring} = \Delta K$

$$mg \sin 37^\circ \times x - \frac{1}{2}kx^2 = 0$$

$$2 \times 10 \times x \times \frac{3}{5} = \frac{1}{2} \times 100 \times x^2, \text{ on solving } x = 0.24 \text{ m}$$

Acceleration at its lowest point,

$$a = \frac{kx - mg \sin 37^\circ}{m} = \frac{100 \times 0.24 - 2 \times 10 \times \frac{3}{5}}{2} = 6 \text{ m/s}^2$$

- (b) Applying work-energy theorem, we get

$$W_{gravity} + W_{friction} + W_{spring} = \Delta K$$

$$mg \sin 37^\circ \times x + \mu mg \cos 37^\circ \times x = \frac{1}{2}kx^2$$

$$\mu mg \cos 37^\circ = mg \sin 37^\circ - \frac{1}{2}kx$$

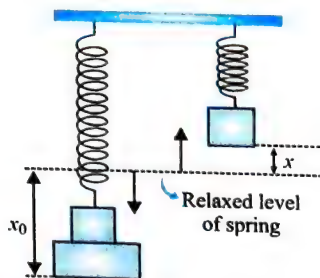
$$\mu \times 2 \times 10 \times \frac{4}{5} = 2 \times 10 \times \frac{3}{5} - \frac{1}{2} \times 100 \times x$$

It is given $x = 0.020 \text{ m}$

$$\text{Hence, } \mu = \frac{12 - 50x}{16} \Rightarrow \mu = \frac{1}{8}$$

12. Initially spring is stretched when both the blocks at equilibrium condition,

Initial stretch of the spring, $kx_0 = (m + 4m)g \Rightarrow x_0 = \frac{(m + 4m)g}{k}$... (i)



When $4m$ block is released, the block m starts move up, gains kinetic energy, crosses relaxed length of spring and then compress the spring before coming to rest. Let final compression of spring is x .

Now applying work energy theorem on block of mass m

$$W_g + W_{sp} = K_f - K_i$$

$$-mg\left(\frac{5mg}{k} + x\right) - \frac{1}{2}k\left(x^2 - \left(\frac{5mg}{k}\right)^2\right) = 0$$

$$\frac{1}{2}kx^2 + mgx - \frac{15m^2g^2}{2k} = 0$$

On solving we get, $x = \frac{3mg}{k}$

Displacement from initial is $\frac{5mg}{k} + \frac{3mg}{k} = \frac{8mg}{k}$

13. Let the force applied by the man, the external agent, be F . The net force acting on the body is zero because it is lowered slowly. Let the elongation of the spring at any instant be x

$\Rightarrow$ The spring pulls the body up by an amount kx

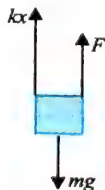
$$\Rightarrow F_{\text{net}} = F + kx - mg = 0$$

$$\Rightarrow F = mg - kx$$

... (i)

The work done dW by F in lowering the body through a distance x_0 is

$$W = \int dW = \int_0^{x_0} (mg - kx) dx$$



$$\Rightarrow W = mgx_0 - \frac{1}{2}kx_0^2$$

... (ii)

The body is lowered (slowly) till it remains in equilibrium. Therefore the external force F is variable and it is zero at the equilibrium of m , since the spring force kx_0 counter balances the weight mg of the body

$$\Rightarrow kx_0 = mg \Rightarrow x_0 = \frac{mg}{k}$$

... (iii)

Putting $x_0 = \frac{mg}{k}$ in (ii), we obtain $W = -\frac{m^2g^2}{2k}$

Exercise 8.3

1. No, For a conservative force, the work done in a round trip should be zero.

2. In a conservative field, $F = -\frac{dU}{dx}$

$$\therefore F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$$

For equilibrium, $F = 0$

$$\text{or } b - 2ax = 0 \quad \text{or } x = \frac{b}{2a}$$

From the given equation, we can see that

$$\frac{d^2U}{dx^2} = 2a \text{ (positive),}$$

i.e., U is minimum.

Therefore, $x = b/2a$ is the stable equilibrium position.

3. Along each step of motion, the frictional force is opposite in direction to the incremental displacement, so in the work $\cos 180^\circ = -1$

(a) $W = (3 \text{ N})(5 \text{ m})(-1) + (3 \text{ N})(5 \text{ m})(-1) = -30.0 \text{ J}$

(b) The distance CO is $(5^2 + 5^2)^{1/2} \text{ m} = 7.07 \text{ m}$

$$W = (3 \text{ N})(5 \text{ m})(-1) + (3 \text{ N})(5 \text{ m})(-1) + (3 \text{ N})(7.07 \text{ m})(-1) = -51.2 \text{ J}$$

(c) $W = (3 \text{ N})(7.07 \text{ m})(-1) + (3 \text{ N})(7.07 \text{ m})(-1) = -42.4 \text{ J}$

(d) The force of friction is a non-conservative force.

4. (a) $U = -\int_0^x (-Ax + Bx^2) dx = \frac{Ax^2}{2} - \frac{Bx^3}{3}$

(b) $\Delta U = \Delta U_{x=3 \text{ m}} - \Delta U_{x=2 \text{ m}}$

$$= \frac{A[(3.00)^2 - (2.00)^2]}{2} - B \frac{[(3.00)^3 - (2.00)^3]}{3}$$

$$= \frac{5.00}{2} A - \frac{19.0}{3} B$$

$$\Delta K = -\Delta U = \left(-\frac{5.00}{2} A + \frac{19.0}{3} B \right)$$

5. $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$$

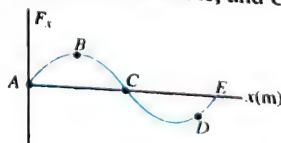
Thus, the force acting at point (x, y) is

$$\vec{F} = F_x \hat{i} + F_y \hat{j} = (7 - 9x^2y) \hat{i} - 3x^3 \hat{j}$$

6. (a) F_x is zero at points A , C , and E ; F_x is positive at point B and negative at point D .

(b) A and E are unstable, and C is stable.

(c)



7. We know force on particle is, $F = -\frac{dU}{dx} \Rightarrow F = -2x + 7$

So, $x = +\frac{7}{2} \text{ m}$, is equilibrium and nature of equilibrium is stable.

If $x > +\frac{7}{2} \text{ m} \Rightarrow F$ is -ve $\Rightarrow$ towards left

If $x < +\frac{7}{2} \text{ m} \Rightarrow F$ is +ve $\Rightarrow$ towards right

That means force always pushes the particle towards $x = +\frac{7}{2} \text{ m}$

Now total mechanical energy is conserved.

So, $K + U = 60 \text{ J}$ (at all time)

$$\Rightarrow K = 60 - (x^2 - 7x + 30) = 30 - x^2 + 7x$$

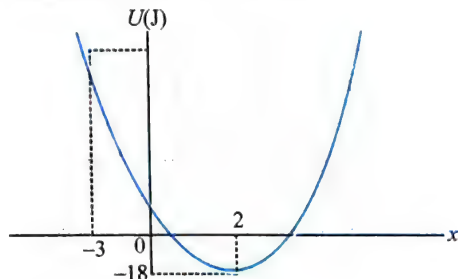
$$\text{If } K = 0 \Rightarrow x = +10 \text{ m and } x = -3 \text{ m}$$

So particle cannot go beyond $(-3 \text{ m}, 10 \text{ m})$.

Motion of the particle is oscillatory about $x = -7/2$ within $(-3 \text{ m}, 10 \text{ m})$

8. (a) $U = 5x^2 - 20x + 2$

The variation of U with x is as shown.



PE is minimum when $\frac{dU}{dx} = 0$.

$$\Rightarrow 10x - 20 = 0 \Rightarrow x = 2 \text{ m}$$

$$\text{At } x = 2 \text{ m}, U = 5(2)^2 - 20(2) + 2 = -18 \text{ J}$$

$$\text{At } x = 3 \text{ m}, U = 5(-3)^2 - 20(-3) + 2 = 107 \text{ J}$$

When particle is released at $x = -3 \text{ m}$, it experiences a force in positive x direction. Its KE is maximum at $x = 2 \text{ m}$.

$$\therefore K_{\max} + (-18) = 107 \Rightarrow K_{\max} = 125 \text{ J}$$

(b) Total energy of the particle is 107 J. When it is at rest, its KE = 0.

$$\therefore U = 107 \Rightarrow 5x^2 - 20x + 2 = 107$$

$$\Rightarrow x^2 - 4x - 21 = 0 \Rightarrow x = -3 \text{ m}, 7 \text{ m} \therefore x_{\max} = 7 \text{ m}$$

9. $F = 2Ax^2y\hat{i} + Axy^2\hat{j}$

$$W = 2A \int x^2 y dx + A \int xy^2 dy$$

From $A \rightarrow B$, $y = 0$. Then $W_1 = 0$.

From $B \rightarrow C$, $x = 1$ ($dx = 0$). Then

$$W_2 = A \int_0^1 y^2 dy = \frac{A}{3}$$

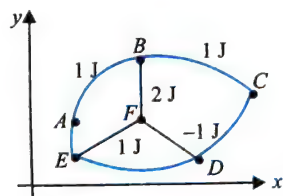
From $C \rightarrow D$, $y = 1$ ($dy = 0$). Then $W_3 = 2A \int_1^0 x^2 dx = -\frac{2A}{3}$.

From $D \rightarrow A$, $x = 0$, $dx = 0$. Then $W_4 = 0$.

$$\text{Then, } W = \Sigma W_i = -\frac{A}{3} \neq 0$$

Hence, the force is non-conservative.

10. $\therefore W_{AC} = 2 \text{ J}$ or $W_{CA} = -2 \text{ J}$



$$\therefore W_{AE} + W_{EF} + W_{FB} + W_{BA} = 0$$

$$\text{or } W_{AE} + 1 + 2 - 1 = 0$$

$$\text{or } W_{AE} = -2 \text{ J}$$

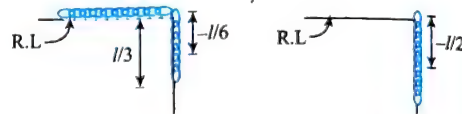
$$\therefore W_{FB} + W_{BC} + W_{CD} + W_{DF} = 0$$

$$\text{or } 2 + 1 + W_{CD} + (-1) = 0$$

$$\text{or } W_{DC} = 2 \text{ J}$$

Exercise 8.4

1. With respect to the top of the table, the initial potential energy of the chain, $U_i = \text{PE of the chain lying on the table} + \text{PE of the overhanging part of the chain}$



(a)

(b)

$$= \frac{2m}{3}g \times 0 + (m/3)g(-l/6) = -\frac{mgl}{18}$$

PE of chain at the instant of slip,

$$U_f = 0 + mg(-l/2) = -\frac{mgl}{2}$$

Since only gravity is acting on the chain, we have

$$-\Delta U = \Delta K$$

$$K = -(U_f - U_i) = \left[\frac{-mgl}{2} - \left(\frac{-mgl}{18} \right) \right] = \frac{4}{9}mgl$$

$$\text{Since } K_i = 0, K_f = \frac{4}{9}mgl.$$

2. Loss in kinetic energy = Gain in potential energy + Work done against friction

$$\text{or } \frac{1}{2}mv_0^2 = mgh + F_f \times d = mgh + \mu mg \times d$$

Substituting $h = 1.1 \text{ m}$ and $\mu = 0.60$, we get

$$d = 1.2 \text{ m}$$

3. Friction is only force which is changing mechanical energy of system.

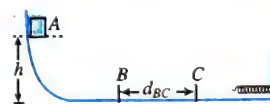
$$W_{\text{friction}} = \Delta \text{ME.}$$

$$-\mu_k mgd \cos \theta = (mgy_{\max} - mgh)$$

$$\text{where } d = \frac{y_{\max}}{\sin \theta} \therefore mgy_{\max} = mgh - \mu_k mgy_{\max} \cot \theta$$

$$\text{Solving, } y_{\max} = \frac{h}{1 + \mu_k \cot \theta}$$

4. $\Delta E_{\text{mech}} = -f dx \Rightarrow E_f - E_i = -f \cdot d_{BC}$



$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}}$$

5. $m = 10 \text{ kg}, k = 400 \text{ N m}^{-1}$

Natural length of spring = 4 m

Decreasing in PE = increasing in KE

$$\frac{1}{2}k \times 1 + mgh = \frac{1}{2}mv^2$$

$$\frac{1}{2} \times 400 \times 1^2 + 10 \times 10 \times 3 = \frac{1}{2} \times 10 v^2$$

$$200 + 300 = 5v^2 \Rightarrow v = 10 \text{ ms}^{-1}$$

6. Let the maximum elongation in the spring be x , when the block is at position 2.

(a) The displacement of the block m is also x .

From conservation of energy, $\Delta K + \Delta U = 0$

$$\Rightarrow \frac{1}{2}kx^2 = mgx$$

$$\Rightarrow x = 2mg/k$$

- (b) Let maximum velocity occur when block moves a distance x .
Using $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(-mgx + \frac{1}{2}kx^2\right) = 0$$

$$\text{or } \frac{1}{2}mv^2 = mgx - \frac{1}{2}kx^2$$

Differentiating above equations w.r.t. x

$$\frac{1}{2}m \cdot 2v \cdot \frac{dv}{dx} = mg \frac{dx}{dt} - \frac{1}{2}k2x \frac{dx}{dt}$$

$$mv \cdot \frac{dv}{dt} = mgv - kv$$

$$\text{For maximum } v, \frac{dv}{dx} = 0 \Rightarrow x = \frac{mg}{k}$$

$$\text{So, } mg\left(\frac{mg}{k}\right) - \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{1}{2}mv_{\max}^2$$

$$\Rightarrow v_{\max} = \left(\sqrt{\frac{m}{k}}\right)g$$

Here we can use $\Delta K + \Delta U = 0$.

Since initially, the system is at rest and finally it comes to rest (instantaneously), the change of KE of the system is zero.

$$\Rightarrow \Delta K = 0$$

Also $\Delta U = 0$.

$$\Rightarrow (\Delta U)_M + (\Delta U)_m = 0$$

Since the block m rises through a height h (say),

$$(\Delta U)_m = mgh$$

Similarly, the change in PE of the mass M is Mgh' , as it falls through a height h' , say

$$\Rightarrow mgh + (-Mgh') = 0 \Rightarrow h' = \frac{m}{M}h \quad \dots(i)$$

When the mass m ascends through a distance h , the length of the string between the pulley and the bar will be equal to $(a + h)$.

$$\therefore h' = \sqrt{(a+h)^2 - a^2} \Rightarrow h' = \sqrt{h(2a+h)} \quad \dots(ii)$$

Eliminating h' from (i) from (ii), we obtain

$$\sqrt{h(2a+h)} = \frac{m}{M}h$$

$$\Rightarrow 2a+h = \frac{m^2}{M^2}h \Rightarrow h = \frac{2a}{\left(\frac{m^2}{M^2} - 1\right)}$$

8. Let the ball m_2 moves through an angle θ . The mass m will fall through a distance $h_1 = R\theta$. The ball m_2 rises through a height h_2 as,

$$h_2 = R \sin \theta.$$

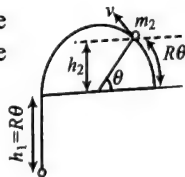
The change in gravitational potential energy of m_1 is

$$\Delta U_1 = -m_1gh_1 = -m_1gR\theta$$

(since m_1 loses its potential energy as it falls down).
The change in gravitational potential energy of m_2 is

$$\Delta U_2 = m_2gh_2 = m_2gR \sin \theta$$

(since m_2 gains potential energy as it rises up)



$\Rightarrow$ Total change in gravitational potential energy,

$$\Delta U = \Delta U_1 + \Delta U_2 \quad \dots(i)$$

$$\Rightarrow \Delta U = -m_1gR\theta + m_2gR \sin \theta = gR(m_2 \sin \theta - m_1\theta).$$

The change in KE of the system ($m_1 + m_2$),

$$\Delta K = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 = \frac{(m_1 + m_2)v^2}{2} \quad \dots(ii)$$

where v = speed of m_1 and m_2 at the positions as shown in the figure

From the principle of conservation of energy we obtain,

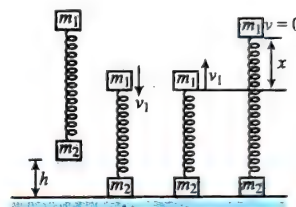
$$\Delta K + \Delta U = 0 \quad \dots(iii)$$

Using (i), (ii) and (iii), we obtain,

$$\frac{1}{2}(m_1 + m_2)v^2 - gR(m_1\theta - m_2 \sin \theta) = 0$$

$$\Rightarrow v = \sqrt{\frac{2gR(m_1\theta - m_2 \sin \theta)}{(m_1 + m_2)}}$$

$$9. v_1 = \sqrt{2gh}$$



For block m_2 to be lifted up

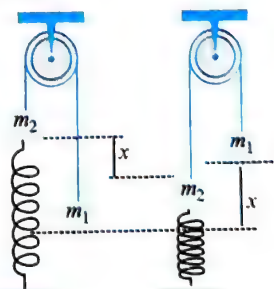
$$kx = m_2g \Rightarrow x = \frac{m_2g}{k}$$

$$\text{Now, } \frac{1}{2}m_1v_1^2 = m_1gx + \frac{1}{2}kx^2$$

$$\text{Solve to get } h = \frac{m_2(2m_1 + m_2)g}{2m_1k}$$

10. Since work done by the constrain forces, tension and normal reaction is zero. The mechanical energy of the system will be conserve

$$\Delta ME = 0 \text{ or } \Delta K + \Delta U = 0$$



Let x = maximum compression of the spring.

$$\text{Then, } \Delta U_{sp} = \frac{k}{2}x^2 \quad \dots(i)$$

As m_2 descends by a distance x ,

$$\Delta U_{gr1} = -m_2gx \quad \dots(ii)$$

As m_1 ascends by a distance x ,

$$\Delta U_{gr2} = m_1gx \quad \dots(iii)$$

Since both m_1 and m_2 come to rest simultaneously,

$$\therefore \Delta K = 0 \quad \dots(iv)$$

Conservation of energy: $\Delta K + \Delta U = 0$

$$[\Delta U_{sp} + \Delta U_{gr1} + \Delta U_{gr2}] + \Delta K = 0 \quad \dots(v)$$

$$\left(\frac{1}{2}kx^2 + m_1gx - m_2gx\right) + 0 = 0 \Rightarrow x = \frac{2(m_2 - m_1)g}{k}$$

11. As the block of mass M descends down, it loses potential energy which appears in the form of kinetic energy of the two blocks, besides doing work against friction.

Now, if the block of mass $2M$ moves by a distance s , and its velocity be v , then the distance descended by block of mass M would be $s/2$ and so its velocity would be $v/2$.

$$\text{Now, loss in potential energy of the system} = Mg\left(\frac{s}{2}\right) \quad \dots(i)$$

$$\text{Work done against friction} = 2M\mu gs \quad \dots(ii)$$

$$\text{Gain in kinetic energy of the two blocks} = \frac{1}{2}(2M)v^2 + \frac{1}{2}M\left(\frac{v}{2}\right)^2 \quad \dots(iii)$$

Law of conservation of energy demands that

$$Mg\left(\frac{s}{2}\right) = 2M\mu gs + Mv^2 + \frac{Mv^2}{8}$$

$$\Rightarrow \frac{gs}{2} = 2\mu gs + \frac{9}{8}v^2$$

$$\Rightarrow \frac{9}{8}v^2 = \frac{gs}{2}(1 - 4\mu)$$

$$\Rightarrow v = \sqrt{\frac{4}{9}gs(1 - 4\mu)}; \text{ velocity of mass, } M = \frac{v}{2}$$

$$\therefore \text{ Required velocity} = \frac{1}{3}\sqrt{gs(1 - 4\mu)}$$

12. Evidently, when the block B descends by a distance l , the pulley P , descends by a distance $l/2$, and consequently, block A ascends by a distance $l/2$. Also, if v be the speed of block A (up), $2v$ would be the speed of block B (down).

Net loss in P.E. of the system

$$= \text{Loss in P.E. of block } B - \text{Gain in P.E. of block } A$$

$$= mgl - mg\frac{l}{2} = mg\frac{l}{2}$$

$$\text{Total gain in K.E.} = \frac{1}{2}mv^2 + \frac{1}{2}m(2v)^2 = \frac{5}{2}mv^2$$

Now, law of conservation of energy demands,

$$\text{Loss in P.E.} = \text{Gain in K.E.}$$

$$\Rightarrow mg\frac{l}{2} = \frac{5}{2}mv^2 \Rightarrow v^2 = \frac{gl}{5}$$

$$\Rightarrow v = \sqrt{\frac{gl}{5}}$$

13. For finding the maximum height H above the end of the track, we need to analyze the projectile motion of the skateboarder after he leaves the track. For this analysis, we will use the principle of conservation of mechanical energy, which applies as friction and air resistance are being ignored. In applying this principle to the projectile motion, however, we will need to know the speed of the skateboarder (v_f) when he leaves the track. Applying the conservation of mechanical energy, we have

$$\Delta K + \Delta U = 0 \text{ or } \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + mgh = 0.$$

Here v_0 is the initial velocity of skateboarder when he enter the track which is equal to 4.0 m/s and $\Delta U = mgh$ where $h = 0.45$ m.

Solving for the final speed at the end of the track gives

$$v_f = \sqrt{v_0^2 + 2gh} = \sqrt{(5.0 \text{ m/s})^2 + 2(10.0 \text{ m/s}^2)(0.45 \text{ m})} = 4.0 \text{ m/s}$$

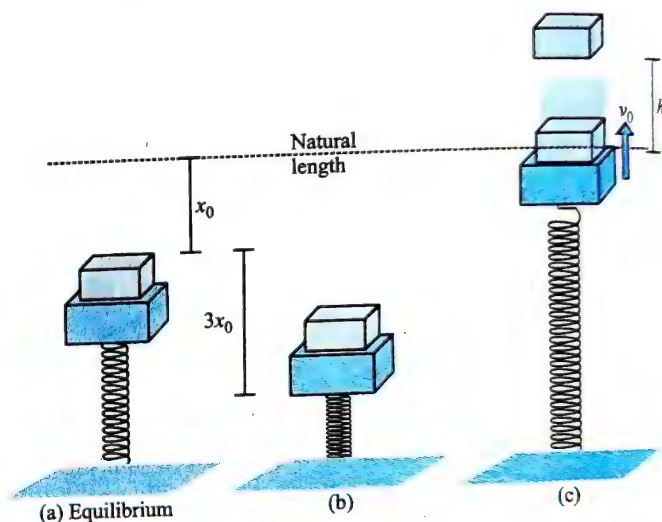
This speed now becomes the initial speed $v_0 = 4.0$ m/s for the projectile motion of the skateboarder after he leaves the track. At the maximum height of her trajectory he is traveling horizontally with a speed v_f that equals the horizontal component of her launch velocity, which is equal to $v_f = (4.0 \text{ m/s}) \cos 60^\circ$. Applying the conservation of mechanical energy again, we have $\Delta K + \Delta U = 0$

$$\text{or, } \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2\right) + mgh = 0$$

and solving for H give

$$H = \frac{v_0^2 - v_f^2}{2g} = \frac{(4.0 \text{ m/s})^2 - [(4.0 \text{ m/s}) \cos 60^\circ]^2}{2(10.0 \text{ m/s}^2)} = 0.60 \text{ m}$$

14. (a) In equilibrium, the spring is compressed by $x_0 = \frac{3Mg}{K}$ [Fig. (a)]



The system is released from position shown in fig. (b). Compression in the spring at this position is $4x_0$. The upper block will lose contact with the lower block when the spring acquires its natural length position.

$$\text{Hence, answer to first part of the question is } 4x_0 = \frac{12Mg}{k}$$

- (b) Let us apply the law of conservation of mechanical energy to get the speed of the block when the spring reaches its natural length.

$$\frac{1}{2}k(4x_0)^2 = \frac{1}{2}3MV_0^2 + 3Mg(4x_0)$$

$$72 \frac{M^2g^2}{k} = \frac{3}{2}MV_0^2 + \frac{36M^2g^2}{k}$$

$$V_0 = \sqrt{\frac{24M}{k}} \cdot g$$

Further height attained by the block of mass $2M$ is given by

$$h = \frac{v_0^2}{2g} = \frac{12Mg}{k}$$

$$\therefore \text{ Height attained above point of release is } H = 4x_0 + h = \frac{24Mg}{k}$$

Exercise 8.5

1. Zero

Explanation: $\vec{F}$ and $\vec{v}$ are perpendicular.

$$\text{Power} = \vec{F} \cdot \vec{v} = Fv \cos 90^\circ = \text{Zero}$$

2. Mass, $m = 20$ metric ton $= 20 \times 1000$ kg; distance, $S = 20$ m; time, $t = 1$ h $= 3600$ s

$$\text{power} = \frac{\text{Work}}{\text{Time}} = \frac{mg \times S}{t} = \frac{20 \times 1000 \times 10 \times 20}{3600} \text{ W} = 1.1 \times 10^3 \text{ W}$$

3. Power, $P = 1 \text{ kW} = 10^3 \text{ W}$, $S = 10 \text{ m}$; time, $t = 1 \text{ s}$;
mass of water, $m = ?$

$$\text{Power} = \frac{mg \times S}{t}$$

$$10^3 = \frac{m \times 10 \times 10}{1} \text{ or } m = \frac{10^3}{10 \times 10} \text{ kg} = 10 \text{ kg}$$

$$\begin{aligned} \text{4. Power of the first coolie} &= \frac{\text{Work}}{\text{Time}} \\ &= \frac{M \times g \times S}{t} = \frac{M \times 9.8 \times 2}{60} \text{ J s}^{-1} \end{aligned}$$

Power of the second coolie is

$$\frac{M \times 9.8 \times 2}{30} \text{ J s}^{-1} = 2 \left(\frac{M \times 9.8 \times 2}{60} \right) \text{ J s}^{-1}$$

$$= 2 \times \text{Power of the first coolie}$$

So, the power of the second coolie is double that of the first one.

Both the coolies spend the same amount of energy.

Alternative method: We know that $W = Pt$

For the same work, $W = P_1 t_1 = P_2 t_2$

$$\text{or } \frac{P_2}{P_1} = \frac{t_1}{t_2} = \frac{1 \text{ min}}{30 \text{ s}} = 2 \text{ or } P_2 = 2P_1$$

5. If $A \text{ m}^2$ is the area, then power $= 200 \times A \text{ W}$

Useful electrical energy produced per second is

$$\frac{20}{100} (200 A) = 40 \times A \text{ W}$$

$$\text{But } 40 A = 8000 \text{ or } A = 200 \text{ m}^2$$

6. Weight of (elevator + passenger) $= mg = 1800 \times 10 \text{ N} = 18000 \text{ N}$

Frictional force $= 4000 \text{ N}$

Total downward force on the elevator is

$$(18000 + 4000) \text{ N} = 22000 \text{ N}$$

Clearly, the motor must have enough power to balance this force.

Now power, $P = Fv = 22,000 \text{ N} \times 2 \text{ ms}^{-1}$

$$= 44,000 \text{ W} = \frac{44000}{746} \text{ hp} = 58.98 \text{ hp}$$

7. The kinetic energy is dependent on the square of the velocity, and work done is equal to the change in kinetic energy. Now, as the change in the kinetic energy of the second person is more than the first person, he does more work. As both of them increase their speeds in the same interval of time, the second person does more work in the same interval of time. Hence, the second person has more power.

8. When 1000 kg of water is lifted from a 12.0 m deep well, work required is $mgh = 1.2 \times 10^5 \text{ J}$

$$\text{Work required per second} = \frac{1.2 \times 10^5}{60} = 2000 \text{ J s}^{-1}$$

Kinetic energy gained by the water per second is

$$\frac{1}{2} mv^2 = \frac{1}{2} \times \frac{1000}{60} \times (20)^2 = 3333.33 \text{ J s}^{-1}$$

Total work required per second by the pump is

$$2000 + 3333.33 = 5333.33 \text{ J s}^{-1}$$

9. The helicopter does work. When the helicopter raises the mass, the potential energy of the body changes. Potential energy gained by the mass $= mgh = 1000 \times 9.8 \times 500 = 4900 \text{ kJ}$

Work done by the helicopter = Gain in potential energy by the body $= 4900 \text{ kJ}$. Power is developed with respect to the ground.

Power developed = work done by the helicopter/time taken to do the work $= 4900 \text{ kJ} / 5 = 980 \text{ kW}$

We assign a positive sign for the power developed as the helicopter serves as the source for delivering the power.

$$10. \text{Power} = \frac{W}{t}$$

$$P = \frac{mgh}{t} = \frac{(700 \text{ N})(10.0 \text{ m})}{8.00 \text{ s}} = 875 \text{ W}$$

$$11. P_{av} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875 \text{ kg}(0.620 \text{ ms}^{-1})^2}{2(21 \times 10^{-3} \text{ s})} = 8.0 \text{ W}$$

12. $h = 50 \text{ m}$, $m = 1.8 \times 10^5 \text{ kg}$, $P = 100 \text{ W}$

$$\text{PE} = mgh = 1.8 \times 10^5 \times 10 \times 50 = 900 \times 10^5 \text{ J h}^{-1}$$

Because half the potential energy is converted into electricity

$$\text{Electrical energy} = \frac{1}{2} \text{PE} = 450 \times 10^5 \text{ J h}^{-1}$$

So, power in watt (J s^{-1}) $= (450 \times 10^5) / (3600)$

$$\text{The number of } 100 \text{ W lamps that can be lit} = \frac{450 \times 10^5}{3600 \times 100} = 125$$

13. In general for a particle moving with constant speed, the power delivered by the total (centripetal force is zero; the reason being, force is always radial and velocity is always tangential, the angle between them being 90°).

However, if the particle has some tangential acceleration too, then the power delivered by all the forces will not be zero, since in this case, tangential forces, besides radial forces, also act.

Therefore, it is wise, to first know, whether the velocity (and hence KE) of the particle changes with time or remains constant)

If v be the instantaneous velocity, then

$$\frac{mv^2}{r} = k^2 r t^2 \Rightarrow v = \frac{krt}{\sqrt{m}}$$

Obviously, the velocity depends on time. Therefore, the power delivered is not zero.

Now, if a_t be the tangential acceleration, then

$$a_t = \frac{dv}{dt} = \frac{kr}{\sqrt{m}}$$

Therefore, tangential force $F_t = ma_t = \sqrt{m}(kr)$

Now, power P delivered is given by

$$P = (\vec{F}_r + \vec{F}_t) \cdot \vec{v} = \vec{F}_r \cdot \vec{v} + \vec{F}_t \cdot \vec{v} \quad [\because \vec{F}_r \text{ is } \perp \text{ to } \vec{v}]$$

$$= \vec{F}_t \cdot \vec{v} \quad [\because \vec{F}_t \text{ is along the same direction as } \vec{v}]$$

$$= \sqrt{m}(kr) \left(\frac{krt}{\sqrt{m}} \right)$$

$$P = k^2 r^2 t$$

$$\text{Alternatively: } E = \frac{1}{2} mv^2 = \frac{1}{2} k^2 r^2 t^2$$

$$P = \frac{dE}{dt} = k^2 r^2 t$$

Exercises

Single Correct Answer Type

1. (2) In case of non-conservative forces, the work done is dissipated as heat, sound, etc., i.e., it does not increase the potential energy. But in case of conservative forces, work done is responsible for increasing or decreasing the potential energy.

2. (2) We know that $dU = -dW$

where dU is the change in potential energy

and dW is the work done by conservative forces

Hence, work done by conservative forces on a system is equal to the negative of the change in potential energy.

3. (1) In the explosion of a bomb or inelastic collision between two bodies as force is internal, momentum is conserved while KE changes. Hence, the KE of a system can be changed without changing its momentum. Similarly, the reverse is also true, e.g., if a force acts perpendicular to motion, work done will be zero and so KE will remain constant. However, the force will change the direction of motion and so the momentum.

Further, body may have energy (i.e., potential energy) without having momentum.

4. (1) For the path AC,

$$W_{AC} = Fs \cos(90^\circ - \theta) = mgs \sin \theta = mgh \quad (\because F = mg)$$

$$\text{For path AB, } W_{AB} = Fa \cos 90^\circ = 0$$

$$\text{For path BC, } W_{BC} = Fh \cos 0^\circ = mgh$$

$$\text{So that } W_{AB} + W_{BC} = mgh = W_{AC}$$

$$\text{i.e., } W_{ABC} = W_{BC}$$

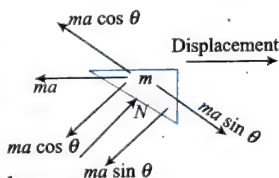
5. (3) Total gravitational energy gained is work done + energy released by the spring = $W + E$

6. (1) ma is pseudo-force.

We find that $mg \sin \theta$ is equal to $ma \cos \theta$. So block remains stationary w.r.t. cage and there is no need of friction.

Statement (i) is correct because angle between N and displacement is

positive. Friction force acting on the block is zero, hence no work will be done by friction, so Statement (ii) is incorrect.



7. (3) At equilibrium position, $x = \frac{mg}{k}$

$$U_{\text{spring}} = \frac{1}{2} kx^2 = \frac{1}{2} k \left(\frac{mg}{k} \right)^2$$

$$= \frac{mgx}{2} = \frac{1}{2} (\text{loss in GPE}) \Rightarrow G = 2S$$

8. (1) Work done by man, $mgL = mg(L - s) + mgs$

where $mg(L - s)$ is the increase in potential energy of the man and mgs is the increase in potential energy of the balloon because the balloon would have been lifted up but for the climbing of the man. So

$$\frac{\text{Increase in PE of man}}{\text{Increase in PE of balloon}} = \frac{mg(L - s)}{mgs} = \frac{L - s}{s}$$

9. (2) Since $k_1 > k_2$, therefore, P will develop greater restoring force than Q . So the applied force on P will be more than the applied force on Q . Extension is same in both the cases. So, more work is done on P .

10. (1) Spring Q has lesser force constant than P . So Q will develop less restoring force than P . As a result, Q will suffer more extension. Since force is same in both the cases, more work will be done on Q .

11. (4) For equilibrium, potential energy has to be minimum, maximum or constant.

12. (1) Total work done is equal to the energy stored in the spring.

13. (3) At point C the potential energy is minimum. Hence, it is a point of stable equilibrium. Also, from C to A , the slope is negative, i.e.,

$$\frac{dU}{dr} < 0$$

Hence, the force of interaction between the particles is repulsive between points C and A .

14. (4) $f(x) = -\frac{dU}{dx}(x)$ or $U(x) = -\int F(x) dx$

Here $F(x) = -kx$, where k is a positive constant.

15. (3) Minimum stopping distance = s

$$\text{Work done against the friction} = W = \mu mgs$$

Initial momentum gained by both toy carts will be same because same force acts for same time.

$$\text{Initial kinetic energy of the toy cart} = \left(\frac{p^2}{2m} \right)$$

$$\text{Therefore, } \mu mgs = \frac{p^2}{2m} \text{ or } s = \left(\frac{p^2}{2\mu gm^2} \right)$$

For the two toy carts, momentum is numerically the same. Further μ and g are the same for the toy carts.

$$\text{So, } \frac{s_1}{s_2} = \left(\frac{m_2}{m_1} \right)^2$$

16. (2) Here $x = (t - 3)^2 = t^2 - 6t + 9$

$$v = \frac{dx}{dt} = 2t - 6$$

$$\text{At } t = 0, v = 2 \times 0 - 6 = -6$$

$$\text{At } t = 6 \text{ s, } v = 2 \times 6 - 6 = +6$$

Initial and final KE are same, hence no work is done.

17. (2) Tension in the string,

$$T = M(g - a) = M \left(g - \frac{g}{2} \right) = Mg$$

$$W = \text{Force} \times \text{Displacement} = -M \left(\frac{g}{2} \right) h$$

Negative sign is because tension is in the upward direction and displacement is in the downward direction.

18. (1) First case: $\frac{1}{2} mv^2 = Fs$... (i)

$$\frac{1}{2} \left(M + \frac{m}{2} \right) v^2 = Fs' \quad \dots (ii)$$

Dividing Eq. (ii) by Eq. (i), we get $\frac{s'}{s} = \frac{3}{2}$ or $s' = 1.5s$

19. (1) From work-energy theorem,

$$\begin{aligned} W = \Delta KE &= K_f - K_i = \frac{1}{2} m(v_f^2 - v_i^2) \\ &= \frac{1}{2} \times 2 [(-20)^2 - (10)^2] = 300 \text{ J} \end{aligned}$$

20. (2) If a liquid of density ρ is flowing through a pipe of cross section A at speed V , the mass coming out per second will be

$$\left(\frac{dm}{dt} \right) = \rho AV$$

In order to get n times water in the same time, we get

$$\left(\frac{dm}{dt}\right)' = n \left(\frac{dm}{dt}\right)$$

i.e., $A'V'\rho' = nAV\rho$

But as pipe and liquid are same, $\rho' = \rho$, $A' = A$

$$V' = nV,$$

$$\text{So } \frac{F'}{F} = \frac{V' \left(\frac{dm}{dt}\right)'}{V \left(\frac{dm}{dt}\right)} = \frac{nV \cdot n \frac{dm}{dt}}{V \left(\frac{dm}{dt}\right)} = n^2$$

$$\text{or } F' = n^2 F$$

$$\frac{P'}{P} = \frac{F'V'}{FV} = \frac{(n^2 F)(nV)}{FV} \text{ or } P' = n^3 P$$

$$(3) P = Fv = m \frac{dv}{dt} v$$

$$\text{or } v \frac{dv}{dt} = \frac{P}{m}$$

$$\text{or } v \frac{dv}{dx} \frac{dx}{dt} = \frac{P}{m}$$

$$\text{or } v^2 \frac{dv}{dx} = \frac{P}{m} \text{ or } v^2 dv = \frac{P}{m} dx$$

$$\text{On integration, we get } \frac{v^3}{3} = \frac{Px}{m} \text{ or } v = \left(\frac{3xP}{m}\right)^{\frac{1}{3}}$$

(3) From work-energy theorem,

$$\Delta KE = W_{\text{net}} \text{ or } K_f - K_i = \int P dt$$

$$\text{or } \frac{1}{2} mv^2 - 0 = \int_0^2 \left(\frac{3}{2} t^2\right) dt$$

$$\text{or } v^2 = \left[\frac{t^3}{2}\right]_0^2 \text{ or } v = 2 \text{ ms}^{-1}$$

(3) Let us assume that the displacement of the body is directly proportional to t^n , i.e.,

$$s = Kt^n; v = \frac{ds}{dt} = Knt^{n-1}$$

$$\text{and } a = \frac{dv}{dt} = Kn(n-1)t^{n-2}$$

$$\text{Force } F = ma = mKn(n-1)t^{n-2}$$

$$\text{Power, } P = Fv = [mKn(n-1)t^{n-2}][Knt^{n-1}]$$

$$= mKn^2(n-1)t^{2n-3}$$

As power is constant, i.e., independent of time, hence

$$2n-3=0 \text{ or } n = \frac{3}{2} \text{ or } s \propto t^{\frac{3}{2}}$$

$$(3) P = Fv = M \frac{dv}{dt} v$$

$$\text{Hence, } v dv = \frac{P}{m} dt$$

On integration, we find $v \propto \sqrt{t}$

(4) Velocity of a projectile at any instant of time (t) is

$$v^2 = v_x^2 + v_y^2 = (u \cos \theta)^2 + \left(u \sin \theta - g \frac{x}{u \cos \theta}\right)^2$$

$$\therefore KE = \frac{1}{2} mu^2 - mgx \tan \theta + \frac{mg^2 x^2}{u^2 \cos^2 \theta}$$

26. (4) The initial potential energy of the object is mgh , which has been used up for work against the force of friction. In returning the body to its initial position, the force performs the same work against friction. In addition, it imparts to the object the initial potential energy. As a result, the total work will be $2mgh$.

27. (3) According to the law of conservation of energy, elastic potential energy stored in the spring = gravitational potential energy acquired by shot

$$\frac{1}{2} kx^2 = mgh \text{ or } h = \frac{kx^2}{2mg}$$

28. (2) $W_{\text{gravity}} = \Delta U_{\text{spring}}$

$$mg \sin \theta (d + 0.25) = \frac{1}{2} k (0.25)^2$$

which gives $d = 37.5 \text{ cm}$

29. (3) Let the particle be dropped from a height h and the spring be compressed by y . According to the conservation of mechanical energy, loss in PE of the particle = gain in elastic potential energy of the spring

$$mg(h+y) = \frac{1}{2} ky^2$$

Now, as the particle and spring are same for second case,

$$\frac{h_1 + y_1}{h_2 + y_2} = \left(\frac{y_1}{y_2}\right)^2 \text{ or } \left(\frac{0.24 + 0.01}{h_2 + 0.04}\right) = \left(\frac{0.01}{0.04}\right)^2$$

Solving, we get $h_2 = 3.96 \text{ m}$

30. (3) Work done by friction:

$$W = (\mu mg \cos \theta) S = (\mu mg \cos \theta) \frac{h}{\sin \theta} = \mu mgh \cot \theta$$

$$\text{Now } \cot \theta_1 = \cot 30^\circ = \sqrt{3}$$

$$\cot \theta_2 = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

i.e., kinetic energy ($KE = mgh - W$) in first case will be less or $K_1 < K_2$.

31. (2) Let x be the extension in the string when the 2-kg block leaves the contact with ground. Then tension in the spring should be equal to the weight of 2-kg block:

$$Kx = 2g \text{ or } x = \frac{2g}{K} = \frac{2 \times 10}{40} = \frac{1}{2} \text{ m}$$

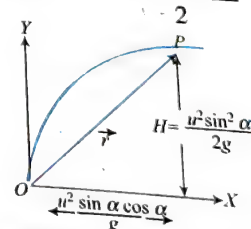
Now from conservation of mechanical energy,

$$mgx = \frac{1}{2} Kx^2 + \frac{1}{2} mv^2$$

$$\Rightarrow \sqrt{2gx - \frac{Kx^2}{m}} = \sqrt{2 \times 10 \times \frac{1}{2} - \frac{40}{4 \times 5}} = 2\sqrt{2} \text{ ms}^{-1}$$

$$32. (4) \vec{r} = \frac{u^2 \sin \alpha \cos \alpha}{g} \hat{i} + \frac{u^2 \sin^2 \alpha}{2g} \hat{j}$$

$$F = -mg \hat{j}, W = \vec{F} \cdot \vec{r} = -\frac{mu^2 \sin^2 \alpha}{2}$$



Alternative method:

Velocity at highest point = $u \cos \alpha$

$$W_{mg} = \Delta KE = \frac{1}{2} m(u \cos \alpha)^2 - \frac{1}{2} mu^2$$

$$= -\frac{mu^2 \sin^2 \alpha}{2}$$

$$33. (4) \text{ Average power } <P> = \frac{<W>}{t} = \frac{mu^2 \sin^2 \alpha}{2 \left(\frac{u \sin \alpha}{g} \right)} = \frac{-mgu \sin \alpha}{2}$$

[Since time taken to go from O to $P = u \sin \alpha / g$]

34. (2) The position of equilibrium corresponds to $F(x) = 0$

$$\text{Since } F(x) = \frac{-dU(x)}{dx}$$

$$\text{so } F(x) = -\frac{d}{dx} \left(\frac{a}{x^4} - \frac{b}{x^2} \right) \text{ or } F(x) = \frac{4a}{x^5} - \frac{2b}{x^3}$$

For equilibrium, $F(x) = 0$, therefore

$$\frac{4a}{x^5} - \frac{2b}{x^3} = 0 \Rightarrow x = \pm \sqrt{\frac{2a}{b}}$$

$$\frac{d^2U(x)}{dx^2} = -\frac{20a}{x^6} + \frac{8b}{x^4}$$

Putting $x = \pm \sqrt{\frac{2a}{b}}$ gives $\frac{d^2U(x)}{dx^2}$ as negative

So U is maximum. Hence, it is position of unstable equilibrium.

35. (2) Maximum frictional force between the slab and the block

$$f_{\max} = \mu N = \mu mg$$

$$= \frac{1}{4} \times 4 \times 10 = 10 \text{ N}$$

Evidently, $f < f_{\max}$

So, the two bodies will move together as a single unit. If a be their combined acceleration, then

$$a = \frac{F}{m+M} = \frac{6}{4+5} = \frac{2}{3} \text{ m s}^{-1}$$

Therefore, frictional force acting can be obtained as

$$f = Ma = \frac{2}{3} \times 5 \text{ N} = \frac{10}{3} \text{ N}$$

$$\text{Using } s = \frac{1}{2} at^2$$

$$s(2) = \frac{1}{2} \times \frac{2}{3} (2)^2 = \frac{4}{3} \text{ and } s(3) = \frac{1}{2} \times \frac{2}{3} (3)^2 = 3$$

Therefore, work done by friction = $f[s(3) - s(2)]$

$$= \frac{10}{3} \left[3 - \frac{4}{3} \right] = \frac{50}{9} = 5.55 \text{ J}$$

36. (4) Here PE = Work done

$$mgH = \mu_{\text{kinetic}} mgS \text{ or } S = \frac{H}{\mu_{\text{kinetic}}}$$

The particle covers the length l or not or covers it repeatedly is determined by the above relation.

37. (2) Since the lift's speed is constant, in the absence of acceleration, there will be no pseudo-force (referring to the lift as the frame of reference) or additional reaction/thrust due to inclined plane (referring to ground as the reference frame) on the particle.

Therefore, friction = $mg \sin \theta$ acting along the plane.

Distance moved by the particle (or lift) in time $t = vt$

Work done in time $t = (mg \sin \theta) vt (\cos 90^\circ - \theta) = mg \sin^2 \theta vt$

38. (3) Given $v = k\sqrt{x}$ or $\frac{dx}{dt} = k\sqrt{x}$ or $x^{-\frac{1}{2}} dx = k dt$

Integrating both sides, we get

$$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = kt + C; \text{ assuming } x(0) = 0$$

Therefore, $C = 0$

$$2\sqrt{x} = kt \Rightarrow x = \frac{k^2 t^2}{4} \text{ or } v = \frac{k^2 t}{2}$$

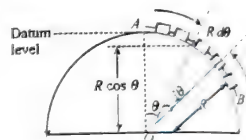
Therefore, work done,

$\Delta W = \text{Increase in KE}$

$$= \frac{1}{2} mv^2 - \frac{1}{2} m(0)^2 = \frac{1}{2} m \left[\frac{k^2 t}{2} \right]^2 = \frac{1}{8} mk^4 t^2$$

39. (4) The mass of elemental length of chain subtending an angle $d\theta$ at

the centre = $\frac{m}{l} (R d\theta)$.



$$\therefore \text{ Its PE} = \frac{m}{l} (R d\theta) g [- (R - R \cos \theta)]$$

The negative sign implies that the elemental length is below the datum level. Therefore,

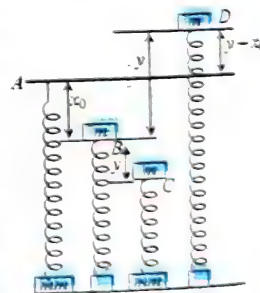
$$dU = \frac{-mgR^2}{l} (1 - \cos \theta) d\theta$$

Therefore, total PE of the chain

$$U = \int_0^{l/R} \frac{-mgR^2}{l} (1 - \cos \theta) d\theta$$

$$\Rightarrow \frac{-mgR^2}{l} [\theta - \sin \theta]_0^{l/R} = \frac{-mgR^2}{l} \left[\frac{l}{R} - \sin \frac{l}{R} \right]$$

40. (2) A is the position of the spring when it is in its normal uncompressed length. The upper disc compresses the spring by x_0 when spring is in equilibrium. So $kx_0 = mg$. Hence, B is the equilibrium position of the spring. Let it be further compressed by y and released. After releasing it can be proved that the spring will go up to the position D so that $BC = BD$. Extension in the spring at this position = $y - x_0$.



Now for lifting up of the lower disc:

$$k(y - x_0) = mg \Rightarrow y = \frac{mg}{k} + x_0$$

$$\Rightarrow y = x_0 + x_0 = 2x_0$$

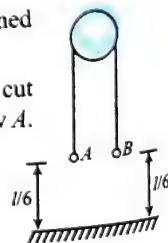
41. (1) Final figure is shown below. It can be obtained as follows.

We can assume that a part of length $l/6$ is cut from the lower portion of side B , and put below A .

$$\text{Mass of this part} = \frac{m}{6}$$

$$\text{This part rises by} = \frac{l}{6}$$

$$\text{Work done} = \text{Increase in PE} = \frac{m}{6l} g \frac{l}{6} = \frac{mgl}{36}$$



48. (3) If A moves down the incline by 1 m, B shall move up by $\frac{1}{2}$ m.

If the speed of B is v , then the speed of A will be $2v$.

From conservation of energy,

Gain in KE = loss in PE

$$\frac{1}{2} m_A (2v)^2 + \frac{1}{2} m_B v^2 = m_A g \times \frac{3}{5} - m_B g \times \frac{1}{2}$$

Solving, we get

$$v = \frac{1}{2} \sqrt{\frac{g}{3}}$$

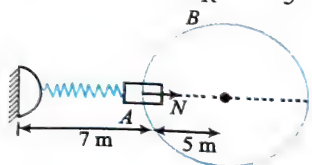
49. (3) Initial extension will be equal to 6 m.

$$\text{Initial energy} = \frac{1}{2} (200)(6)^2 = 3600 \text{ J}$$

$$\text{Reaching } A, \text{ we get } \frac{1}{2} m v^2 = 3600 \text{ J}$$

$$m v^2 = 7200 \text{ J}$$

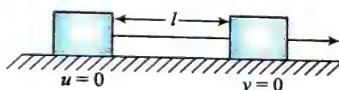
$$\text{From FBD at } A, \text{ we get } N = \frac{m v^2}{R} = \frac{7200}{5} = 1440 \text{ N}$$



- (2) Applying work-energy theorem on the block, we get

$$F l - \frac{1}{2} k l^2 = 0$$

$$l = \frac{2F}{k} \text{ or work done} = F l = \frac{2F^2}{k}$$



- (3) Work done against friction must equal to the initial kinetic energy.

$$\frac{1}{2} m v^2 = \int_1^{\infty} \mu m g dx \Rightarrow \frac{v^2}{2} = \mu g \int_1^{\infty} \frac{1}{x^2} dx$$

$$\frac{v^2}{2} = \mu g \left[-\frac{1}{x} \right]_1^{\infty}$$

$$v^2 = 2gA \Rightarrow v = \sqrt{2gA}$$

- (4) **Statement I:** Work done by gravity is same for motion from A to J and B to M for equal mass. So KE will be equal.

Statement II: Acceleration = $g \sin \theta$

$$\sin \theta_A > \sin \theta_B \Rightarrow \frac{h}{l} > \frac{h}{2l}$$

Statement III: $W_g + W_{\text{ext}} = 0$ (Because the block moves slowly)

$$W_{\text{ext}} = -W_g$$

From B to O : W_g is positive, so $W_{\text{ext}} < 0$.

- (1) Let at any time, the speed of the block along the incline upwards be v .

Then from Newton's second law,

$$\frac{P}{v} - mg \sin \theta - \mu mg \cos \theta = \frac{m dv}{dt}$$

The speed is maximum when $\frac{dv}{dt} = 0$

$$v_{\text{max}} = \frac{P}{mg \sin \theta + \mu mg \cos \theta}$$

48. (3) The spring will exert maximum force when the ball is at its lowest position. If the ball has descended through a distance x to reach the position,

$$mgx = (1/2) Kx^2 \text{ or } x = 2mg/K \quad \dots(i)$$

For the block B to leave contact spring force,

$$Kx = Mg \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), $m = M/2$

49. (2) **Force method:**

$$\text{Form } P \text{ to } Q: v^2 = 0^2 + 2a_1 s_1$$

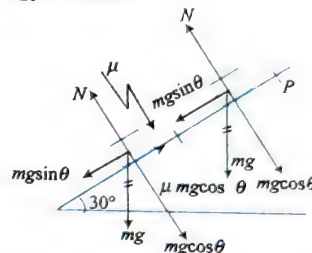
$$\text{where } a_1 = g \sin 30^\circ, s_1 = 1 \text{ m}$$

$$\text{and from } Q \text{ to } R: 0^2 = v^2 + 2a_2 s_2$$

$$\text{where } a_2 = g \sin 30^\circ - \mu g \cos 30^\circ, s_2 = 2 \text{ m}$$

$$\text{solve to get } \mu = \sqrt{3}/2$$

Work-energy method:



In terms of energy considerations you can summarize the whole process as loss in gravitational potential energy of the block = work done against friction, or

$$(mg) \times (3 \sin 30^\circ) = (\mu mg \cos \theta) \times 2$$

$$3 \times 1/2 \mu \times \frac{\sqrt{3}}{2} \times 2 \Rightarrow \mu = \frac{\sqrt{3}}{2}$$

$$50. (2) F_x = -\frac{\partial U}{\partial X} = \sin(x+y) = \frac{1}{\sqrt{2}}$$

$$F_y = -\frac{\partial U}{\partial Y} = \sin(x+y) = \frac{1}{\sqrt{2}}$$

$$\vec{F} = \frac{1}{\sqrt{2}} (\hat{i} + \hat{j})$$

51. (1) As long as the block of mass m remains stationary, the block of mass M released from rest let it comes down by distance x , applying conservation of mechanical energy

$$\Delta K + \Delta U = 0 \Rightarrow 0 + (\Delta U_{\text{gravity}} + \Delta U_{\text{spring}}) = 0$$

$$\left(-Mgx + \frac{1}{2} kx^2 \right) = 0 \Rightarrow x = \frac{2Mg}{k}$$

Thus at the maximum extension in spring is

$$kx = 2Mg \quad \dots(1)$$

For block of mass m to just move up the incline

$$kx = mg \sin \theta + \mu mg \cos \theta \quad \dots(2)$$

$$2Mg = mg \times \frac{3}{5} + \frac{3}{4} mg \times \frac{4}{5}$$

$$\text{or } M = \frac{3}{5} m$$

$$52. (1) \text{ Area under } P-x \text{ graph} = \int p dx = \left(m \frac{dv}{dt} \right) v dx = \int_1^v m v^2 dv$$

$$= \left[\frac{m v^3}{3} \right]_1^v = \frac{10}{7 \times 3} (v^3 - 1)$$

$$\text{From the graph, area} = \frac{1}{2} (2 + 4) \times 10 = 30$$

$$\text{or } \frac{10}{7 \times 3} (v^3 - 1) = 30$$

$$\text{or } v = 4 \text{ ms}^{-1}$$

Alternative method:

From graph

$$P = 0.2x + 2$$

$$mv \frac{dv}{dx} = 0.2x + 2$$

$$mv^2 dv = (0.2x + 2) dx$$

Now integrating both sides, we get

$$\int_1^v mv^2 dv = \int_0^{10} (0.2x + 2) dx$$

$$v = 4 \text{ ms}^{-1}$$

53. (1) From work-energy theorem, for upward motion

$$\frac{1}{2} m (16)^2 = mgh + W \text{ (work done by air resistance)}$$

for downward motion,

$$\frac{1}{2} m (8)^2 = mgh - W \Rightarrow \frac{1}{2} [(16)^2 + (8)^2] = 2gh$$

$$\text{or } h = 8 \text{ m}$$

54. (4) If the particle is released at the origin, it will try to go in the direction of force. Here dU/dx is positive and hence force is negative; as a result, it will move towards negative x -axis.

When the particle is released at $x = 2 + \Delta$, it will reach the point of least possible potential energy (-15 J) where it will have maximum kinetic energy.

$$\frac{1}{2} mv_{\max}^2 = 25$$

$$v_{\max} = 5 \text{ ms}^{-1}$$

The particle will now perform oscillatory motion with $x = 5$ as mean position.

$$\text{In (c), } E_i = u_i + k_i = 15 + 6 = 21 \text{ J}$$

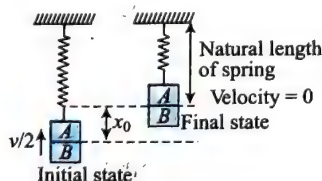
$$\text{At } x = 10, U_f = 20$$

$$K_f = 1 \neq 0$$

So the particle crosses $x = 10$.

55. (2) The initial extension in spring is $x_0 = mg/k$

Just after collision of B with A , the speed of the combined mass is $v/2$.



For the spring to just attain natural length, the combined mass must rise up by $x_0 = mg/k$ and come to rest.

Applying conservation of energy between initial and final states,

$$\frac{1}{2} 2m \left(\frac{v}{2} \right)^2 + \frac{1}{2} k \left(\frac{mg}{k} \right)^2 = 2mg \left(\frac{mg}{k} \right)$$

$$\text{Solving, we get } v = \sqrt{\frac{6mg^2}{k}}$$

56. (4) Power $= Fv \propto v$

$$\Rightarrow mv \frac{dv}{dt} = kv, \text{ where } k = \text{constant}$$

$$\Rightarrow m \frac{dv}{dt} = k \Rightarrow mv \frac{dv}{dx} = k$$

$$\Rightarrow v dv = \frac{k}{m} dx$$

$$\text{Integrating both sides, we get } \frac{v^2}{2} = \frac{k}{m} x$$

Hence, displacement is proportional to the square of instantaneous velocity.

57. (3) $f = kt$

$$m \int_0^v dv = k \int_0^t t dt$$

$$mv = kt^2$$

$$v = \frac{k}{m} t^2$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m \frac{k^2}{m^2} t^4 = \left(\frac{k^2}{2m} \right) t^4$$

$$KE \propto t^4$$

58. (4) Given $\vec{F} = (xy)^2 \hat{i} + (x^2y) \hat{j}$

$$W = \int F_x dx + \int F_y dy$$

$$= \int xy^2 dx + \int x^2 y dy$$

$$= \frac{1}{2} \int d(x^2 y^2) = \left[\frac{x^2 y^2}{2} \right]_{(0,0)}^{(4,0)} = 0$$

59. (4) The z -component of the force and the x -component of displacement are ineffective here.

$$dW = F_y dy = 3xy dy$$

$$(\because z = 0)$$

$$= 6x^4 dx$$

$$(\because y = x^2)$$

Integrating between $x = 0$ and $x = 2$ gives the result.

60. (3) As the particle moves only under the action of conservative force, its mechanical energy must be conserved. So

$$\Delta T + \Delta U = 0 \text{ (T stands for kinetic energy)}$$

$$\text{or } \Delta T = -\Delta U = U_i - U_f$$

$$= \alpha(3^2 + 3^2) - \alpha(1^2 + 1^2) = 16\alpha$$

61. (1) According to the work-energy theorem,

$$\frac{1}{2} mv_0^2 = mg\alpha \int_0^x x dx + \frac{1}{2} kx^2$$

$$\text{Solving we get, } x = v_0 \sqrt{\frac{m}{k + \alpha mg}}$$

62. (2) The speed is maximum when acceleration is zero.

Let displacement of block is x_0 when the speed of block is maximum.

At equilibrium, applying Newton's law to the block along the incline

$$mg \sin \theta = \mu mg \cos \theta + kx_0$$

$$(mg \sin \theta - \mu mg \cos \theta) = kx_0$$

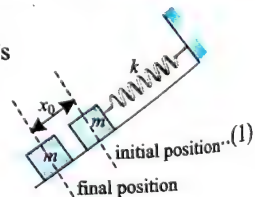
Applying work-energy theorem

to block between initial and

final position, we get

$$(KE)_f = (KE)_i + mgx_0 \sin \theta - \frac{1}{2} kx_0^2 - \mu mg x_0 \cos \theta$$

$$\frac{1}{2} mv_{\max}^2 = 0 + mgx_0 \sin \theta - \frac{1}{2} kx_0^2 - \mu mg x_0 \cos \theta \quad \dots(2)$$



$$\frac{1}{2}mv_{\max}^2 = (mg \sin \theta - \mu mg \cos \theta)x_0 - \frac{1}{2}kx_0^2$$

Solving (1) and (2), we get

$$v_{\max} = (\sin \theta - \mu \cos \theta)g \sqrt{\frac{m}{k}}$$

$$63. (1) F = \frac{k}{r^2}$$

$$U = -\int \vec{F} \cdot d\vec{r} = -\int \vec{F} d\vec{r} \cos \pi$$

$$\int F dr = \frac{k}{r^2} dr = -\frac{k}{r}; \quad V_2 = \text{radial velocity} = \frac{dr}{dt}$$

$$\text{Hence, KE due to this velocity} = \frac{1}{2} \left(\frac{dr}{dt} \right)^2 m$$

$$V_1 = \text{tangential velocity and } mV_1 r = L$$

$$\therefore V_1 = \frac{L}{mr}$$

$$\text{KE due to tangential velocity} = \frac{1}{2}mv_1^2$$

$$\left(\frac{L}{mr} \right)^2 mV_1^2 = \frac{1}{2}m \frac{L^2}{m^2 r^2} = \frac{1}{2} \frac{L^2}{mr^2}$$

$$\text{Total energy} = \text{KE}_{\text{total}} + \text{PE}$$

$$= \frac{1}{2}m \left(\frac{dr}{dt} \right)^2 + \frac{1}{2} \frac{L^2}{mr^2} - \frac{k}{r}$$

64. (3) Since the pendulum started with no kinetic energy, conservation of energy implies that the potential energy at $Q_{\max}$ must be equal to the original potential energy, i.e., the vertical position will be same. Therefore,

$$L \cos \alpha = l + (L-l) \cos \theta$$

$$\Rightarrow \cos \theta = \frac{L \cos \alpha - l}{L - l}$$

$$\Rightarrow \theta = \cos^{-1} \left[\frac{L \cos \alpha - l}{L - l} \right]$$

65. (1) Let the velocity of wedge be v .

$$\text{Loss in PE} = \text{Gain in KE}$$

$$mgR \cos 45^\circ = \frac{1}{2}mv^2 + \frac{1}{2}m(v_1 \cos 45^\circ - v)^2 + \frac{1}{2}m(v_1 \sin 45^\circ)^2$$

$$\text{From conservation of linear momentum}$$

$$m(v_1 \cos 45^\circ - v) = mv$$

Here v_1 is the velocity of ball w.r.t. wedge. Solve to get

$$v = \left(\frac{gR}{3\sqrt{2}} \right)^{1/2}$$

66. (1) Given force is a constant force and work done is path-independent under a constant force. Hence $W_1 = W_2$.

$$67. (1) mg \sin \theta = \frac{3}{5}mg$$

$$f_{\max} = \mu mg \cos \theta = \left(\frac{3}{4} \right) \left(\frac{4}{5} \right) mg = \frac{3}{5}mg$$

$$f_{\max} = mg \sin \theta = \frac{3}{5}mg$$

Block will move upwards when

$$kx_0 = f_{\max} + mg \sin \theta = \frac{6}{5}mg \quad \dots(i)$$

From conservation of mechanical energy

$$Mgx_0 = \frac{1}{2}kx_0^2$$

$$M = \frac{kx_0}{2g} = \frac{\frac{6}{5}mg}{2g} = \frac{3}{5}m$$

68. (1) As the block of mass M , descends down, it loses potential energy, which appears in the form of kinetic energy of the two blocks, besides doing work against friction.

Now, if the block of mass $2M$ moves by a distance s , and its velocity be v , then the distance descended by block of mass M would be $s/2$ and so its velocity would be $v/2$.

$$\text{Now, loss in potential energy of the system} = Mg \left(\frac{s}{2} \right) \quad \dots(ii)$$

$$\text{Work done against friction} = 2M\mu gs \quad \dots(iii)$$

Gain in kinetic energy of the two blocks

$$= \frac{1}{2}(2M)v^2 + \frac{1}{2}M \left(\frac{v}{2} \right)^2 \quad \dots(iii)$$

Law of conservation of energy demands that:

$$Mg \left(\frac{s}{2} \right) = 2M\mu gs + Mv^2 + \frac{Mv^2}{8}$$

$$\frac{gs}{2} = 2\mu gs + \frac{9}{8}v^2$$

$$\frac{9}{8}v^2 = \frac{gs}{2} (1 - 4\mu) \Rightarrow v = \sqrt{\frac{4}{9}gs(1 - 4\mu)}$$

$$\text{Velocity of mass, } M = \frac{v}{2}$$

$$\text{Required velocity} = \frac{1}{3} \sqrt{gs(1 - 4\mu)}$$

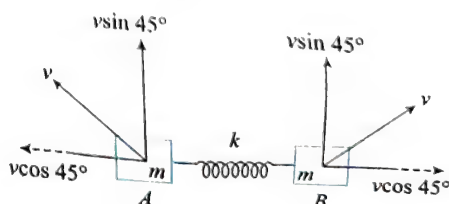
69. (3) As $W_{\text{ext}} = \Delta(\text{ME})$; ME = Mechanical energy. Mechanical energy will keep on increasing up to the instant the W_{ext} is positive, which will happen till there is no compression in the spring.

First the spring gets extended to a maximum and after which the extension decreases up to the natural length. After that there is a compression in the spring, results in a -ve external work (so as to move the end of spring at constant speed u). Hence, maximum energy stored is at the natural length and $\text{ME}_{\max} = (1/2)mv^2$

At the natural length $v = 2u$, since the block is moving at this instant at a speed u with respect to the other end of the spring. Hence, $\text{ME}_{\max} = (1/2)m(2u)^2 = 2mu^2$.

70. (1)

$$\frac{1}{2}kx^2 = 2 \left[\frac{1}{2}m \left(\frac{v}{\sqrt{2}} \right)^2 \right] \Rightarrow \frac{1}{2}kx^2 = \frac{mv^2}{2}$$



$$\Rightarrow x = v \sqrt{\frac{m}{k}} \quad [\text{The velocities along } y \text{ axis do not change}]$$

71. (2) $mv \frac{dv}{dx} = (ma - kx)$

$$\int_0^0 mv dv = \int_0^x (ma - kx) dx$$

$$0 = max - \frac{kx^2}{2} \Rightarrow x = \frac{2ma}{k}$$



72. (1) The friction force and external force will do work on the block

$$W_{\text{total}} = W_f + W_F = -\mu mgx + \int_0^x (a - bx^2) dx$$

As total work is equal to zero;

$$0 = (-\mu mg + a)x - \frac{bx^3}{3} \Rightarrow x = \sqrt{\frac{3(a - \mu mg)}{b}}$$

73. (1) From given graphs:

$$\vec{F} = \left(\frac{3}{4}x + 10\right)\hat{i} + \left(20 - \frac{4}{3}y\right)\hat{j} + \left(\frac{4}{3}z - 16\right)\hat{k}$$

$$W = \int \vec{F} \cdot d\vec{s}$$

$$= \int_{(0,5,12)}^{(4,20,0)} \left[\left(\frac{3}{4}x + 10\right)\hat{i} + \left(20 - \frac{4}{3}y\right)\hat{j} + \left(\frac{4}{3}z - 16\right)\hat{k} \right] \times [dx\hat{i} + dy\hat{j} + dz\hat{k}]$$

$$= 192 \text{ J}$$

74. (2) $W_{\text{spring}} + W_{100\text{N}} = \Delta k \text{ (on A)}$

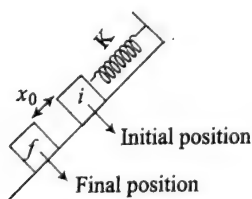
$$W_{\text{spring}} + (100) \left(\frac{10}{100}\right) = \frac{1}{2}(2)(2)^2$$

$$W_{\text{spring}} = 4 - 10 = -6 \text{ J}$$

75. (4) The speed is maximum when acceleration is least.

Let displacement of block is x_0 when the speed of block is maximum.

At equilibrium, applying Newton's law to the block along the incline



$$mg \sin \theta = \mu mg \cos \theta + kx_0 \quad \dots(1)$$

Applying work energy theorem to block between initial and final position is

$$k_f = k_i + mgx_0 \sin \theta - \frac{1}{2}kx_0^2 - \mu mgx_0 \cos \theta \quad \dots(2)$$

Solving (1) and (2), we get

$$V_{\text{max}} = (\sin \theta - \mu \cos \theta) g \sqrt{\frac{m}{k}}$$

Multiple Correct Answers Type

1. (1, 2)

The change in momentum for each time interval t_0 is $\Delta p = Ft_0$ which is constant.

But the change in kinetic energy in each time interval t_0 is different.

$$\text{First } t_0 \text{ interval: } \Delta K_1 = \frac{\Delta p^2}{2m} = \frac{F^2 t_0^2}{2m}$$

$$\text{Second } t_0 \text{ interval: } \Delta K_2 = \frac{(2\Delta p)^2 - (\Delta p)^2}{2m} = \frac{3F^2 t_0^2}{2m}$$

The change in kinetic energy is $\Delta K = Fx_0 = \text{constant}$.

$$\text{First } x_0 \text{ displacement: } \Delta p_1 = \sqrt{2m \Delta K_1} = \sqrt{2m(Fx_0)}$$

Second x_0 displacement:

$$\Delta p_2 = \sqrt{2m(2Fx_0)} - \sqrt{2m(Fx_0)} = 0.414 \sqrt{2m(Fx_0)}$$

2. (2, 4)

The statement given in option (2) is a very standard concept related to the work-energy theorem. Work-energy theorem can be applied to any system.

Work-energy theorem can be applied to non-inertial frames of reference also, provided the work done by inertial forces is also considered on right-hand side of the work-energy theorem.

3. (2, 3)

Consider a system of two point particles which are free to move on a smooth horizontal surface, then the work done by electrical forces acting on the two particles is non-zero. But it is not necessarily true always. For a rigid body, there is no relative displacement of particles, hence, internal forces do not do any work.

4. (2, 3, 4)

Single force on a particle may change the magnitude and or direction of velocity, so momentum changes necessarily but kinetic energy may or may not change.

From work-energy theorem,

$$\Delta K = W_{\text{ext}} + W_{\text{int}} + W_{\text{conservative}} + W_{\text{NC}}$$

Kinetic energy can be increased due to work done by internal forces also.

5. (1, 2, 4, 5)

Initially, the spring has an elongation, $\Delta L = mg/k$, where m is the mass of hanging block and k is the force constant of the spring. It means, initially spring has some strain energy.

When the boy pushes the block upwards, the potential energy of the block increases and work is done by the boy during lifting of the block but spring becomes strain free. It means, initially, strain energy stored in spring is lost. Hence, the work done by the boy will be equal to increase in gravitational potential energy of the block minus loss in strain energy stored in the spring. Hence, only (3) is correct, i.e., all other options (1), (2), (4) and (5) are not correct.

6. (1, 2, 3)

In options (1) and (3), no work is done because the force of gravity in option (1) and the centripetal force in option (3) are perpendicular to the direction of displacement. In option (2), no net force acts on the raindrop since it is falling with a uniform velocity, hence no work is done. In (4), the mass has to do work against gravity. Hence, the correct answers are (1), (2) and (3).

7. (1, 2, 3)

According to the work-energy theorem for a non-conservative system, we have $W_c + W_{nc} = \Delta K$

Also, we know that the work done by a conservative force equals the decrease in potential energy, so $W_c = -\Delta U$

$$W_{nc} - \Delta U = \Delta K$$

8. (1, 3)

Since, force on the vehicle is constant, it will move with a constant acceleration.

Let this acceleration be a .

Then at time t , its velocity will be equal to $v = at$.

Hence, at time t , the kinetic energy,

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} ma^2 t^2$$

It means, graph between E and t will be a parabola of increasing slope. Hence, option (3) is correct.

The power associated with the force is equal to

$$P = Fv = Fat.$$

Hence, the graph between power and time will be a straight line passing through the origin. Hence, option (1) is also correct.

9. (1, 3)

If the body slips over a rough surface such that normal reaction of the surface has to balance only the normal component of weight of the body, then the energy lost against friction depends only upon the horizontal component of displacement and is equal to μmgx . It does not depend upon the shape of the surface.

If the vertical component of displacement of the body is equal to y , increase in its gravitational potential energy will be equal to $mg y$.

Hence, total work done by the force will be equal to $\mu mgx + mg y$.

Hence, options (1) and (3) are correct.

10. (1, 2, 3, 4)

At any time t , $v = \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$

$$E = \frac{1}{2} mv^2 = \frac{1}{2} m(u^2 + g^2 t^2 - 2ugt \sin \theta)$$

Hence, E - T graph is parabolic $E = \frac{p^2}{2m}$

Hence, E - p^2 graph is straight line through origin

$$E = \frac{1}{2} mu^2 - mg y$$

Putting $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$, we get

$$E = \frac{1}{2} mu^2 - mg x \tan \theta + \frac{mx^2 x^2}{2u^2 \cos^2 \theta}$$

Hence E - y graph is a straight line and E - x graph is parabolic.

11. (2, 3)

Let the mass of the block hanging from the spring be m . Then initial elongation of the spring will be equal to mg/k . When the force F is applied to pull the block down, the work done by F and further loss of gravitational potential energy of the block is used to increase strain energy of this spring.

$$(F\delta + mg\delta) = \left[\frac{1}{2} k \left(\frac{mg}{k} + \delta \right)^2 - \frac{m^2 g^2}{2k} \right] \quad \dots(i)$$

From Eq. (i), $\delta = 2F/k$

Hence, option (2) is correct.

Since a constant force is applied on the block to pull it down, during this process, work done by force, $W = F\delta$. Hence, option (3) is also correct.

Increase in energy of the spring is equal to

$$\left[\frac{1}{2} k \left(\frac{mg}{k} + \delta \right)^2 - \frac{m^2 g^2}{2k} \right]$$

From this equation, increase in energy is not equal to $\frac{1}{2} k\delta^2$.

Hence, option (4) is wrong.

12. (1, 2, 4)

When the plank is shifted, the cord elongates and becomes inclined with vertical. Therefore, a tension is developed in the cord and that tension has two components, a vertical component and a horizontal component: Horizontal component tries to slide the block leftwards, relative to the plank. But friction F_2 prevents that slipping. Hence, the block moves to the right with the plank.

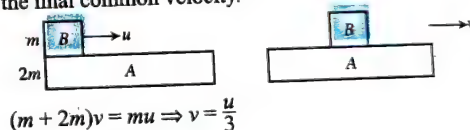
Since the block does not slip over the plank, no energy is lost against friction F_2 . Hence, work done by the force F is used to overcome loss against friction F_1 and to elongate the cord. Therefore, option (1) is correct.

The cord elongates due to displacement of the block and the block gets displaced due to friction F_2 acting on it. Hence, the friction F_2 is responsible for the elongation of the cord. Therefore, strain energy stored in the cord is equal to work done by the friction on the block or work done against friction acting on the upper surface of the plank. Hence, the total work done by F is equal to energy lost against friction F_1 plus work done against friction acting on upper surface of the plank. Therefore, option (2) is correct. Hence, it is obvious that option (3) is wrong.

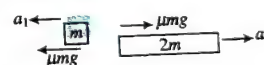
Since, strain energy stored in the cord is equal to work done by friction acting on the block, option (4) becomes same as option (2). Hence, it is also correct.

13. (2, 4)

v is the final common velocity.



$$(m + 2m)v = mu \Rightarrow v = \frac{u}{3}$$



$$a_1 = \frac{\mu mg}{m} = \mu g, a_2 = \frac{\mu mg}{2m} = \frac{\mu g}{2}$$

Acceleration of B relative to A:

$$a_1 + a_2 = \mu g + \frac{\mu g}{2} = \frac{3}{2} \mu g \text{ towards left}$$

Work done against friction:

$$W_1 = K_i - K_f = \frac{1}{2} mu^2 - \frac{1}{2} 3mv^2 = \frac{mu^2}{3}$$

$$\text{Final KE} = \frac{1}{2} 3mv^2 = \frac{mu^2}{6}$$

Hence, (3) is not correct.

Option (4) is correct, because momentum is always conserved as there is no external force.

14. (1, 2, 3, 4)

$$\Delta KE = \frac{1}{2} mv^2 = \frac{1}{2} \times 1 \times 2^2 = 2 \text{ J}$$

$$W_{\text{cons}} = -\Delta U = -5 \text{ J} \Rightarrow \Delta U = 5 \text{ J}$$

$$W_{\text{ext}} = \Delta U + \Delta KE = 5 + 2 = 7 \text{ J}$$

15. (1, 3, 4)

Option (1) is correct because at 3, force is opposite to displacement. At b , u is minimum.

Option (2) is incorrect because at 2, there is net force and hence no equilibrium.

Option (3) is correct because at 1, 3 and b force is zero.

At b , $\frac{du}{dx} = -F = 0$

Option (4) is correct. Positive work is done after position 1. Hence, KE will increase. So KE is least at 1.

16. (1, 2, 3, 4)

Since potential energy of the particle is equal to ϕ , x -component of the force acting on the particle is equal to

$$F_x = -\frac{d\phi}{dx}$$

and y -component of the force on the particle is equal to

$$F_y = -\frac{d\phi}{dy}$$

Hence, force acting on the particle is $\vec{F} = -(3\hat{i} + 4\hat{j})\text{N}$

It means, force acting on the particle is constant. Hence, the particle moves with constant acceleration. So option (1) is correct.

Acceleration of the particle, $\vec{a} = \frac{\vec{F}}{m_2} = -(3\hat{i} + 4\hat{j})\text{ms}^{-1}$

Its magnitude is $a = \sqrt{3^2 + 4^2} = 5\text{ms}^{-2}$

Since, the particle was initially at rest at (6, 4), position of the particle at time t is given by

$$x = 6 + \frac{1}{2}a_x t^2 = \left(6 - \frac{3}{2}t^2\right)\text{m}$$

and $y = 4 + \frac{1}{2}a_y t^2 = (4 - 2t^2)\text{m}$

when the particle crosses the x -axis, $y = 0$,

$$t_1 = \sqrt{2}\text{s}$$

Displacement of the particle during this time,

$$s_1 = \frac{1}{2}at^2 = 5\text{m}$$

Hence, work done by the force, up to this instant

$$Fs_1 = 5 \times 5\text{J} = 25\text{J}$$

Hence, option (2) is also correct.

The particle crosses y -axis when $x = 0$.

Hence, $6 - \frac{3}{2}t_2^2 = 0$ or $t_2 = 2\text{s}$

Speed of the particle at this instant will be

$$v = at_2 = 5 \times 2 = 10\text{ms}^{-1}$$

Hence, option (3) is also correct.

At $t = 4\text{s}$, $x = 6 - \frac{3}{2}(4)^2 = -18\text{m}$

and $y = 4 - 2(4)^2 = -28\text{m}$

Hence, option (4) is also correct.

17. (1, 2)

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx^2$$

$$v_{\max} = \sqrt{\frac{k}{m}} x = \sqrt{\frac{400}{1}} \times 1 = 2\text{ms}^{-1}$$

As the maximum speed is 2 m/s, hence the range of speed should be between 0 and 2 m/s. Hence, options (1) and (2) are correct.

18. (1, 2)

Work done from $x = 0$ to $x = 2\text{m}$,

$$W = (5 + 10) \times \frac{2}{2} = 15\text{J}$$

Work done from $x = 4$ to $x = 6\text{m}$,

$$W = -10 \times \frac{2}{2} = -10\text{J}$$

Work done from $x = 0$ to $x = 6\text{m}$,

$$W = 15 - 10 + 10 = 15\text{J} = \Delta\text{KE}$$

19. (1, 2, 3)

$$W = \int F_x dx + \int F_y dy = \int x^2 dx + \int y^2 dy = \frac{x^3}{3} + \frac{y^3}{3}$$

$$W_{OA} = \left[\frac{x^3}{3}\right]_0^a + \left[\frac{y^3}{3}\right]_0^0 = \frac{a^3}{3}$$

$$W_{AB} = \left[\frac{x^3}{3}\right]_a^a + \left[\frac{y^3}{3}\right]_0^a = \frac{a^3}{3}$$

$$W_{BC} = \left[\frac{x^3}{3}\right]_a^0 + \left[\frac{y^3}{3}\right]_a^a = -\frac{a^3}{3}$$

$$W_{CO} = \left[\frac{x^3}{3}\right]_0^0 + \left[\frac{y^3}{3}\right]_a^0 = -\frac{a^3}{3}$$

on the entire closed path:

$$W = W_{OA} + W_{AB} + W_{BC} + W_{CD} = 0$$

20. (3, 4)

Work done by $\vec{F}_2$ can be positive, zero or negative depending upon the displacement of particle. Hence, (1) and (2) are wrong.

For (3) and (4):

$$W_{F_1} + W_{F_2} + \dots + W_{F_n} = \Delta\text{KE} = 0 \quad \dots(i)$$

$$\Rightarrow W_{F_2} = -(W_{F_1} + W_{F_3} + \dots + W_{F_n})$$

Hence (3) is correct.

If F_2 is conservative force, $W_{F_2} = -\Delta U$

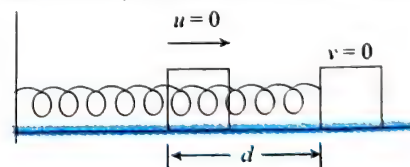
$$\Rightarrow W_{F_1} + W_{F_3} + \dots + W_{F_n} = \Delta U$$

Hence, (4) is correct.

21. (1, 3)

For the system to lose entire mechanical energy, block should come at rest, at the position when there is no deformation in spring.

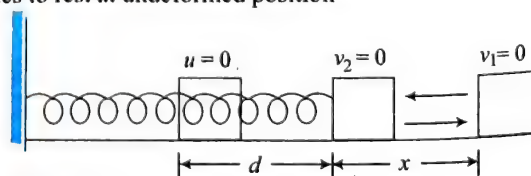
Case (i): Block directly comes to rest at undeformed position



$$\frac{1}{2}kd^2 = \mu mgd$$

$$\Rightarrow d = \frac{2\mu mg}{k} = \frac{2 \times 0.4 \times 1 \times 10}{200} = 4\text{cm}$$

Case (ii): Block goes beyond undeformed position once and then comes to rest at undeformed position



From (2) to (3)

$$\frac{1}{2}kx^2 = \mu mgx \Rightarrow x = 4 \text{ cm}$$

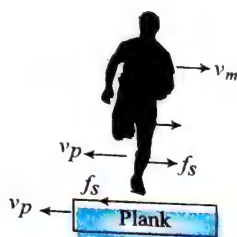
From (1) to (3)

$$\mu mg(d+x) = \frac{1}{2}k(d^2 - x^2)$$

Solve to get $d = 8 \text{ cm}$.

Similarly for the next case, we will get $d = 12 \text{ cm}$ and so on.

22. (1, 3, 4)



At contact point of man and plank, the relative motion is zero. As plank moves backward, man's foot also moves backwards, but friction on man acts forward. Hence work done by friction on man is negative. But with respect to plank, foot will be at rest, hence no work done by friction on man w.r.t. plank.

23. (1, 3)

$$\text{For (1): } mgy = \frac{1}{2}ky^2 \Rightarrow y = \frac{2mg}{k}$$

Hence, (1) is correct, (2) is wrong.

$$\text{For (3): } y_{eq} = \frac{mg}{k}$$

Let velocity on reaching y_{eq} is v when there was no air resistance.

Then

$$mgy_{eq} = \frac{1}{2}ky_{eq}^2 + \frac{1}{2}mv^2$$

$$\Rightarrow mg\left(\frac{mg}{k}\right) = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{m^2g^2}{2k}$$

Now this KE will be lost when finally mass settles down at y_{eq} .

Hence, (3) is correct, (4) is wrong.

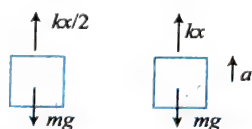
24. (2, 3, 4)

Apply conservation of energy:

loss in PE of ball = gain in PE of spring

$$mgx = \frac{1}{2}kx^2 \Rightarrow x = 2mg/k$$

At position $x/2$:



$kx/2 = mg$. So net force is zero. Hence, no acceleration at lowest position: $kx - mg = ma \Rightarrow a = g$

25. (1, 2, 3)

The work done by friction on the block is equal to change in KE

$$W_f = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{mv_0}{m+M}\right) - \frac{1}{2}mv_0^2 = -ve$$

So, choice (1) is correct.

The work done by friction on the plank

$$W_2 = \frac{1}{2}mv^2 - 0 = +ve$$

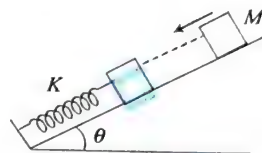
So choice (2) is correct.

Net work done by friction

$$W = W_1 + W_2 = -ve$$

So choice (3) is correct.

26. (1, 2, 3)



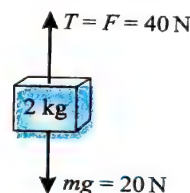
(1) Gravitational potential energy

$$U = mgh = mg(-x \sin \theta) \left[\sin \theta = \frac{h}{x}\right] = -mgx \sin \theta$$

(2) Elastic potential energy = $\frac{1}{2}kx^2$

(3) Total potential energy $\frac{1}{2}kx^2 - mgx \sin \theta$

27. (1, 2, 4)



Free body diagram of block is as shown in figure.

From work-energy theorem

$$W_{net} = \Delta KE \text{ or } (40 - 20)s = 40$$

or $s = 2 \text{ m}$

Work done by gravity is $-20 \times 2 = -40 \text{ J}$

and work done by tension is $40 \times 2 = 80 \text{ J}$

28. (1, 2, 3)

Maximum extension will be at the moment when both masses stop momentarily after going down. Applying W-E theorem from starting to that instant,

$$k_f - k_i = W_{gr.} + W_{sp} + W_{ten}$$

$$0 - 0 = 2Mgx + \left(-\frac{1}{2}Kx^2\right) + 0$$

$$x = \frac{4Mg}{K}$$

System will have maximum KE when net force on the system becomes zero.

$$\therefore 2Mg = T \text{ and } T = kx$$

$$\Rightarrow x = \frac{2Mg}{K}$$

Hence, KE will be maximum when $2M$ mass has gone down by $2Mg/K$.

Applying W/E theorem,

$$k_f - 0 = 2Mg \frac{2Mg}{K} - \frac{1}{2}K \cdot \frac{4M^2g^2}{K^2}$$

$$k_f = \frac{2M^2g^2}{K^2}$$

Maximum energy of spring

$$\frac{1}{2} K \cdot \left(\frac{4Mg}{K} \right)^2 = \frac{8M^2 g^2}{K}$$

Therefore maximum spring energy = $4 \times$ maximum KE

When KE is maximum $x = \frac{2Mg}{K}$.

Spring energy = $\frac{1}{2} \cdot K \cdot \frac{4M^2 g^2}{K^2} = \frac{2M^2 g^2}{K^2}$ i.e. (4) is wrong.

29. (2, 4)

At point 'C', the potential energy is minimum, hence it is a point of stable equilibrium.

Also, from E to F, the slope is negative i.e., $\frac{dU}{dr} < 0$

Hence, the force of interaction between the particles is repulsive between points E and F.

30. (1, 2, 3)

If the particle is released at the origin, it will try to go in the direction of force. Here $\frac{du}{dx}$ is positive and hence force is negative, as a result it will move towards -ve x-axis.

When the particle is released at $x = 2 + \Delta$; it will reach the point of least possible potential energy (-15 J) where it will have maximum kinetic energy.

$$\therefore \frac{1}{2} m v_{\max}^2 = 25 \Rightarrow v_{\max} = 5 \text{ m/s}$$

The particle will now perform oscillatory motion between -15 J $\leq U \leq 15$ J, because reaching $U = +15$ J, the kinetic energy and hence speed becomes zero.

$$-15 \text{ J} \leq U \leq 15 \text{ J}, U = +15 \text{ J},$$

$$\text{In (C); } E_i = U_i + K_i = 15 + 6 = 21 \text{ J}$$

$$\text{At } x = 10 \text{ m; } U_f = 20 \text{ J} \Rightarrow K_f = 1 \text{ J} \neq 0 \Rightarrow x = 10 \text{ m}$$

$\Rightarrow$ The particle cross $x = 10 \text{ m}$.

Linked Comprehension Type

For Problems 1-4

1. (4) At $x = 5 \text{ m}$, $U = 20 + (5-2)^2 = 29 \text{ J}$, $K = 20 \text{ J}$

$$\text{Mechanical energy} = E = U + K = 29 + 20 = 49 \text{ J}$$

2. (1) The greatest or least value of x will be at that point where K is zero and U is maximum.

$$U_{\max} = 49 \text{ J} \Rightarrow 20 + (x-2)^2 = 49 \Rightarrow (x-2)^2 = 29 \\ \Rightarrow x-2 = \pm\sqrt{29} \Rightarrow x = -3.38 \text{ m}, 7.38 \text{ m}$$

3. (4) KE is maximum, where U is minimum and

$$U_{\min} = 20 \text{ J at } x = 2 \text{ m,}$$

$$K_{\max} = E - U_{\min} = 49 - 20 = 29 \text{ J}$$

$$4. (3) F(x) = -\frac{dU}{dx} = -2(x-2) = 2(2-x)$$

For Problems 5 and 6

5. (3) From work-energy theorem, kinetic energy of the block at $x = x$ is

$$K = \int_0^x 4 - x^2 dx = 4x - \frac{x^3}{3}$$

For K to be maximum, $\frac{dK}{dx} = 0$

$$\text{or } 4 - x^2 = 0 \text{ or } x = \pm 2 \text{ m}$$

At $x = +2 \text{ m}$, $\frac{d^2 K}{dx^2}$ is negative, i.e., kinetic energy K is maximum and

$$K_{\max} = 4 \times 2 - \frac{(2)^3}{3} \cong 5.33 \text{ J}$$

6. (1) KE of the particle at $x = x$ is $K = 4x - \frac{x^3}{3}$

At maximum displacement, velocity will be zero or

$$K = 0, \text{ i.e., } x = 2\sqrt{3} \text{ m}$$

For Problems 7-10

7. (2), 8. (2), 9. (2), 10. (1)

$$a = \frac{F_{\max} - mg \sin \theta}{m} \\ = \frac{\mu mg \cos \theta - mg \sin \theta}{m} = \frac{g}{3}$$

$$AB = vt + \frac{1}{2} at^2$$

$$\frac{h}{\sin 30^\circ} = \sqrt{\frac{gh}{6}} t + \frac{1}{2} \frac{g}{3} t^2$$

$$gt^2 + \sqrt{6gh} t - 12h = 0$$

$$\text{Solving, we get } t = \sqrt{6 \frac{h}{g}}$$

$$v_f = v_i + at = \sqrt{\frac{gh}{6}} + \frac{g}{3} \sqrt{6 \frac{h}{g}} = 3 \sqrt{\frac{gh}{6}}$$

$$\Delta KE = \frac{1}{2} m (v_f^2 - v_i^2) = \frac{2}{3} mgh$$

Distance moved by conveyor belt till the boy reaches B:

$$s = v_i t = \sqrt{\frac{gh}{6}} \sqrt{\frac{6h}{g}} = h$$

w.r.t. conveyor belt, distance moved by the boy,

$$2h - h = h$$

Work done by gravity w.r.t. conveyor,

$$W_{mg} = -mgh \sin 30^\circ = -\frac{mgh}{2}$$

From the frame of belt: $v_i = 0$, $v_f = at = 2 \sqrt{\frac{gh}{6}}$

$$W_{\text{boy}} = \Delta KE + \Delta PE$$

$$= \frac{1}{2} m (v_f^2 - v_i^2) + mg \frac{h}{2}$$

$$= \frac{1}{2} m \left[4 \frac{gh}{6} \right] + \frac{mgh}{2} = \frac{5mgh}{6}$$

From the frame of ground:

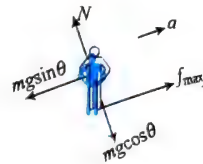
$$W_{\text{boy}} + W_f + W_{mg} = \Delta KE$$

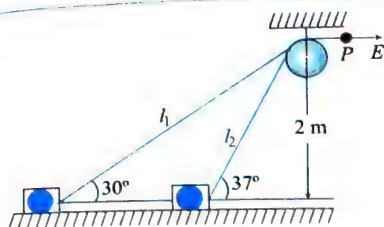
$$W_f = \frac{2}{3} mgh + mgh - \frac{5}{6} mgh = \frac{5}{6} mgh$$

For Problems 11-13

$$11. (2) l_1 = \frac{2}{\sin 30^\circ} = 4 \text{ m}, l_2 = \frac{2}{\sin 37^\circ} = \frac{10}{3} \text{ m}$$

$$\text{Displacement of point P: } l_1 - l_2 = 4 - \frac{10}{3} = \frac{2}{3} \text{ m}$$





Work done, $W = F(l_1 - l_2) = 50 \times \frac{2}{3} = \frac{100}{3} \text{ J}$

12. (3) $W = \frac{1}{2} m (v^2 - u^2)$, $u = 0$

$$\Rightarrow \frac{100}{3} = \frac{1}{2} \cdot 10 v^2 \Rightarrow v^2 = \frac{20}{3} \Rightarrow v = \sqrt{\frac{20}{3}} \text{ ms}^{-1}$$

13. (4) Initial acceleration = $\frac{F \cos 30^\circ}{m}$

Final acceleration = $\frac{F \cos 37^\circ}{m}$

Ratio = $\frac{\cos 30^\circ}{\cos 37^\circ} = \frac{\sqrt{3} \times 5}{2 \times 4} = \frac{5\sqrt{3}}{8}$

For Problems 14–16

14. (3) Applying work–energy theorem on the block for any compression x ,

$$W_{\text{ext}} + W_g + W_{\text{spring}} = \Delta \text{KE}$$

$$Fx + 0 - \frac{1}{2} Kx^2 = \frac{1}{2} mv^2$$

KE versus x graph is an inverted parabola.

15. (1) $W_{\text{ext}} = Fx$

16. (3) From beginning to end of motion: $\Delta \text{KE} = 0$

$$x = \frac{2F}{K} \text{ (from work–energy theorem)}$$

First half corresponds to $0 \leq x \leq \left(\frac{F}{K}\right)$ Work done by force during first half:

$$W_1 = F \frac{x}{2} = \frac{F^2}{K}$$

Energy in spring, $U_1 = \frac{1}{2} K \left(\frac{x}{2}\right)^2 = \frac{F^2}{2K}$

For Problems 17–19

17. (1) The concept used in solving these questions is that kinetic energy can never be negative. It would always be non-negative.

Total mechanical energy = $\text{KE} + U$

Here if $E = 25 \text{ J}$, then $K = 25 - U$

For K to be non-negative, $U < 25 \text{ J}$, which is the case, for $-\infty < x < -5$ and $5 < x < \infty$.

18. (4) The concept is same as above.

If $E = -40 \text{ J}$, $K = -40 - U$.

It means $U < -40$ for K to be positive, but from given variations, it is clear that minimum value of U is -35 J . So this is not possible.

19. (4) Here $E = 60 \text{ J}$ and as the system is isolated, the total mechanical energy remains conserved. It is clear from the given variation that U varies from -35 J to 50 J . So, $K = E - U$ will be from 10 J to 95 J .

For Problems 20–25

20. (3) For path OMA :

Path OM : $\vec{F} = y^2 \hat{i}$, $dx = 0$

$$W_{OM} = \int_0^6 \vec{F} \cdot d\vec{x} = 0$$

Path MA : $\vec{F} = (36\hat{i} + x\hat{j}) \text{ N}$

$$W_{MA} = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy, dy = 0$$

$$= 36 \times 6 + 0 = 216 \text{ J}$$

$$W_{OMA} = 216 \text{ J}$$

For path OLA :

Path OL : $\vec{F} = \hat{j}$, $dy = 0$

$$W_{OL} = 0$$

Path LA : $\vec{F} = (y^2\hat{i} + 6\hat{j}) \text{ N}$

$$W_{LA} = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy$$

$$= 0 + 6 \times (y_2 - y_1) = 6 \times 6 = 36 \text{ J}$$

$$\therefore W_{OMA} \neq W_{OLA}$$

Hence, force is not conservative.

21. (2)

22. (2) $W_{OA} = \int F_x dx + \int F_y dy$

$$= \int_0^6 y^2 dx + \int_0^6 x dy; \text{ for this path: } y = x$$

$$= \int_0^6 x^2 dx + \int_0^6 y dy = \left[\frac{x^3}{3} \right]_0^6 + \left[\frac{y^2}{2} \right]_0^6 = 90 \text{ J}$$

23. (4) Here $W_{OMA} = W_{OLA}$

Hence, force is conservative.

24. (1) $a = \frac{(4\hat{i} + 3\hat{j})}{2}$

$$v^2 = u^2 + 2ax$$

$$= 4 \times 6 + 2 \frac{(4\hat{i} + 3\hat{j})}{2} \cdot (4\hat{i} - 8\hat{j}) = 24 - 8 = 16$$

$$v = 4 \text{ ms}^{-1}$$

25. (3) In conservative force, $W = -\Delta U = -(U_f - U_i)$

$$-8 = -(U_f - 16)$$

$$\Rightarrow U_f = 24 \text{ J}$$

For Problems 26 and 27

26. (3)

(a) The magnitude of the work done by friction is the kinetic energy of the package at point B, or $\mu_k mgL = \frac{1}{2} mv_B^2$.

$$\text{or } \mu_k = \frac{(1/2)v_B^2}{gL} = \frac{(1/2)(5 \text{ m/s})^2}{(10 \text{ m/s}^2)(2.5 \text{ m})} = 0.50$$

27. (2) $W_{\text{other}} = K_B - U_A$

$$= \frac{1}{2}(0.200 \text{ kg})(5 \text{ m/s})^2 - (0.200 \text{ kg})(10 \text{ m/s}^2)(1.60 \text{ m})$$

$$= -1 \text{ J}$$

Equivalently, since

$$K_A = K_B = 0, U_A + W_{AB} + W_{BC} = 0,$$

or $W_{AB} = -U_A - W_{BC} = mg(-1.5 \text{ m}) - (0.300)(-3.00 \text{ m})$

$$= -0.832 \text{ J}$$

For Problems 28–32

28. (4) Since $kx^2/2 = mv^2/2$

$$\Rightarrow v = x\sqrt{(k/m)} = 10 \text{ m/s}$$

29. (4) $kx^2/2 = mv^2/2 + \mu mgx$

$$\Rightarrow v = \sqrt{100 - 4x}$$

30. (2) $v = \sqrt{100 - 4x} = 0 \Rightarrow x = 25 \text{ m}$

31. (3) Using $v = u + at$

$$\Rightarrow v = 10 - \mu gt = 10 - 2t$$

32. (4) $v = 10 - 2t = 0 \Rightarrow t = 5 \text{ s}$

For Problems 33 and 34

33. (1), 34. (4)

$$\left(\frac{1}{2}M + m\right)v_0^2 = \frac{1}{2}kx_m^2$$

$$kx_m = \mu_s g$$

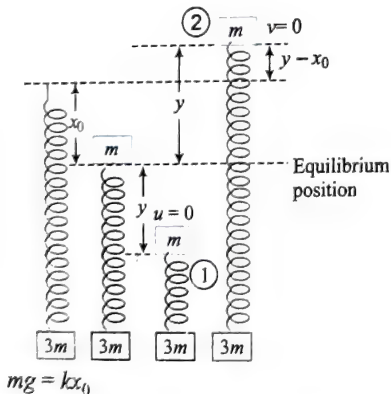
$$x_m = \mu_s/k$$

$$(m + M)v_0^2 = \frac{k\mu_s^2 g^2}{k}$$

$$k = \left(\frac{\mu_s g}{v_0}\right)^2 (M + m)$$

For Problems 35–37

35. (3)



For the lower block to lose contact with ground, $k(y - x_0) = 3mg$

$$\Rightarrow y = \frac{3mg}{k} + x_0 = \frac{4mg}{k}$$

36. (1) For minimum normal reaction on $3m$ from ground to be mg , maximum tension in the string should be $2mg$.

$$k(y - x_0) = 2mg \Rightarrow y = \frac{2mg}{k} + x_0 = \frac{3mg}{k}$$

37. (4)

Here, $y = \frac{5mg}{k}$ at position (2), normal reaction will be mg , but in this case velocity will not be zero at position (2). Let velocity at position (2) is v .

$$(KE + PE)_1 = (KE + PE)_2$$

Taking normal length of spring as reference level,

$$0 - mg\left(x_0 + \frac{5mg}{k}\right) + \frac{1}{2}k\left(x_0 + \frac{5mg}{k}\right)^2$$

$$= \frac{1}{2}mv^2 + mg\left[\frac{2mg}{k}\right] + \frac{1}{2}\left(\frac{2mg}{k}\right)^2$$

$$\Rightarrow -\frac{6m^2g^2}{k} + \frac{18m^2g^2}{k} = \frac{1}{2}mv^2 + \frac{4m^2g^2}{k}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{8m^2g^2}{k} \Rightarrow v = 4g\sqrt{\frac{m}{k}}$$

Matrix Match Type

1. i $\rightarrow$ b; ii $\rightarrow$ c; iii $\rightarrow$ a; iv $\rightarrow$ c

As the force and direction of movement is in the same direction, so work done by lifting force is positive.

Due to constant velocity total work must be zero.

As gravity acts downwards, so work done by it is negative.

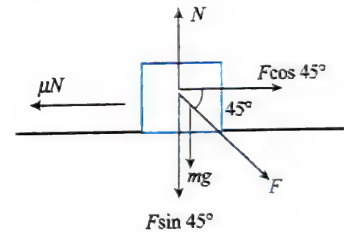
Here we have only two forces, lifting force and gravity, which are operational. As the object is moving with constant velocity, net work done by these forces has to be zero.

2. i $\rightarrow$ a; ii $\rightarrow$ b, c, d; iii $\rightarrow$ b, c, d; iv $\rightarrow$ c

In OA, velocity increases, hence KE increases. So overall work done is positive. If a number of forces are acting on the body, then it is possible that some forces do positive work, some negative and some zero.

In BC, velocity is constant, hence there is no change in KE. So net work done by all the forces is zero.

3. i $\rightarrow$ b; ii $\rightarrow$ a; iii $\rightarrow$ c; iv $\rightarrow$ a



$$F \cos 45^\circ = \mu mg + \mu F \sin 45^\circ$$

$$F = 75\sqrt{2} \text{ N}$$

Surface is exerting friction and normal force, but work is done by friction only

$$= -75\sqrt{2} \times \frac{1}{\sqrt{2}} \times 20$$

$$= -1500 \text{ J}$$

Work done by gravity is zero as it is perpendicular to the direction of motion.

Work done by man on the block in pushing it through 10 m is

$$w = 75\sqrt{2} \times \frac{1}{\sqrt{2}} \times 10 = 750 \text{ J}$$

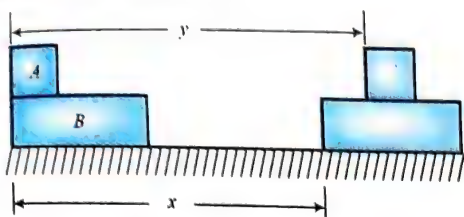
As the block is moving with constant velocity, net force on the block must be zero.

4. i $\rightarrow$ a; ii $\rightarrow$ b; iii $\rightarrow$ b; iv $\rightarrow$ b

Friction forces on A and B will be acting as shown in figure. Both will be moving towards right after giving the impulse.



- (i) Work done by friction on B should be positive.
 (ii) Work done by friction on A should be negative.
 (iii) Negative work done on A will be more than positive work on B by friction. This is because displacement of A will be more than that of B during any time.



- (iv) In the frame of A , B will go towards left by a displacement of $y-x$. Hence work will be negative.

5. i $\rightarrow$ a, c; ii $\rightarrow$ b, c; iii $\rightarrow$ c; iv $\rightarrow$ b, d

$$\sqrt{5gr} = \sqrt{5 \times 10 \times 2} = 10 \text{ ms}^{-1}$$

$$\sqrt{2gr} = \sqrt{2 \times 10 \times 2} = \sqrt{40} \text{ ms}^{-1}$$

- (i) 3.5 ms^{-1} is less than $\sqrt{2gr}$. So the particle will stop before the string becomes horizontal.
 (ii) 8 ms^{-1} lies between $\sqrt{2gr}$ and $\sqrt{5gr}$. The particle will cross the horizontal position but will not be able to reach the highest point.
 (iii) Tension is maximum at the lowest point. Hence,

$$15 = mg + \frac{mv^2}{r} \Rightarrow v_2 = \sqrt{40} \text{ ms}^{-1}$$

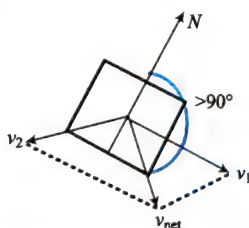
In this case particle will just be able to reach the horizontal level. Here both speed and tension become zero.

(iv) $30 = mg + \frac{mv^2}{r} \Rightarrow v_2 = 10 \text{ ms}^{-1}$

Here particle will be able to reach the highest point. Tension will become zero at the highest point. Hence (b) and (d) are matching.

6. i $\rightarrow$ b, d; ii $\rightarrow$ a, d; iii $\rightarrow$ c, d; iv $\rightarrow$ a, d

- (i) Angle between velocity of the block and normal reaction on the block is obtuse. Hence, work done by normal reaction on the block is negative.



As the block falls by vertical distance h , from work-energy theorem,

Work done by mg + work done by N = KE of the block

$$\therefore \text{work done by } N = \frac{1}{2} mv^2 - mgh < 0$$

$$\therefore \frac{1}{2} mv^2 < mgh$$

- (ii) Work done by normal reaction on wedge is positive.
 Since loss in PE of the block = KE of wedge + KE of block,
 work done by normal reaction on the wedge = KE of the wedge.
 Therefore, work done by N on the wedge is less than mgh .
 (iii) Net work done by normal reaction on the block and wedge is zero.

- (iv) The net work done by all forces on the block is positive, because its kinetic energy has increased.

Also KE of the block is less than mgh .

Hence, net work done on the block = final KE of the block $< mgh$.

7. i $\rightarrow$ c; ii $\rightarrow$ b; iii $\rightarrow$ d; iv $\rightarrow$ a

Vehicle is stopped, but due to inertia bob will acquire the velocity v_0 w.r.t. vehicle.

$$h = l - l \cos 60^\circ = \frac{l}{2} = 2.5 \text{ m}$$

$$mgh = \frac{1}{2} mv_0^2$$

$$v_0 = \sqrt{2gh} = \sqrt{2 \times 10 \times 2.5} = 5\sqrt{2} \text{ ms}^{-1}$$

Net force at the lowest point is

$$\frac{mv_0^2}{l} = \frac{2 \times (5\sqrt{2})^2}{5} = 20 \text{ N}$$

Acceleration of the bob at the lowest point,

$$\frac{v_0^2}{l} = \frac{(5\sqrt{2})^2}{5} = 10 \text{ ms}^{-2}$$

At point B: $T = mg \cos 60^\circ$

$$\text{Net force} = mg \sin 60^\circ = 2 \times 10 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ N}$$

$$\text{Acceleration} = g \sin 60^\circ = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ ms}^{-2}$$

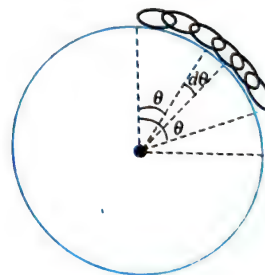
8. i $\rightarrow$ c; ii $\rightarrow$ d; iii $\rightarrow$ a; iv $\rightarrow$ b

Gravitational PE w.r.t. the center of sphere

$$= \int_0^{l/R} \frac{M}{l} g R^2 \cos \theta d\theta = \frac{M}{l} g R^2 \sin \left(\frac{l}{R} \right)$$

When the chain has been released and it has slid through angle θ , from mechanical energy theorem, we have

$$u_i - u_f = k_f - k_i \Rightarrow u_i - u_f = k_f$$



$$\frac{M}{l} g R^2 \sin \left(\frac{l}{R} \right) - \int_0^{\theta + \frac{l}{R}} \frac{M}{l} g R^2 \cos \theta d\theta = \frac{1}{2} mv^2$$

If v is the velocity at the moment when angle traversed is θ , then

$$\text{KE} = \frac{mR^2g}{l} \left[\sin \left(\frac{l}{R} \right) + \sin \theta - \sin \left(\theta + \frac{l}{R} \right) \right]$$

From the expression of KE, v can be obtained and differentiated in order to obtain tangential acceleration which will be ($\theta = 0^\circ$)

$$a_t = \frac{Rg}{l} \left[1 - \cos \left(\frac{l}{R} \right) \right] \quad \left[\text{use } v = R \frac{d\theta}{dt} \right]$$

[Differentiate w.r.t. time where $\frac{d\theta}{dt} = \omega$]

$$\text{Radial acceleration} = \frac{v^2}{R}$$

Hence, we have

$$a_r = \frac{2Rg}{l} \left[\sin\left(\frac{l}{R}\right) + \sin\theta - \sin\left(\theta + \frac{l}{R}\right) \right]$$

9. (2)

- (i) Forces are conservative so E is constant, KE will increase up to equilibrium and then decrease.
 (ii) Gravitation force is conservative, so E is constant, U decreases and KE increases.

$$U = mgv - \frac{1}{2}mv^2 = \frac{1}{2}m(gt)^2$$

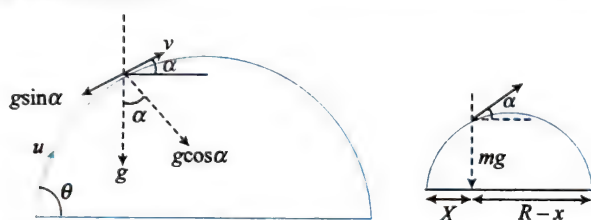
- (iii) $U = mgh = \text{constant } E = U + KE$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m(0 + at)^2 \text{ or } KE = \frac{1}{2}m\left(\frac{F}{m} \times t\right)^2$$

- (iv) Here velocity is constant, so KE is constant. As the block comes down U decreases so E decreases.

10. (2) $a_r = g \cos \alpha$, $a_t = g \sin \alpha$

First decreases and then increases. So radial acceleration increases and then decreases, Tangential acceleration decreases and then increases.



Power due to gravity = $\vec{m}g \cdot \vec{v}$

$$= mgv \cos\left(\frac{\pi}{2} + \alpha\right) = -mgv \sin \alpha$$

$v \sin$ first decreases and then increases.

So power (power is negative here) increases and then decreases
 Total acceleration is g which always remains constant.

11. (3) For 0 to t_1 , slope is constant, speed is constant and so work done is zero.

For t_1 to t_2 , slope is zero, speed is zero and so work done is zero.

For t_2 to t_3 , slope is increasing, speed is increasing and so work done is positive.

For t_3 to t_4 , slope is decreasing, speed is decreasing and so work done is negative.

12. (1) As no external force is acting on the system, the velocity of centre of mass will move with constant speed. The blocks A and B will oscillate about centre of mass. Hence, the velocity of blocks A or B may be zero at certain instant of time.

As centre of mass moves with constant velocity, hence the kinetic energy of system of blocks can never be zero. At the time when the spring is in its natural length, the potential energy is zero. The potential energy will be maximum when the deformation of the spring is maximum.

13. (1) $(W_F)_{OAC} = \int (xy dx + x^2 y dy)$

$$= \int_0^A (xy dx + x^2 y dy) + \int_A^C (xy dx + x^2 y dy)$$

On OA path;

$y = 0$, $dy = 0$ and on AC path

$x = 1$, $dx = 0$

$$\text{So } (W_F)_{OAC} = \int_0^A (0 \cdot dx + 0 \cdot dy) + \int_{y=0}^{y=4} (0 + 1 \cdot y dy) = 8 \text{ J}$$

$$(W_F)_{OBC} = 0 + \int_B^C (xy dx + x^2 y dy)$$

$$= \int_{x=0}^1 \{x \cdot 4 dx + x^2 \cdot 4(0)\} = 2 \text{ J}$$

$$(W_F)_{ODC} = \int_{y=4x^2}^1 (xy dx + x^2 y dy)$$

$$= \int_0^1 (x \cdot 4x^2 dx + x^2 \cdot 4x^2 \cdot 8x dx) = 1 + \frac{32}{6} = \frac{19}{3} \text{ J}$$

Numerical Value Type

1. (7) $5.6 = \frac{1}{2}k(0.3)^2$

$$W = \frac{1}{2}k[(0.3 + 0.15)^2 - 0.3^2]$$

Solve to get: $W = 7 \text{ J}$

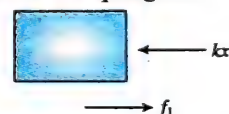
2. (2) Maximum elongation is given by

$$x_{\max} = \frac{2(F_1 m_2 + F_2 m_1)}{K(m_1 + m_2)}$$

Here $F_1 = F_2 = F$; $m_1 = m$ and $m_2 = M$

Put the values and solve to get $x_{\max} = 2F/K$

3. (7) Let compression in the spring be x .



$$f_l \geq kx \Rightarrow (1/2) \times 0.14 \times 10 > 10x$$

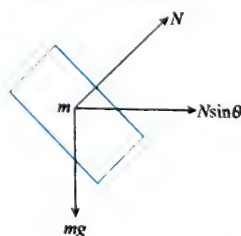
$$\Rightarrow x \leq 0.07 \text{ m} = 7 \text{ cm}$$

4. (4) As the centre of gravity of rope comes up by $h/2$ and that of bucket by h , required work would be

$$W = \sqrt{ng}r + Mgh = 4000 \text{ J} = 4 \text{ kJ}$$

5. (5) If block does not slip then, $a = g \tan \theta$

$$\Rightarrow 10 = 10 \tan \theta \Rightarrow \theta = 45^\circ$$



$$N \sin \theta = ma \Rightarrow N \times \frac{1}{\sqrt{2}} = 0.1 \times 10 \Rightarrow N = \sqrt{2} \text{ N}$$

$$S = \frac{1}{2} \times at^2 = \frac{1}{2} \times 10(1)^2 = 5 \text{ m}$$

$$W_N = NS \cos \theta = \sqrt{2} \times 5 \times 1/\sqrt{2} = 5 \text{ J}$$

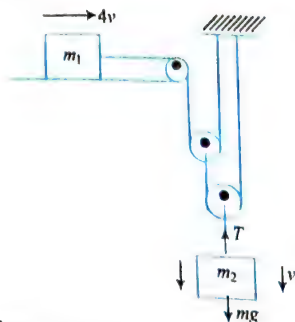
$$\text{Alternatively: } W_N = \frac{1}{2}mv^2 = \frac{1}{2}(0.1)(at)^2 = 5 \text{ J}$$

6. (4) From constraints we can show that if at any time velocity of m_2 is v , then velocity of m_1 will be $4v$. Similarly, for displacement. Applying work-energy theorem, $K_f - K_i = \text{Work done} = W$

$$\frac{1}{2}m_2 v^2 + \frac{1}{2}m_1 (4v)^2 = m_2 g \times y - \mu m_1 g (4y);$$

where $y = 9 \text{ m}$

$$\Rightarrow v = 4 \text{ ms}^{-1}$$



7. (2) At equilibrium, $(k_1 + k_2)x = mg$

$$\Rightarrow k_1 + k_2 = \frac{mg}{x} \quad \dots(i)$$

$$(x = 40 \text{ cm}, x_1 = 48 \text{ cm})$$

In case of constant acceleration,

$$(k_1 + k_2)x_1 - mg = ma \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $a = 2 \text{ ms}^{-2}$

8. (9) Let x be the elongation in the spring when it returns back.

$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + mg(x + x_0)$$

On solving, we get

$$kx = -mg \pm (mg - kx_0) = -2mg + kx_0$$

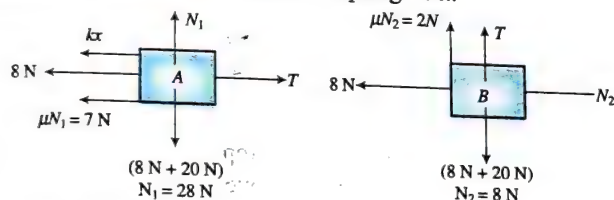
(ignoring negative sign)

And also lower block will bounce up if

$$kx \geq mg \Rightarrow -2mg + kx_0 \geq mg$$

$$\text{So } x_0 = \frac{3mg}{k} \Rightarrow x_0 = 9 \text{ cm}$$

9. (10) Let the maximum extension in spring be x .



Apply work-energy theorem: (Net work done by tension will be zero)

$$-\frac{1}{2}kx^2 - 8x - 7x + 28x - 2x = \Delta KE = 0$$

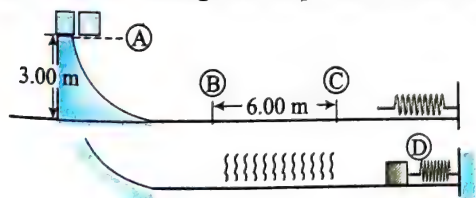
$$\Rightarrow -\frac{1}{2} \times 220x^2 + 11x = 0 \Rightarrow x = 0.1 \text{ m} = 10 \text{ cm}$$

10. (0.5) KE of car converts into elastic potential energy of springs.

$$\frac{1}{2}mv^2 = \frac{1}{2}k_1\left(\frac{50}{100}\right)^2 + \frac{1}{2}k_2\left(\frac{20}{100}\right)^2$$

solve to get $v = 0.5 \text{ ms}^{-1}$

11. (4) The easiest way to solve this problem about a chain-reaction process is by considering the energy changes experienced by the block between the point of release (initial) and the point of full compression of the spring (final). Recall that the change in potential energy (gravitational and elastic) plus the change in kinetic energy must equal the work done on the block by non-conservative forces. We choose the gravitational potential energy to be zero along the flat portion of the track.



There is zero spring potential energy in situation (A) and (D) and zero gravitational potential energy in situation (D). Putting the energy equation into symbols:

$$K_D - K_A - U_{gA} + U_{sD} = -f_k d_{BC}$$

Expanding into specific variables:

$$0 - 0 - mgy_A + \frac{1}{2}kx^2 = -f_k d_{BC}$$

The friction force is $f_k = \mu_k mgd$, so

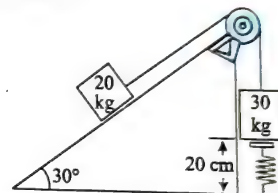
$$mgy_A - \frac{1}{2}kx^2 = \mu_k mgd$$

Solving for the unknown variable μ_k gives

$$\mu_k = \frac{y_A}{d} - \frac{kx^2}{2mgd} = \frac{3.00 \text{ m}}{6.00 \text{ m}} - \frac{(3000 \text{ N/m})(0.20 \text{ m})^2}{2(10.0 \text{ kg})(10 \text{ m/s}^2)(6.00 \text{ m})} = 0.40$$

Hence, $x = 4$.

12. (3) We choose the zero configuration of potential energy for the 30.0-kg block to be at the unstretched position of the spring, and for the 20.0-kg block to be at its lowest point on the incline, just before the system is released from rest. From conservation of energy, we have



$$(K + U)_i = (K + U)_f$$

$$0 + (30.0 \text{ kg})(10 \text{ m/s}^2)(0.200 \text{ m}) + \frac{1}{2}(250 \text{ N/m})(0.200 \text{ m})^2 = \frac{1}{2}(20.0 \text{ kg} + 30.0 \text{ kg})v^2$$

$$+ (20.0 \text{ kg})(10 \text{ m/s}^2)(0.200 \text{ m}) \sin 30^\circ$$

$$60 \text{ J} + 5.00 \text{ J} = (25.0 \text{ kg})v^2 + 20 \text{ J}$$

$$v = \frac{3}{\sqrt{5}} \text{ m/s}$$

13. (2) If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg . The extension of the spring, from $|\vec{F}_s| = kx$, must be mg/k . Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2$$

$$= 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)(-4m^2)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take

$$M = m(3 - 1) = 2m$$

$$\text{Hence, } \frac{M}{m} = 2.$$

14. (5) Let v_1 and v_2 are the velocities along the spring of A and B when spring regains its natural length then

$$2v_1 = 8v_2 \quad \dots(i)$$

$$\frac{1}{2}2v_1^2 + \frac{1}{2}8v_2^2 = \frac{1}{2}1000\left(\frac{20}{100}\right)^2 \quad \dots(ii)$$

$$\text{From (i) and (ii), } v_1 = 4 \text{ m/s, } v_2 = 1 \text{ m/s}$$

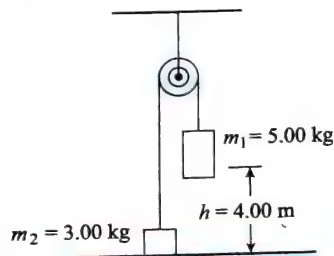
$$\text{Ground frame velocity of A} = \sqrt{v^2 + v_1^2} = \sqrt{3^2 + 4^2} = 5 \text{ m/s}$$

$$15. (7) H = 3 + \frac{u_B^2 \sin^2 \theta}{2g} = 3 + \frac{2g(h - h_B) \sin^2 30^\circ}{2g}$$

$$4(\text{given}) = 3 + h \sin^2 30^\circ - h_B \sin^2 30^\circ$$

$$\Rightarrow 4 = 3 + \frac{h}{4} - \frac{(3)}{4} \Rightarrow h = 7 \text{ m}$$

16. (5) We assign height $y = 0$ to the table top. Using conservation of energy for the system of the Earth and the two objects:



Then total mechanical energy of the system remains constant and the energy version of the isolated system model gives

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_{fa}$$

At the initial point, K_{Ai} and K_{Bi} are zero and we define the gravitational potential energy of the system as zero. Thus the total initial energy is zero, and we have

$$0 = \frac{1}{2}(m_1 + m_2)v_{fa}^2 + m_2gh + m_1g(-h)$$

Here we have used the fact that because the cord does not stretch, the two blocks have the same speed. The heavier mass moves down, losing gravitational potential energy, as the lighter mass moves up, gaining gravitational potential energy.

$$\text{Simplifying, } (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v_{fa}^2$$

$$\Rightarrow v_{fa} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}} = \sqrt{\frac{2(5.0 \text{ kg} - 3.0 \text{ kg})10(4.0 \text{ m})}{(5.0 \text{ kg} + 3.0 \text{ kg})}} = \sqrt{20} \text{ m/s}$$

Now we apply conservation of mechanical energy for the system of the 3.0-kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.0-kg object reaches its highest position in its free fall.

$$\Delta K + \Delta U = 0 \Rightarrow \Delta K = -\Delta U$$

$$0 - \frac{1}{2}m_2v^2 = -m_2g\Delta y$$

$$\Rightarrow \Delta y = \frac{v^2}{2g} = 1.0 \text{ m}$$

$$\text{Hence, } y_{\max} = 4.0 \text{ m} + \Delta y = 5.0 \text{ m}$$

17. (10) From given graphs:

$$a_x = \frac{3}{4} \text{ and } a_y = -\left(\frac{3}{4}t + 1\right)$$

$$\Rightarrow v_x = \frac{3}{8}t^2 + C$$

$$\text{At } t = 0: vx = -3 \Rightarrow C = -3$$

$$\therefore v_x = \frac{3}{8}t^2 - 3$$

$$\Rightarrow dx = \left(\frac{3}{8}t^2 - 3\right) dt \quad \dots(1)$$

$$\text{Similarly } dy = \left(-\frac{3}{8}t^2 - t + 4\right) dt \quad \dots(2)$$

$$\text{As } dW = \vec{F} \cdot d\vec{s} = \vec{F} \cdot (dx\hat{i} + dy\hat{j})$$

$$\therefore \int_0^W dw = \int_0^4 \left[\frac{3}{4}t\hat{i} - \left(\frac{3}{4}t + 1\right)\hat{j} \right] \cdot \left[\left(\frac{3}{8}t^2 - 3\right)\hat{i} + \left(-\frac{3}{8}t^2 - t + 4\right)\hat{j} \right] dt$$

$$\therefore W = 10 \text{ J}$$

18. (20) Maximum chance of slipping occurs when spring is maximum compressed. At this moment, as force exerted by the spring is maximum, acceleration of the system is maximum. Hence maximum friction force is required at this moment.

By W/E theorem

$$\frac{1}{2}(M + m)V^2 = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{(M + m)V^2}{K}}$$

$$\text{Now for upper block } a_m = \frac{kx_m}{M + m}$$

Force on upper block is provided by the friction force. Therefore

$$\mu mg \geq \frac{kx_m \cdot m}{M + m}$$

$$\text{For limiting value } V = \mu g \sqrt{\frac{M + m}{k}}$$

$$\text{Using values } V_{\max} = 20 \text{ cm/s}$$

$$19. (15) \vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -6\hat{i} + 8\hat{j}$$

$$\vec{a} = -3\hat{i} + 4\hat{j} \text{ has some direction as that of } \vec{u} = -1.5\hat{i} + 2\hat{j}$$

$$|\vec{a}| = 5, |\vec{u}| = \frac{5}{2}$$

Since $\vec{u}$ and $\vec{a}$ are in same direction, particle will move along a straight line

$$S = \frac{5}{2} \times 2 + \frac{1}{2} \times 5 \times 2^2 = 15 \text{ m}$$

20. (30) Draw FBD of B to get extension in spring. Instant when block B just loses contact with ground, the net force on it becomes zero.

$$kx - T \cos \theta = 0 \quad \dots(i)$$

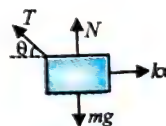
$$T \sin \theta + N - mg = 0 \quad \dots(ii)$$

To lift the block B, $N = 0$, from (i) and (ii), we get

$$\frac{kx}{\cos \theta} \sin \theta = mg$$

$$x = \frac{mg}{k \tan \theta} = \frac{80}{k \times (4/3)} = \frac{60}{k}$$

If spring has to just extend till this value at their extension it should be at rest.



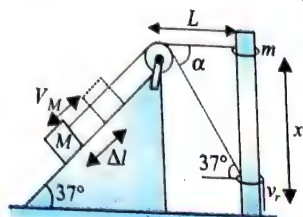
Now we apply work energy theorem to get

$$F_x = \frac{1}{2} kx^2 \Rightarrow F = 30 \text{ N}$$

2. (0) Let x is the vertical distance covered by the ring. Then

$$x = L \tan 37^\circ = 0.7 \times \frac{3}{4}$$

$$\Delta l = L \sec 37^\circ - L = L(\sec 37^\circ - 1) \Rightarrow \frac{L}{4} = \Delta l$$



Δl = distance moved by block M

Now, from constraint relation

$$v_M = v_r \cos 37^\circ = \frac{4}{5} v_r \quad \dots(ii)$$

v_r = velocity of ring,

v_M = velocity of the block at this instant

From work-energy theorem, we get $W_{\text{gravity}} = \Delta KE$

$$-mgx + Mg\Delta l \sin 37^\circ + \frac{1}{2} m v_r^2 + \frac{1}{2} M v_M^2 = 0 \quad \dots(iii)$$

On solving equations (i) and (ii), we get

$$v_r = 0 \text{ m/s}$$

$$\text{Similarly, } W_2 = \frac{F^2}{2k_1} \Rightarrow W \propto \frac{1}{K}$$

$$W_1 > W_2 \Rightarrow k_1 < k_2$$

Statement II is true.

$$\text{Now, } W_1 = \frac{1}{2} k_1 X^2$$

$$W_2 = \frac{1}{2} k_2 X^2$$

So, $W_2 > W_1$. Hence, statement I is false.

3. (1) Restoring force on rubber-band, $F = ax + bx^2$

Work done in stretching the rubber-band by a small amount dx ,
 $dW = Fdx$

Net work done in stretching the rubber-band by L is

$$W = \int dW = \int_0^L F dx = \int_0^L (ax + bx^2) dx$$

$$\Rightarrow W = \left[a \frac{x^2}{2} + b \frac{x^3}{3} \right]_0^L = \frac{aL^2}{2} + \frac{bL^3}{3}$$

4. (3) From work-energy theorem, $W_g + W_{f_1} + W_{f_2} = \Delta K$

$$mgh + 2W = 0 \quad \left[\because W_{f_1} = W_{f_2} = W \right]$$

$$\Rightarrow W = -\frac{mgh}{2} \text{ hence } W_{f_1} = -\frac{mgh}{2}$$

But work done by friction,

$$W_{f_1} = -\mu N_1 (PQ) = -\mu mg \cos 30^\circ (4)$$

$$\text{Hence, } -\mu mg \cos 30^\circ (4) = -\frac{mgh}{2} \Rightarrow \mu = 0.29$$

$$\text{As } W_{f_1} = W_{f_2} \text{ hence } -\mu mgx = -\frac{mgh}{2}$$

$$\Rightarrow x = \frac{h}{2\mu} = 3.5 \text{ m}$$

5. (4) The change in potential energy during exercise

$$\Delta PE = 1000 \times mgh$$

$$= 1000 \times 10 \times 9.8 \times 1 = 98000 \text{ J}$$

$$\text{Mechanical energy per unit mass} = \frac{2}{100} \times 3.8 \times 10^7$$

$$= 7.6 \times 10^5 \text{ J/kg}$$

$$\therefore \text{Fat burn} = \frac{9800}{7.6 \times 10^5} = 12.89 \times 10^{-3} \text{ kg}$$

$$6. (1) \frac{K_f}{K_i} = \frac{\frac{1}{2} m v_0^2}{\frac{1}{2} m v_0^2} = \frac{1}{4}$$

$$\text{which gives } v_f = \frac{v_0}{2} \text{ and } -kv^2 = \frac{mdv}{dt}$$

$$\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = \int_0^{t_0} \frac{-kdt}{m}$$

$$\left[-\frac{1}{v} \right]_{v_0}^{\frac{v_0}{2}} = \frac{-k}{m} t_0$$

On solving we get

$$k = \frac{m}{v_0 t_0} = \frac{10^{-2}}{10 \times 10} = 10^{-4} \text{ kg m}^{-1}$$

Archives

EE Main

Single Correct Answer Type

$$1. (3) U = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$\text{Force, } F = -\frac{dU}{dx} = -\frac{d}{dx} \left(\frac{a}{x^{12}} - \frac{b}{x^6} \right)$$

$$= -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7} \right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7} \right]$$

At equilibrium, $F = 0$.

$$\therefore \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0 \text{ or } x^6 = \frac{2a}{b}$$

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b} \right)^2} - \frac{b}{\left(\frac{2a}{b} \right)}$$

$$= \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

$$U_{(x=\infty)} = 0$$

$$D = [U_{(x=\infty)} - U_{\text{at equilibrium}}]$$

$$= \left[0 - \left(-\frac{b^2}{4a} \right) \right] = \frac{b^2}{4a}$$

$$2. (1) k_1 x_1 = k_2 x_2 = F$$

$$W_1 = \frac{1}{2} k_1 x_1^2 = \frac{(k_1 x_1)^2}{2k_1} = \frac{F^2}{2k_1}$$

7. (3) $F = 6t = ma$

$$\Rightarrow a = 6t$$

$$\Rightarrow \frac{dv}{dt} = 6t \quad \int_0^v dv = \int_0^t 6t \, dt$$

$$v = (3t^2)_0^1 = 3 \text{ m/s}$$

From work energy theorem

$$W_F = \Delta KE = \frac{1}{2}m(v^2 - u^2)$$

$$= \frac{1}{2}(1)(9 - 0) = 4.5 \text{ J}$$

JEE Advanced

Single Correct Answer Type

1. (3) Area under $F-t$ curve = 4.5 kg ms^{-1}

$$KE = \frac{1}{2}(2)\left(\frac{4.5}{2}\right)^2 = 5.06 \text{ J}$$

2. (4) $dw = \vec{F} \cdot \vec{dr} = \vec{F} \cdot (dx\hat{i} + dy\hat{j})$

$$= K \int \frac{xdx}{(x^2 + y^2)^{3/2}} + \frac{ydy}{(x^2 + y^2)^{3/2}}$$

$$x^2 + y^2 = a^2$$

$$w = \frac{k}{a^3} \int_a^0 xdx + \int_0^a ydy = \frac{k}{a^3} \left(\frac{-a^2}{2} + \frac{a^2}{2} \right) = 0$$

3. (2) During motion $K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$

$$\frac{d(KE)}{dt} = mv \frac{dv}{dt} = v \left(m \frac{dB}{dt} \right)$$

Multiple Correct Answers Type

1. (1, 2, 4)

We are given $\frac{dk}{dt} = \gamma t$; as $k = \frac{1}{2}mv^2$

Now differentiating both sides, we get

$$\therefore \frac{dk}{dt} = mv \frac{dv}{dt} = \gamma t$$

$$\therefore m \int_0^v v dv = \gamma \int_0^t t dt$$

$$\Rightarrow m \left[\frac{v^2}{2} \right]_0^v = \gamma \left[\frac{t^2}{2} \right]_0^t$$

$$\text{or } \frac{mv^2}{2} = \frac{\gamma t^2}{2}$$

$$\text{which gives } v = \sqrt{\frac{\gamma}{m}} t \quad \text{or } v \propto t \quad \dots(i)$$

Hence option (2) is correct

$$\text{as } a = \frac{dv}{dt}$$

$$\text{From (i) } a = \sqrt{\frac{\gamma}{m}} = \text{constant}$$

$$\text{Since } F = ma$$

$$\therefore F = m \sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m} = \text{constant}$$

Hence option (1) is correct.

As force is constant by magnitude and direction wise. Hence, it is conservative.

Also from (i),

$$\frac{dx}{dt} = \sqrt{\frac{\gamma}{m}} \cdot t \quad \text{or } dx = \sqrt{\frac{\gamma}{m}} \cdot t dt$$

$$\int_0^x dx = \sqrt{\frac{\gamma}{m}} \int_0^t t dt$$

$$\text{or } x = \sqrt{\frac{\gamma}{m}} \cdot \frac{t^2}{2} \quad \text{or } x \propto t^2$$

Matrix Match Type

1. a $\rightarrow$ p, q, r, t; b $\rightarrow$ q, s; c $\rightarrow$ p, q, r, s; d $\rightarrow$ p, r, t

The particle will experience an attractive force towards $x = 0$ in the region $|x| < a$

if $0 < x < a$

$$\Rightarrow F = -ve \text{ and if, } a < x < 0$$

$$\Rightarrow F = +ve$$

The particle will oscillate about $x = -a$ if U is min at $x = -a$ and U at

$$x = -a < \frac{U_0}{4}$$

$$\text{Also } F = -\frac{dU}{dx}$$

$$\Rightarrow (a) \quad F = \frac{2U_0x}{a} \left[1 - \left(\frac{x}{a} \right)^2 \right]$$

$$(b) \quad F = -\frac{U_0x}{a^2}$$

$$(c) \quad F = -\frac{U_0x}{a^2} \left(1 - \frac{x^2}{a^2} \right) e^{-\frac{x^2}{a^2}}$$

$$(d) \quad F = -\frac{U_0}{2a} \left(1 - \frac{x^2}{a^2} \right)$$

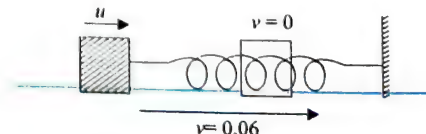
Numerical Value Type

1. (8) $2mg - T = 2ma$

$$T - mg = ma; \quad a = \frac{g}{3}; \quad T = \frac{4mg}{3}$$

$$W = T \times s = T \times \frac{1}{2} at^2 = 8 \text{ J}$$

2. (4)



$$\Rightarrow \frac{1}{2} mu^2 = \mu mg \times 0.06 + \frac{1}{2} Kx^2$$



$$\frac{1}{2} \times 0.18u^2 = 0.1 \times 1.18 \times 10 \times 0.06 + \frac{1}{2} \times 2 \times (0.06)^2$$

$$u = 0.4 \text{ ms}^{-1}$$

$$0.4 = \frac{N}{10} \Rightarrow N = 4$$

$$3. (5) \text{ Power} = \frac{dW}{dt} \Rightarrow W = 0.5 \times 5 = 2.5 = \text{KE}_f - \text{KE}_i$$

$$2.5 = \frac{M}{2}(v_f^2 - v_i^2) \rightarrow v_f = 5$$

4. (5) Using work energy theorem,

$$W_{mg} + W_F = \Delta \text{KE}$$

$$-mgh + Fd = \Delta \text{KE}$$

$$-1 \times 10 \times 4 + 18(5)$$

$$= \Delta \text{KE}$$

$$\Delta \text{KE} = 50$$

$$\therefore n = 5$$

Chapter 9

Concept Application Exercises

Exercise 9.1

1. Angular velocity of A with respect to O is

$$\omega_{AC} = \frac{(v_{AO})_{\perp}}{r_{AO}} = \frac{v}{r} = \omega$$

$$\therefore \omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2}$$

$$\text{and } \omega_{AC} = \frac{(v_{AC})_{\perp}}{r_{AC}} = \frac{v}{3r} = \frac{\omega}{3}$$

2. Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_{\perp}}{r_{AO}} = \frac{v}{r} = \omega$$

$$\text{Now, } \omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}} \Rightarrow v_{AB} = 2v$$

Since v_{AB} is perpendicular to $r_{AB} \Rightarrow (v_{AB})_{\perp} = v_{AB} = 2v$; $r_{AB} = 2r$

$$\Rightarrow \omega_{AB} = \frac{(v_{AB})_{\perp}}{r_{AB}} = \frac{2v}{2r} = \omega$$

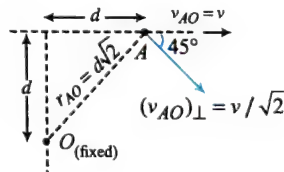
3. Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_{\perp}}{r_{AO}}$$

$$v_{AO} = v, (v_{AO})_{\perp} = \frac{v}{\sqrt{2}}$$

$$r_{AO} = d\sqrt{2}$$

$$\omega_{AO} = \frac{(v_{AO})_{\perp}}{r_{AO}} = \frac{v/\sqrt{2}}{d\sqrt{2}} = \frac{v}{2d}$$



4. Since speed increases uniformly, average tangential acceleration is equal to instantaneous tangential acceleration.

Therefore, the instantaneous tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6.0 - 5.0}{2.0} \text{ ms}^{-2} = 0.5 \text{ ms}^{-2}$$

$$\text{The angular acceleration is } \alpha = \frac{a_t}{r} = \frac{0.5 \text{ ms}^{-2}}{20 \text{ cm}} = 2.5 \text{ rad s}^{-2}$$

5. The distance covered in completing the circle is $2\pi r = 2 \times 10 \text{ cm}$. The linear speed is

$$v = \frac{2\pi r}{t} = \frac{2\pi \times 10 \text{ cm}}{4 \text{ s}} = 5 \text{ cm s}^{-1}$$

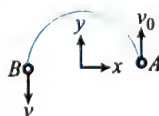
$$\text{The acceleration is } a = \frac{v^2}{r} = \frac{(5 \text{ cm s}^{-1})^2}{10 \text{ cm}} = 2.5 \text{ cm s}^{-2}$$

6. The average acceleration of the particle when it revolves half of the circle is

$$a_{av} = \frac{|\vec{v} - \vec{v}_0|}{\Delta T}$$

where $\vec{v} = -v\hat{j}$, $\vec{v}_0 = v_0\hat{j}$, and $\Delta T = T$ (given).

$$\text{Then } a_{av} = \frac{v + v_0}{T}$$



Let us express $v + v_0$ in terms of R and T .

As the particle moves with constant angular acceleration its average angular velocity is

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\omega + \omega_0}{2}$$

where $\omega = \frac{v}{R}$, $\omega_0 = \frac{v_0}{R}$, $\Delta\theta = \pi$, and $\Delta t = T$

$$\text{Then, } v + v_0 = \frac{2\pi R}{T}$$

Substituting the value of $v + v_0$ from Eq. (ii) in Eq. (i), we have

$$a_{av} = \frac{2\pi R}{T^2}$$

7. We have $v = v_0 - kt$ (given). Then, the tangential acceleration of the

particle is $a_t = \frac{dv}{dt} = -k$

The angular acceleration of the particle is $\alpha = \frac{a_t}{R} = \frac{-k}{R}$

If $\alpha = \omega$, we have $\frac{k}{R} = \frac{v_0 - kt}{R}$

$$\text{Then, } t = \frac{v_0 - k}{k}$$

$$8. \omega_{BA} = \frac{v_{BA\perp}}{AB},$$

where $v_{BA\perp} = v \cos \theta$ and $AB = AC \cos \theta$

because ABC is a right-angled triangle.

$$\text{Then, } \omega_{BA} = \frac{v}{AC} = \frac{v}{2R}$$

Substituting $\frac{v}{R} = \omega_{BO}$, we have $\omega_{BA} = \frac{1}{2} \omega_{BO} = \left(-\frac{1}{2}\omega\right)$

Alternative procedure:

$$\phi = 2\theta$$

$$\text{Then, } \frac{d\phi}{dt} (-\omega_{BO}) = 2 \frac{d\theta}{dt}, \text{ where } \frac{d\theta}{dt} = \omega_{BA}$$

This gives $\omega_{BO} = 2\omega_{BA}$.

9. Let the satellites 1 and 2 describe angles θ_1 and θ_2 at the center of earth during time t , respectively. Suppose 2 is ahead of 1 by an angle θ . Hence, we can write

$$\theta = \theta_2 - \theta_1$$

They will be again lie on same line after a minimum time T' say. For this $\theta = 2\pi$. This gives

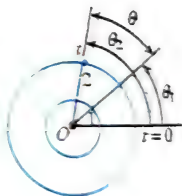
$$\theta_2 - \theta_1 = 2\pi. \text{ Dividing } T' \text{ in both sides, we have } \frac{\theta_2}{T'} - \frac{\theta_1}{T'} = \frac{2\pi}{T'}$$

Substituting $\frac{\theta_2}{T'} = \omega_2$ and $\frac{\theta_1}{T'} = \omega_1$, we have

$$\frac{2\pi}{T'} = \omega_2 - \omega_1$$

Since, $\omega_2 = \frac{2\pi}{T_2}$ and $\omega_1 = \frac{2\pi}{T_1}$, then $\frac{1}{T'} = \frac{1}{T_2} - \frac{1}{T_1}$

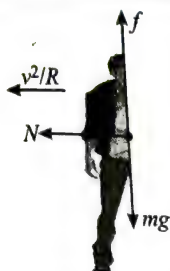
$$\text{This gives } T' = \frac{T_1 T_2}{T_1 - T_2}$$



Exercise 9.2

1. Here friction (f) is upward and opposes tendency to move down. Also normal force is radially towards the centre to provide centripetal acceleration.

$$(F_{\text{net}})_x = \frac{mv^2}{R} \Rightarrow N = \frac{mv^2}{R}$$

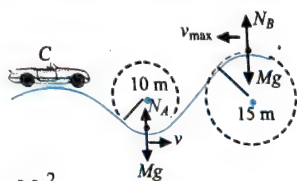


Also as man does not fall down, $f = mg \leq \mu N$

$$\Rightarrow mg \leq \mu \frac{mv^2}{R} \Rightarrow v \geq \sqrt{\frac{gR}{\mu}}$$

$$\Rightarrow v_{\min} = \sqrt{\frac{gR}{\mu}} = \sqrt{\frac{10 \times 2}{0.2}} = 10 \text{ m/s}$$

2. (a) $v = 20.0 \text{ ms}^{-1}$, N_A = force of track on roller coaster, and $R = 10 \text{ m}$.



$$N_A = Mg + \frac{Mv^2}{R}$$

$$= (500 \text{ kg})(10 \text{ ms}^{-2}) + \frac{(500 \text{ kg})(20 \text{ m/s})^2}{10.0 \text{ m}}$$

$$N_A = 5000 \text{ N} + 20,000 \text{ N} = 2.50 \times 10^4 \text{ N}$$

(b) At B point, $Mg - N_B = \frac{Mv^2}{R}$

For maximum velocity at B, N_B should be zero

$$Mg = \frac{Mv_{\max}^2}{R}$$

$$v_{\max} = \sqrt{Rg} = \sqrt{15.0(10.0)} = 5\sqrt{6} \text{ ms}^{-1}$$

3. (a) $N - mg = -\frac{mv^2}{r}$

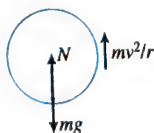
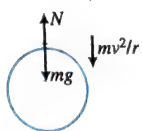
To feel weightless at top, $N = 0$

$$mg = \frac{mv^2}{r}$$

$$\Rightarrow v = \sqrt{rg} = \sqrt{200} = 10\sqrt{2} \text{ ms}^{-1}$$

(b) $N - mg = \frac{mv^2}{r}$

$$N = m \left(g + \frac{v^2}{r} \right) = 1200 \text{ N}$$



4. (a) Since the object of mass m_2 is in equilibrium,

$$\sum F_y = T - m_2g = 0 \quad \text{or} \quad T = m_2g$$

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = m_2g$.

(c) From $F_c = \frac{m_1v^2}{R}$, we have $v = \sqrt{\frac{RF_c}{m_1}} = \sqrt{\left(\frac{m_2}{m_1}\right)gR}$

(d) The puck will spiral inward, gaining speed as it does so. It gains speed because the extra-large string tension produces forward tangential acceleration as well as inward radial acceleration of the puck, pulling at an angle of less than 90° to the direction of the inward spiraling velocity.

(e) The puck will spiral outward, slowing down as it does so.

5. (a) Force on the particle when the fan is at full speed

$$F_n = m\omega^2 R = \left(\frac{1}{1000}\right) \left(\frac{2\pi \times 1500}{60}\right)^2 \left(\frac{60}{100}\right) = \frac{3}{2}\pi^2 \text{ N}$$

(b) The friction force exerts this force.

(c) $F_{\text{particle}} = F_n = \frac{3}{2}\pi^2 \text{ N}$

6. (a) If the block does not slip $f \leq \mu N$

$$m\omega^2 L \leq \mu mg \Rightarrow \omega \leq \left(\frac{\mu g}{L}\right)^{1/2}$$

(b) It is the case of non-uniform circular motion.

$$F_{\text{tangential}} = m \frac{dv}{dt} = m \frac{d(\omega L)}{dt} = mL \frac{d\omega}{dt} = m\alpha L \quad \dots(i)$$

$$F_{\text{radial}} = m\omega^2 L \quad \dots(ii)$$

$$\text{Net force, } F_{\text{net}} = \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2}$$

$$\text{Friction force, } f = F_{\text{net}} \Rightarrow f = \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2}$$

$$f \leq \mu N \Rightarrow \sqrt{(m\alpha L)^2 + (m\omega^2 L)^2} \leq \mu mg$$

$$m^2\alpha^2 L^2 + m^2\omega^4 L^2 \leq \mu^2 m^2 g^2$$

$$\Rightarrow \omega \leq \left[\left(\frac{\mu g}{L} \right)^2 - \alpha^2 \right]^{1/4}$$

7. Let the car loses the contact at angle θ with the vertical

$$mg \cos \theta - N = \frac{mv^2}{R}$$

$$N = mg \cos \theta - \frac{mv^2}{R}$$

...(i)

For losing the contact, $N = 0$

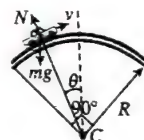
$$\Rightarrow v = \sqrt{Rg \cos \theta} \quad [\text{from Eq. (i)}]$$

For minimum speed, $\cos \theta$ should be minimum so that θ should be maximum.

$$\theta_{\max} = 45^\circ \Rightarrow \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$v_{\min} = \left(\frac{Rg}{\sqrt{2}} \right)^{1/2}$$

So that if car cannot lose the contact at initial or final point, car cannot lose the contact anywhere.



8. Here the motion of the particle is constrained with in the groove radial direction. During rotation of the table, when particle is at a distance x from the center of the table as shown in figure its acceleration is given as

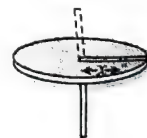
$$a = \omega^2 r$$

$$v \frac{dv}{dx} = \omega^2 x$$

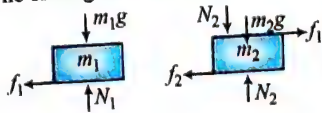
$$v dv = \omega^2 x dx$$

$$\text{Integrating } \int_0^v v dv = \int_l^L \omega^2 x dx$$

$$\left[\frac{v^2}{2} \right]_0^v = \omega^2 \left[\frac{x^2}{2} \right]_l^L \Rightarrow v = \omega (L^2 - l^2)^{1/2}$$



9. (a) $f_1 = m_1 \omega^2 R$
 $f_2 - f_1 = m_2 \omega^2 R$
 $f_2 = f_1 + m_2 \omega^2 R = (m_1 + m_2) \omega^2 R$
 (b) If there is no sliding between m_1 and m_2 , therefore,



$$f_1 \leq f_{1\max}$$

$$m_1 \omega^2 R \leq \mu_1 m_1 g$$

$$\Rightarrow \omega \leq \sqrt{\frac{\mu_1 g}{R}}$$

If there is no sliding between m_2 and ground.

$$f_2 \leq \mu_2 N_2$$

$$(m_1 + m_2) \omega^2 R \leq \mu_2 (m_1 + m_2) g$$

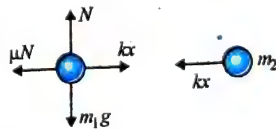
$$\omega \leq \sqrt{\frac{\mu_2 g}{R}}$$

As $\mu_1 < \mu_2$, hence, maximum angular velocity will be $\sqrt{\frac{\mu_1 g}{R}}$ (i)

10. From FBD of m_2 : $kx = m_2 \omega^2 (l_0 + x)$

From FBD of m_1 : $kx = \mu N = \mu m_1 g$

$$\Rightarrow x = \frac{\mu m_1 g}{k} \quad \dots (ii)$$



- (a) Substituting the value of x in Eq. (i)

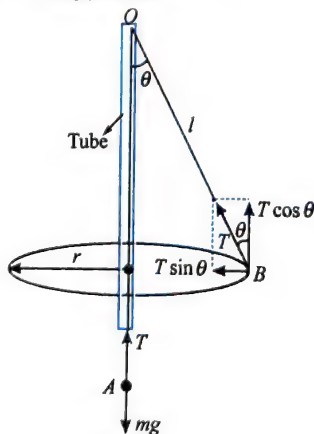
$$k \left(\frac{\mu m_1 g}{k} \right) = m_2 \omega^2 \left(l_0 + \frac{\mu m_1 g}{k} \right)$$

$$\Rightarrow \omega^2 = \frac{\mu m_1 g}{m_2 \left(l_0 + \frac{\mu m_1 g}{k} \right)} \Rightarrow \omega = \sqrt{\frac{\mu k m_1 g}{m_2 (l_0 k + \mu m_1 g)}}$$

- (b) Acceleration of m_2 , $a_2 = \frac{kx}{m_2} = \frac{\mu m_1 g}{m_2}$

Exercise 9.3

1. A large mass (M) and a small mass (m) hang at the end A and B respectively of a string passing over a tube at O as shown in figure.



$$T \sin \theta = m r \omega^2 \text{ or } \omega = \sqrt{\frac{T \sin \theta}{m r}}$$

$$\text{or } \omega = \sqrt{\frac{M g \sin \theta}{m l \sin \theta}} = \sqrt{\frac{M g}{m l}} \quad (\text{as } r = l \sin \theta \text{ and } T = M g)$$

$$\text{Thus, } v = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{M g}{m l}}$$

2. (a) The car is going around an unbanked curve, so the centripetal force must be horizontal. The static frictional force is the only horizontal force, so it serves as the centripetal force. The maximum centripetal force occurs when friction reaches to its maximum value. Therefore, the maximum speed v the car can have without slipping is related to

$$F_c = \mu N = \frac{m v^2}{r} \quad \dots (i)$$

In part (a), the car is subject to two downward-pointing forces, its weight W and the downforce F_v . The vertical acceleration of the car is zero, so the upward normal force must balance the two downward forces: $N = mg + F_v$. Combining this relation with Eq. (i), we obtain an expression for the coefficient of static friction:

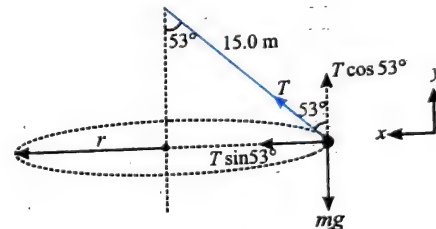
$$\mu_s = \frac{\frac{m v^2}{r}}{N} = \frac{\frac{m v^2}{r}}{mg + F_v} = \frac{m v^2}{r(mg + F_v)} \quad \dots (ii)$$

$$\mu_s = \frac{(900)(50)^2}{(200)[(900)(10.0) + 11000]} = \frac{9}{16}$$

- (b) The downforce is now absent ($F_v = 0$). Solving Eq. (ii) for the speed of the car, we find that

$$v = \sqrt{\mu_s r g} = \sqrt{\frac{9}{16}(200)(10.0)} = 15\sqrt{5} \text{ m/s}$$

3.



- (a) In vertical direction $\Sigma F_y = 0$, so

$$T \cos 53^\circ = mg \quad \dots (i)$$

Solving for the magnitude T of the tension gives

$$T = \frac{mg}{\cos 53^\circ} = \frac{(200)(10)}{(3/5)} = \frac{10000 \text{ N}}{3}$$

- (b) In horizontal direction, $\Sigma F_x = 0$, so

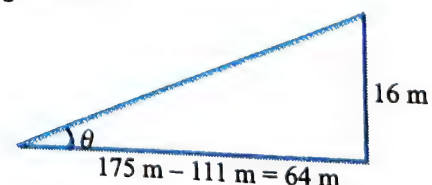
$$T \sin 53^\circ = m a_c = m \left(\frac{v^2}{r} \right) \quad \dots (ii)$$

From the drawing we see that the radius r of the circular path is $r = (15.0 \text{ m}) \sin 53^\circ = 12.0 \text{ m}$. Solving Eq. (ii) for the speed v gives

$$v = \sqrt{\frac{r T \sin 53^\circ}{m}} = \sqrt{\frac{(12)(2500) \sin 53^\circ}{150}} = 4\sqrt{10} \text{ m/s}$$

4. The angle θ at which a friction-free curve is banked depends on the radius r of the curve and the speed v with which the curve is to be negotiated, $\tan \theta = v^2 / rg$. For known values of θ and r , the safe speed is, $v = \sqrt{rg \tan \theta}$

The drawing at the right shows a cross-section of the track. From the drawing we have



$$\tan \theta = \frac{16 \text{ m}}{64 \text{ m}} = 0.25$$

- (a) Therefore, the smallest speed at which cars can move on this track without relying on friction is

$$v_{\min} = \sqrt{(111)(10)(0.25)} = \frac{\sqrt{1110}}{2} \text{ m/s}$$

- (b) Similarly, the largest speed is

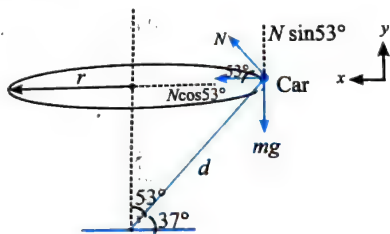
$$v_{\max} = \sqrt{(175)(10)(0.25)} = \frac{5\sqrt{70}}{2} \text{ m/s}$$

5. The distance d is related to the radius r of the circle on which the car travels by $d = r/\sin 53^\circ$ (see figure).

We can obtain the radius by noting that the car experiences a centripetal force that is directed toward the center of the circular path. This force is provided by the component, $N \cos 53^\circ$, of the normal force that is parallel to the radius. Setting this force equal to the mass m of the car times the centripetal acceleration ($a_c = v^2/r$) gives $F_N \cos 50.0^\circ = ma_c = mv^2/r$. Solving for the radius r and substituting it into the relation $d = r/\sin 50.0^\circ$ gives

$$N \cos 53^\circ = \frac{mv^2}{r} \quad \dots(i)$$

$$r = d \sin 53^\circ \quad \dots(ii)$$



$$\text{From Eqs. (i) and (ii), } d = \frac{mv^2}{(N \cos 53^\circ)(\sin 53^\circ)} \quad \dots(iii)$$

The magnitude N of the normal force can be obtained by observing that the car has no vertical acceleration, so the net force in the vertical direction must be zero, $\sum F_y = 0$.

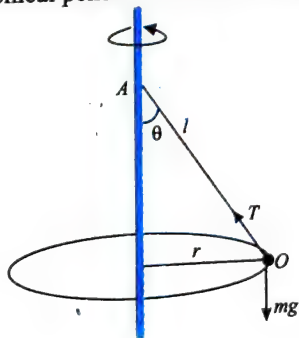
$$N \sin 53^\circ = mg \quad \dots(iv)$$

Solving Eqs. (iii) and (iv), we get

$$d = \frac{mv^2}{\left(\frac{mg}{\sin 53^\circ}\right)(\cos 53^\circ)(\sin 53^\circ)}$$

$$\Rightarrow d = \frac{v^2}{g \cos 53^\circ} = \frac{(30)^2}{(10) \cos 53^\circ} = 150 \text{ m}$$

6. (a) Consider a conical pendulum with one string (AO) only.



$$r = l \sin \theta \quad \dots(i)$$

$$T \sin \theta = m \omega^2 r$$

$$T \sin \theta = m \omega^2 l \sin \theta$$

$$\Rightarrow T = m \omega^2 l$$

$$\text{and } T \cos \theta = mg$$

$$\Rightarrow \cos \theta = \frac{mg}{m \omega^2 l} = \frac{g}{\omega^2 l} \quad \dots(iii)$$

For a conical pendulum to be possible, $\theta > 0^\circ$

$$\Rightarrow \cos \theta < 1 \Rightarrow \frac{g}{\omega^2 l} < 1$$

$$\sqrt{\frac{g}{l}} < \omega$$

Hence, particle will stay at B if $\omega \leq \sqrt{\frac{g}{l}}$

- (b) When $\omega > \sqrt{\frac{g}{l}}$, the particle begins to rotate in a circle and the tension in string becomes greater than mg .

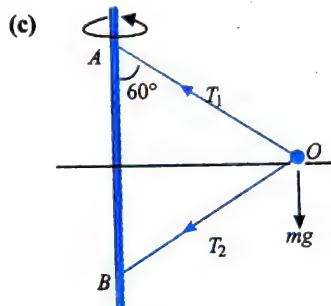
The second string will get taut only when $\theta = 60^\circ$

$$\text{From Eq. (iii), } \omega^2 = \frac{g}{l \cos 60^\circ} = \frac{2g}{l}$$

$$\omega = \sqrt{\frac{2g}{l}}$$

If $\omega > \sqrt{\frac{2g}{l}}$, there is tension in BO

Required range is $\sqrt{\frac{g}{l}} < \omega \leq \sqrt{\frac{2g}{l}}$



$$T_1 \cos 60^\circ = T_2 \cos 60^\circ + mg$$

$$T_1 - T_2 = 2mg \quad \dots(iv)$$

$$\text{and } (T_1 + T_2) \sin 60^\circ = m \omega^2 r$$

$$\frac{\sqrt{3}}{2} (T_1 + T_2) = m \omega^2 \frac{\sqrt{3}}{2} l$$

$$\Rightarrow T_1 + T_2 = m \omega^2 l \quad \dots(v)$$

For, $T_1 = 2T_2$, Eqs. (iv) and (v) give $T_2 = mg$ and $3T_2 = m \omega^2 l$

$$\Rightarrow 6mg = m \omega^2 l \Rightarrow \omega = \sqrt{\frac{6g}{l}}$$

$$7. r = R \sin \theta, N \cos \theta = mg \quad \dots(i)$$

$$N \sin \theta = m \omega^2 r = m \omega^2 R \sin \theta$$

$$\Rightarrow N = m \omega^2 R \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\cos \theta = \frac{g}{\omega^2 R} \Rightarrow \theta = \cos^{-1} \left(\frac{g}{\omega^2 R} \right)$$

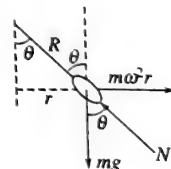
8. The free body diagram of the block is

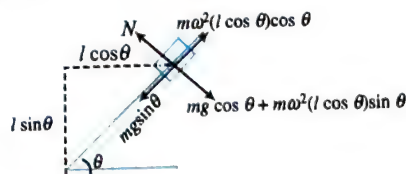
- (a) For block not to slide along wedge, applying Newton's second law along incline, we get

$$mg \sin \theta = m \omega^2 (l \cos \theta) \cos \theta$$

$$\omega = \sqrt{\frac{g \sin \theta}{l \cos^2 \theta}}$$

- (b) Applying Newton's second law along normal to the wedge, we get





$$N = mg \cos \theta + m\omega^2(l \cos \theta) \sin \theta$$

$$= mg \cos \theta + m \frac{g \sin \theta}{l \cos^2 \theta} l \cos \theta \sin \theta = mg \sec \theta$$

Exercise 9.4

1. Let the velocity at B be
- v
- , then

$$\frac{1}{2}mv^2 = mg(h - 2R) \Rightarrow v = \sqrt{2g(h - 2R)} \quad \dots(i)$$

$$N + mg = \frac{mv^2}{R} \quad \dots(ii)$$

Put $N = mg$, solve Eqs. (i) and (ii) to get $h = 3R$

2. To complete the circle of radius
- r
- , the velocity at the top
- $\geq \sqrt{rg}$
- .

From A to B;

Loss in GPE = Gain in KE

$$mg(h - 2r) = \frac{1}{2}mv^2$$

$$mg(h - 2r) = \frac{1}{2}m(\sqrt{rg})^2 \Rightarrow h = \frac{5}{2}r$$

Hence, h must be at least equal to $2.5r$.

3. Here tension in the rod at the topmost point of circle can be zero or negative for completing the loop. So velocity at the topmost point is zero.

From energy conservation,

$$\frac{1}{2}mv^2 + \frac{1}{2}m\frac{v^2}{4} = mg(2l) + mg(4l) + 0$$

$$v = \sqrt{\frac{48gl}{5}}$$

4. (a)
- $l = 2a$
- , Velocity at lowest point from energy conservation

$$0 + mg2a = \frac{1}{2}mv^2$$

$$v = \sqrt{4ga}$$

Here the radius of circle is a about nail and velocity at lowest point is not sufficient to complete the loop. Therefore, motion of bob is the combination of circular and projectile motion. Because the velocity at lowest point lies between $\sqrt{3ga}$ and $\sqrt{5ga}$.

- (b)
- $l = 2.5a$
- ; Velocity at lowest point from energy conservation

$$0 + mg(2.5a) = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{5ga}$$

here radius of circle is a about nail and velocity at lowest point is just sufficient to complete the loop so that here looping the loop about nail.

5. By COME theorem,

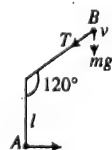
$$u = \sqrt{6gl}$$

$$\frac{1}{2}mu^2 = mgl(1 - \cos 120^\circ) + \frac{1}{2}mv^2$$

$$\text{gives } v = \sqrt{3gl}$$

$$\text{At point B, } T + mg \cos 60^\circ = \frac{mv^2}{l}$$

$$\text{By putting } v = \sqrt{3gl}, \text{ we get } T = \frac{5}{2}mg$$



$$6. \frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + mgr$$

$$\Rightarrow \frac{mv_0^2}{r} = 2mg + 2mg = 4mg$$

$$N = mg + \frac{mv_0^2}{r} = 5mg$$

7. At topmost point,
- $T + 2mg = \frac{mv^2}{r}$

In critical condition, $v^2 = 2rg$

Applying conservation of energy (or work energy theorem) between top and bottom points.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg2r + mg.2r$$

$$\Rightarrow u^2 = v^2 + 8gr = 10gr$$

$$\therefore u = \sqrt{10gr}$$

8. Apply conservation of energy between A and B.

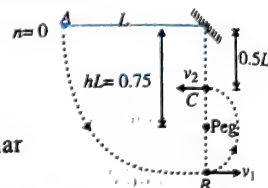
$$\frac{1}{2}mv_1^2 = mgL \Rightarrow v_1 = \sqrt{2gL}$$

We see that $v_1 > \sqrt{5g(0.25L)}$

So the ball will complete the circular motion about peg.

Apply conservation of energy between A and C:

$$\frac{1}{2}mv_2^2 = mg(0.5L) \Rightarrow v_2 = \sqrt{gL}$$



- 9.
- $T = mg \cos \theta + \frac{mv^2}{l}$

$$\Rightarrow mg = mg \cos \theta + \frac{mv^2}{l}$$

$$mgh = \frac{1}{2}m(\sqrt{gl})^2 - \frac{1}{2}mv^2$$

$$mg(l - l \cos \theta) = \frac{1}{2}mgl - \frac{1}{2}mv^2$$

From Eqs. (i) and (ii), $v = \sqrt{\frac{gl}{3}}$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

10. As shown in figure at point B,
- $T = 0$
- .

Thus, we have

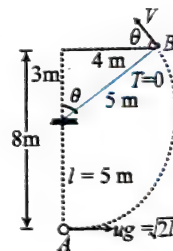
$$\frac{mv^2}{l} = mg \cos \theta$$

$$v = \sqrt{gl \cos \theta} = \sqrt{29.4} = 5.42 \text{ ms}^{-1}$$

After point B, particle will move in a projectile trajectory with this initial velocity. Let h be the maximum height it further rises. It is given

as

$$h = \frac{v^2 \sin^2 \theta}{2g} = \frac{29.4 \times (4/5)^2}{2 \times 9.8} = 0.96 \text{ m}$$



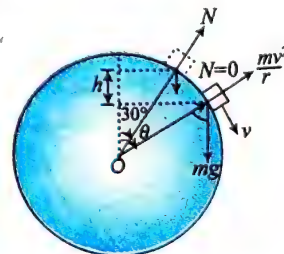
11. (a) At the time of release, the speed of box is zero. It will push the sphere only with the normal component of its weight. Along radial direction, we have

$$N = mg \cos 30^\circ \Rightarrow N = \frac{\sqrt{3}}{2}mg$$

- (b) The situation is shown in figure

Let the box lose contact with the sphere at an angle θ from the vertical. At this instant, its normal reaction becomes zero. Thus, we have at this point

$$\frac{mv^2}{R} = mg \cos \theta$$



$$v = \sqrt{Rg \cos \theta} \quad \dots(i)$$

Velocity of particle at this point can be given by energy conservation as it has fallen a distance h , we have

$$h = R(\cos 30^\circ - \cos \theta)$$

$$\text{and } v = \sqrt{2gh} = \sqrt{2gR \left(\frac{\sqrt{3}}{2} - \cos \theta \right)} \quad \dots(ii)$$

Equating Eqs. (i) and (ii), we have

$$\cos \theta = \sqrt{3} - 2 \cos \theta \Rightarrow \cos \theta = \frac{1}{\sqrt{3}} \text{ or } \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

Angular displacement of the box before leaving the sphere is

$$\cos^{-1} \left(\frac{1}{\sqrt{3}} \right) - \frac{\pi}{6}$$

$$\text{Distance travelled by the box is } \left[\cos^{-1} \left(\frac{1}{\sqrt{3}} \right) - \frac{\pi}{6} \right] R$$

2. Let the initial compression of the spring be x and when the spring is released, the stored compressional energy in the spring is given to the block and it shoots on the track, which ends in a vertical circle of radius r . When the mass reaches the point P , the weight of block at this point is in vertical downward direction and the block pushes the track with the only force mv^2/r .

At P , the force on track should be equal to the weight of the block, thus

$$\frac{mv^2}{r} = mg \text{ or } v = \sqrt{rg}$$

Now we apply work-energy theorem from starting point of the spring to the final P for the block.

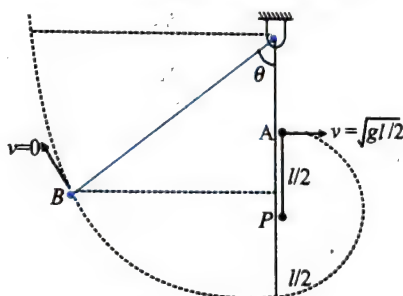
As initial kinetic energy of the block is zero and at P it is $mv^2/2$, thus we have

$$0 + \frac{1}{2} kx^2 - mgr = \frac{1}{2} mv^2$$

$$\frac{1}{2} kx^2 - mgr = \frac{1}{2} m (rg)$$

$$kx^2 = 3mgr \Rightarrow x = \sqrt{\frac{3mgr}{k}}$$

13. Apply conservation of energy between A and B .



$$\frac{1}{2} mv^2 + mgl = mg \frac{3l}{2} [1 - \cos \theta]$$

$$v^2 + 2gl = 3gl[1 - \cos \theta]$$

$$\text{Put } v = \sqrt{\frac{gl}{2}}$$

$$\frac{gl}{2} + 2gl = 3gl[1 - \cos \theta]$$

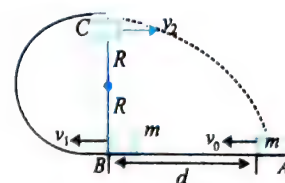
$$\frac{5}{2} = 3[1 - \cos \theta]$$

$$\Rightarrow \frac{5}{6} = 1 - \cos \theta \Rightarrow \cos \theta = \frac{1}{6} \Rightarrow \theta = \cos^{-1} \left(\frac{1}{6} \right)$$

14. (a) From C to A , time taken

$$t = \sqrt{\frac{2(2R)}{g}} = \sqrt{\frac{4R}{g}}$$

$$v_2 = \frac{d}{t} = d \sqrt{\frac{g}{4R}}$$



$$\text{From } B \text{ to } C: \frac{1}{2} mv_1^2 = \frac{1}{2} mv_2^2 + mg(2R)$$

$$\Rightarrow v_1^2 = v_2^2 + 4gR = \frac{d^2 g}{4R} + 4gR$$

$$\text{From } A \text{ to } B: v_1^2 = v_0^2 - 2\mu gd \Rightarrow v_0^2 = v_1^2 + 2\mu gd$$

$$\Rightarrow v_0 = \sqrt{\frac{d^2 g}{4R} + 2g(\mu d + 2R)}$$

$$(b) \text{ For minimum } v_0, v_1 = \sqrt{5gR}$$

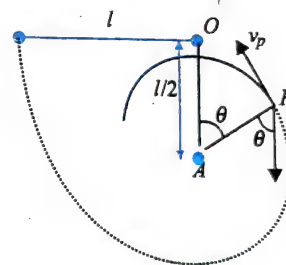
15. If at point P , tension is zero, then $mg \cos \theta = \frac{mv^2}{r}$

Using conservation of energy, $v^2 = gl(1 - \cos \theta)$

$$\therefore mg \cos \theta = \frac{mgl}{l/2} (1 - \cos \theta)$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{2}{3} \right)$$

$$\therefore \text{Height of point } P = \frac{l}{2} + \frac{l}{2} \cos \theta = \frac{5l}{6}, \text{ from lowest point.}$$



$$v^2 = gl \left(1 - \frac{2}{3} \right) = \frac{gl}{3} \Rightarrow v = \sqrt{\frac{gl}{3}}$$

Now the particle describes parabolic path.

The height attained by the particle, from point P .

$$h = \frac{(v \sin \theta)^2}{2g} = \frac{5l}{54}$$

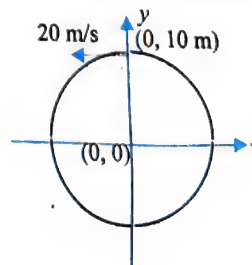
Therefore, highest point from lowest point will be

$$\left(\frac{5l}{6} + \frac{5l}{54} \right) = \frac{50l}{54}$$

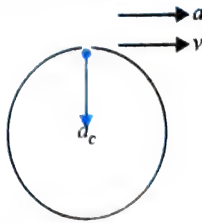
Exercises

Single Correct Answer Type

1. (2) From right hand rule, the velocity vector $\vec{v} = (-20\hat{i})\text{m/s}$



2. (3)



3. (2) $\vec{a}_r$ must be towards the centre of the circle. Therefore, (3) is wrong. $\vec{a}_T$ must be opposite to $\vec{v}$, hence options (1) and (4) are wrong. Correct answer (2).

4. (4) $\vec{v}$ and $\vec{a}_T$ are along tangent and $\vec{\omega}$ is perpendicular to plane of circular motion.

5. (3) $a_t = 2 \text{ ms}^{-2}$, $v = u + a_t t = 1 + 2 \times 2 = 5 \text{ ms}^{-1}$

$$a_c = \frac{v^2}{r} = \frac{5^2}{25} = 1 \text{ ms}^{-2}$$

$$\text{Net acceleration} = \sqrt{a_c^2 + a_t^2} = \sqrt{1^2 + 2^2} = \sqrt{5} \text{ ms}^{-2}$$

6. (3) $a_c = \frac{v^2}{r}$ i.e., constant in magnitude if v is constant.

7. (1) Angular velocity is always directed perpendicular to the plane of the circular path. Hence, required change in angle is 0° .

8. (2) Angular acceleration and angular velocity are along the axis of circular path. So they cannot be perpendicular to each other.

9. (4) $\alpha = (\omega_2 - \omega_1)/t = (400 - 100)/5 = 60 \text{ rev min}^{-2}$

$$= \frac{60 \times 2\pi}{(60)^2} = \frac{2\pi}{60} \text{ rad s}^{-2}$$

$$a_t = \alpha r = \frac{2\pi}{60} \times \frac{50}{100} = \frac{\pi}{60} \text{ ms}^{-2}$$

10. (1) $\tan \theta = \frac{v^2}{Rg} \Rightarrow \frac{h}{b} = \frac{v^2}{Rg} \Rightarrow h = \frac{v^2 b}{Rg}$

11. (1) $v_{\max} = \sqrt{\frac{Rg(\tan \theta + \mu)}{1 - \mu \tan \theta}}$

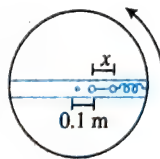
12. (2) $k = 10^2 \text{ N cm}^{-1} = 10^4 \text{ N m}^{-1}$.

Let the ball move distance x away from the center as shown in figure.

$$kx = m\omega^2(0.1 + x)$$

$$\Rightarrow 10^4 x = \frac{90}{1000} \times (10^2)^2 \times (0.1 + x)$$

$$\text{Solve to get } x \approx 10^{-2}$$

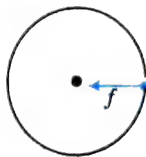


13. (2) $f_f = \mu mg$, friction will provide the necessary centripetal force $f = m\omega^2 r$

$$f \leq f_f \Rightarrow m\omega^2 r \leq \mu mg$$

$$\Rightarrow \mu \geq \frac{\omega^2 r}{g} = \frac{2^2 \times 50 / 100}{10}$$

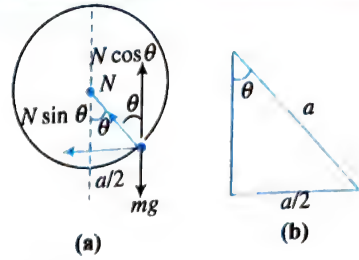
$$\Rightarrow \mu \geq 0.2$$



14. (1) $mg + T_2 = \frac{mv_2^2}{R}$, $T_1 - mg = \frac{mv_1^2}{R}$

Never confuse the actual force on a body (tension, force of gravity, etc.) with the effects they produce: ma , $\frac{mv^2}{R}$, etc.

15. (3)



$$N \cos \theta = mg \quad \dots(i)$$

$$N \sin \theta = \frac{m\omega^2 a}{2} \quad \dots(ii)$$

$$\text{From Eqs. (ii)/(i), } \tan \theta = \frac{\omega^2 a}{2g} \Rightarrow \omega^2 = \frac{2g \tan \theta}{a}$$

$$\text{Now, } \sin \theta = \frac{a}{2} \times \frac{1}{a} = \frac{1}{2} \text{ or } \theta = 30^\circ$$

$$\text{or } \omega^2 = \frac{2g}{\sqrt{3}a}$$

16. (3) In conical pendulum,

$$T \cos \theta = mg = \text{constant}$$

$$\Rightarrow T_1 \cos \theta_1 = T_2 \cos \theta_2$$

$$\frac{T_1}{T_2} = \frac{\cos \theta_1}{\cos \theta_2} = \frac{\cos 60^\circ}{\cos 45^\circ}$$

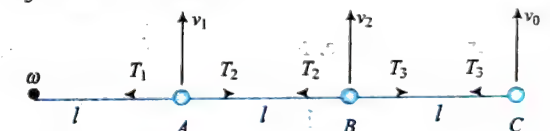
$$= \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{1}{2}$$

17. (1) According to question,

$$v_0 = \omega 3l, v_2 = \omega 2l = \frac{2v_0}{3} \text{ and } v_1 = \omega l = \frac{v_0}{3}$$

$$T_3 = m\omega^2 3l = 3m\omega^2 l$$



$$T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = 5m\omega^2 l$$

$$T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = 6m\omega^2 l$$

$$T_3 : T_2 : T_1 = 3 : 5 : 6$$

18. (4) $T = m\omega_1^2 l_1 \quad \dots(i)$

$$T \sin \theta = m\omega_2^2 l_2 \sin \theta \Rightarrow T = m\omega_2^2 l_2 \quad \dots(ii)$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{\omega_2^2}{\omega_1^2}$$

19. (4) FBD: Let N_1 and N_2 be the normal reactions offered by the vertical and horizontal surfaces respectively. N_1 pushes the block towards the centre of circle and N_2 equilibrate the block in vertical by nullifying the weight mg .

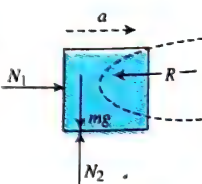
Force equation:

$$N_1 = ma_r = \frac{mv^2}{R} \quad \dots(i)$$

$$N_2 = mg \quad \dots(ii)$$

Hence, the net reaction (contact) force is $\vec{N} = \vec{N}_1 + \vec{N}_2$

$$|\vec{N}| = |\vec{N}_1 + \vec{N}_2| = \sqrt{N_2^2 + N_1^2}$$



Substituting N_1 and N_2 , we have $N = m\sqrt{\frac{v^4}{R^2} + g^2}$

20. (2) When the particle comes with the contact of hemisphere surface, it will slide and friction will be of kinetic nature. This kinetic friction will provide tangential acceleration to the particle.

Tangential acceleration of the particle,

$$a_t = \frac{\mu mg}{m} = \mu g = \frac{g}{2} \leftarrow$$

Normal acceleration of the particle,

$$a_n = \frac{v^2}{R} = \frac{gR}{R} = g \uparrow$$

Net acceleration of the particle,

$$a_{\text{net}} = \sqrt{a_t^2 + a_n^2} = \sqrt{\frac{g^2}{4} + g^2} = \frac{\sqrt{5}g}{2} \swarrow$$

21. (4) As car travels with constant speed. The net force on the car should be towards the centre. The resultant of F_{air} and F_B may towards the centre.

22. (4) Change in linear momentum of block

$$|\Delta p| = 2mv$$

Time taken to move from A to B,

$$\Delta t = \frac{\pi R}{v}$$

$$\text{Force exerted on the block} = \frac{\Delta p}{\Delta t} = \frac{2mv^2}{\pi R}$$

23. (4) Maximum velocity for car A

$$v_{A,\text{max}} = \sqrt{\mu gr} = \sqrt{0.1 \times 10 \times 100} = 10 \text{ ms}^{-1}$$

$$v_{B,\text{max}} = \sqrt{\mu_B g R_B} = \sqrt{0.2 \times 10 \times 200} = 20 \text{ ms}^{-1}$$

Time taken by cars and A and B

$$t_A = \frac{200\pi + \pi \cdot 100}{100} = 3\pi \text{ s}; t_B = \frac{200\pi + \pi \times 200}{20} = 2\pi \text{ s}$$

24. (4) F is centrifugal force. Net force is along AB. So seat will be aligned along AB.

25. (2) Given that $K = as^2$ or $\frac{1}{2}mv^2 = as^2$

$$\text{or } mv^2 = 2as^2$$

Differentiating w.r.t. time, we get

$$m(2v) \times \frac{dv}{dt} = 2a \times 2s \times \frac{ds}{dt}$$

$$\text{But } \frac{ds}{dt} = v$$

$$\text{So } 2m \frac{dv}{dt} = 4as \text{ or } m \frac{dv}{dt} = 2as$$

$$\text{Now, } m \frac{dv}{dt} = \text{tangential force} = F_t$$

$$F_t = 2as$$

$$\text{Centripetal force } F_r = \frac{mv^2}{R} = \frac{2as^2}{R}$$

$$F_{\text{net}} = \sqrt{F_t^2 + F_r^2} = \sqrt{(2as)^2 + \left(\frac{2as^2}{R}\right)^2} = 2as \sqrt{1 + \frac{s^2}{R^2}}$$

26. (3) $a_c = \frac{v^2}{r} = k^2 r t^2$ or $v = krt$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}mk^2r^2t^2$$

By work-energy theorem,

$$W = \Delta K = \frac{1}{2}mk^2r^2t^2 - 0$$

$$P = \frac{dW}{dt} = mk^2r^2t$$

Alternative method: $a_t = \frac{dv}{dt} = kr$

Power is given by tangential force only. So, power = $F_t v = ma_t v = mk^2r^2t$

Power of centripetal force is zero.

27. (3) Given $a_n = kt^2$ or $\frac{v^2}{r} = kt^2$ or $v^2 = krt^2$

Therefore, average power delivered = $\frac{\text{Total work done}}{\text{Total time elapsed}} = \frac{\text{Increase in KE}}{\text{Total time elapsed}}$

$$\text{or } \langle P \rangle = \frac{\frac{1}{2}m(v^2 - 0^2)}{t_0} = \frac{m}{2} \frac{krt_0^2}{t_0} = \frac{mkr t_0}{2}$$

Alternative: $v = \sqrt{kr}t \Rightarrow a_t = \sqrt{kr}$

$$P = F_t v = ma_t v = mkr t$$

$$\langle P \rangle = \frac{\int_0^{t_0} P dt}{t_0} = \frac{1}{2}mkr t_0$$

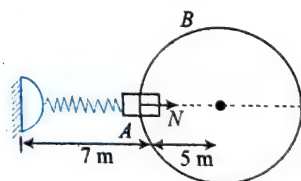
28. (3) Initial extension will be equal to 6 m.

$$\text{Initial energy} = \frac{1}{2}(200)(6)^2 = 3600 \text{ J}$$

$$\text{Reaching A, we get } \frac{1}{2}mv^2 = 3600 \text{ J}$$

$$mv^2 = 7200 \text{ J}$$

$$\text{From FBD at A, we get } N = \frac{mv^2}{R} = \frac{7200}{5} = 1440 \text{ N}$$



29. (1) KE of blocks at B = PE at A - PE at B

$$\frac{1}{2}mv^2 = mgh - mg2r = mg(h - 2r)$$

$$v^2 = 2g(h - 2r)$$

$$\text{Also, } \frac{mv^2}{r} = xmg + mg$$

$$\text{or } v^2 = (x + 1)rg$$

$$\text{Equating Eqs. (i) and (ii), we get } 2g(h - 2r) = (x + 1)rg \quad \dots(ii)$$

$$\text{or } 2gh = (x + 1)gr + 4gr = (x + 5)gr$$

$$h = \left(\frac{x + 5}{2}\right)r$$

30. (4) $N_A + mg \cos 60^\circ = \frac{mv_A^2}{R}$

$$N_A = \frac{mv_A^2}{R} - mg \cos 60^\circ$$

$$= 6mg - 0.5mg = 5.5mg$$

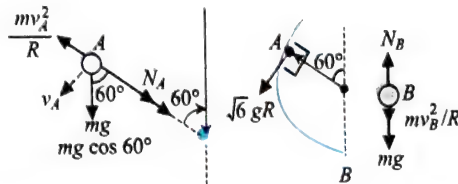
$$\frac{1}{2}mv_A^2 + mgR(1 + \cos 60^\circ) = \frac{1}{2}mv_B^2$$

$$v_A^2 + 2gR \left(1 + \frac{1}{2}\right) = v_B^2$$

$$6gR + 3gR = v_B^2$$

$$v_B^2 = 9gR \Rightarrow v_B = \sqrt{9gR}$$

$$N_B = mg + \frac{mv_B^2}{R} = 10mg$$



31. (3) Conservation of energy gives

$$\frac{1}{2}mv^2 = mgR \sin \theta \Rightarrow \frac{mv^2}{R} = 2mg \sin \theta = F_C$$

$$\text{Now, } N - mg \sin \theta = F_C \Rightarrow N = 3mg \sin \theta$$

$$\therefore \frac{F_C}{N} = \frac{2}{3}$$

32. (3) Velocity at lowest position = $\sqrt{2gl(1 - \cos 60^\circ)} = \sqrt{gl}$

$$T = mg + \frac{mv^2}{l} = mg + \frac{m}{l}gl = 2mg$$

$$\therefore T = \mu 4mg \Rightarrow \mu = \frac{1}{2}$$

33. (1) $mgh = \frac{1}{2}mv^2 + 2mgR$

$$gh = \frac{v^2}{2} + 2gR$$

$$\text{Required velocity at the top, } v = \sqrt{gR}$$

$$gh = \frac{gR}{2} + 2gR \Rightarrow h = \frac{5}{2}R$$

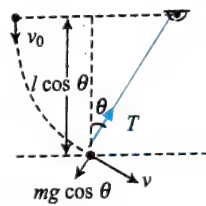
34. (2) $v^2 = v_0^2 + 2gl \cos \theta$

$$T = mg \cos \theta + \frac{mv^2}{l}$$

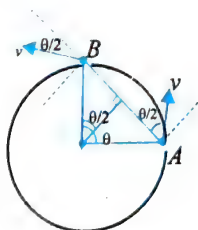
$$\text{Given } T = 2mg$$

$$2mg = mg \cos \theta + \frac{m}{l}(v_0^2 + 2gl \cos \theta)$$

$$\Rightarrow \cos \theta = \frac{1}{4}$$



35. (3)



$$\omega_{B,A} = \frac{\text{velocity of B w.r.t. A perpendicular to line AB}}{\text{length of AB}}$$

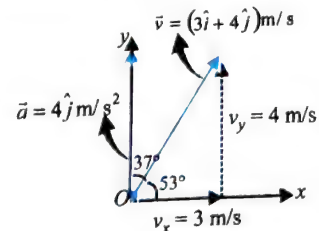
$$\omega_{B,A} = \frac{2v \sin(\theta/2)}{2R \sin(\theta/2)} = \frac{v}{R}$$

36. (4) $\frac{dy}{dx} = \tan \theta = x \Rightarrow \sin \theta = \frac{x}{\sqrt{x^2 + 1}}$

$$\text{Tangential acceleration} = g \sin \theta = \frac{gx}{\sqrt{x^2 + 1}}$$

37. (1) At $t = 1s$, $\vec{v} = (3\hat{i} + 4\hat{j})m/s \Rightarrow v = \sqrt{3^2 + 4^2} = 5m/s$

$$\text{Normal acceleration, } a_n = \frac{v^2}{R}$$



$$\text{Total acceleration, } \vec{a} = \frac{d\vec{v}}{dt} = 4\hat{j}$$

The direction of tangential acceleration should be along its velocity

$$a_t = 4 \cos 37^\circ = \frac{16}{5}, a_n = \frac{12}{5} \Rightarrow \frac{a_t}{a_n} = \frac{16}{12} = \frac{4}{3}$$

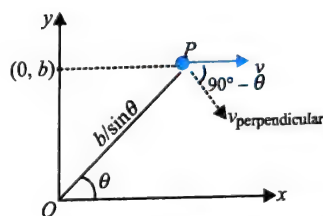
38. (1) Angular acceleration, $\alpha = \frac{a_t}{R} = \frac{5}{100} = \frac{1}{20} \text{ rad/s}^2$

$$\text{But } \alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta} \Rightarrow \int_0^{2\pi} d\theta = 20 \int_0^\omega \omega d\omega$$

$$[\theta]_0^{2\pi} = 20 \left[\frac{\omega^2}{2} \right]_0^\omega \Rightarrow \omega = \sqrt{\frac{\pi}{5}} \text{ rad/s}$$

$$\Rightarrow a_r = \omega^2 R = 20 \pi \text{ m/s}^2$$

39. (1)



$$\omega_0 = \frac{\text{velocity perpendicular to line OP}}{\text{length of OP}}$$

$$\omega_0 = \frac{v \sin \theta}{b / \sin \theta} = \frac{v \sin^2 \theta}{b}$$

40. (4) $\omega_1 = \frac{v_1}{R_1} = 2 \text{ rad/s}$

$$\omega_2 = \frac{v_2}{R_2} = \frac{15}{3} = 5 \text{ rad/s}$$

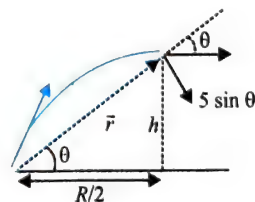
$$\theta_2 = \theta_1 + \pi$$

$$\omega_2 t = \omega_1 t + \pi \Rightarrow t = \frac{\pi}{\omega_2 - \omega_1} = \frac{\pi}{5 - 2} = \frac{\pi}{3} \text{ s}$$

41. (1) $\tan \theta = \frac{v^2}{Rg} \Rightarrow v = \sqrt{Rg \tan \theta} \Rightarrow v \propto \sqrt{R}$

$$T = \frac{2\pi R}{v} \Rightarrow T \propto \sqrt{R}$$

42. (1)



$$\omega = \frac{5 \sin \theta}{r}; \tan \theta = \frac{h}{(R/2)} = 1$$

$$\text{So, } \omega = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ rad/s}$$

43. (1) Velocity of ball when it enters the circular track $= \sqrt{2gh}$

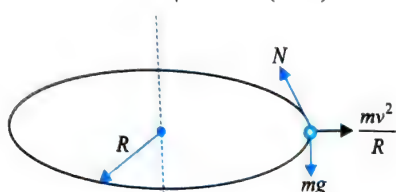
$$\therefore N = \frac{mv^2}{R} = \frac{2mgh}{R}$$

44. (3) $mg \cos 53^\circ + T = m\omega^2 R \sin 53^\circ$... (i)

$$T = mg$$

From (i) and (ii), we get $\omega = 10 \text{ rad/s}$

45. (3) Net normal force, $N = \sqrt{(mg)^2 + \left(\frac{mv^2}{R}\right)^2}$



$$f_k = \mu N$$

46. (1) Suppose velocity at an instant is v . Then $N = \frac{mv^2}{r}$... (i)

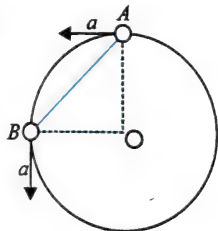
$$\text{and } -\mu N = m \frac{dv}{dt}$$

$$\text{From Eqs. (i) and (ii), } -\frac{\mu v^2}{r} = \frac{dv}{dt}$$

$$\int_{v_0}^v \frac{dv}{v^2} = \frac{\mu}{r} \int_0^t dt$$

On solving we get v at time t .

47. (3)



Both rings will have equal acceleration as shown

$$Mg - T \cos 45^\circ = ma$$

$$T \sin 45^\circ = ma \Rightarrow a = g/2$$

48. (4) Let velocity at end A is v .

The components of velocities at ends, along the length of the rod should be equal, $10 \cos 37^\circ = v \cos \theta$... (i)

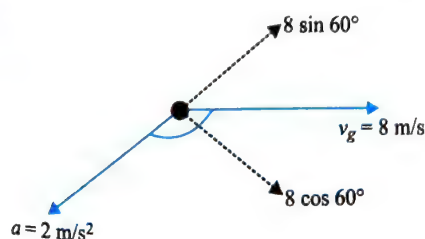
And angular velocity, $\omega = \frac{v \sin \theta + 6}{L}$... (ii)

From Eqs. (i) and (ii), we get $v = 8\sqrt{2} \text{ m/s}$

Now from Eq. (i),

$$\cos \theta = \frac{10 \cos 37^\circ}{v} = \frac{10 \times 4/5}{8\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

49. (3)

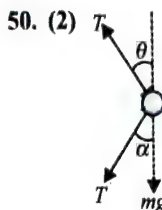


Radius of curvature is minimum when $a_N = 2 \text{ m/s}^2$

So, velocity component along acceleration should be zero.

At certain instant velocity parallel to acceleration will be zero then velocity $= 8 \cos 60^\circ$

$$\text{Radius of curvature} = \frac{v^2}{a_\perp} = \frac{(8 \cos 60^\circ)^2}{2} = 8 \text{ m}$$



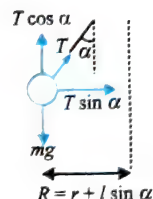
Tension will be same in both parts.

$$T \cos \theta = mg + T \cos \alpha \Rightarrow \alpha > \theta$$

51. (3) $T \sin \alpha = m\omega^2 R \Rightarrow T \cos \alpha = mg$

$$\tan \alpha = \frac{\omega^2 R}{g}$$

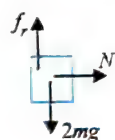
$$\omega = \sqrt{\frac{g \tan \alpha}{R}} = \sqrt{\frac{g \tan \alpha}{r + l \sin \alpha}}$$



52. (1) $f_r = \mu N = 2mg \Rightarrow N = \frac{2mg}{\mu}$... (i)

$$N = 2m\omega^2 l$$

$$\text{From Eqs. (i) and (ii), } \omega = \sqrt{\frac{g}{\mu l}}$$



53. (4) $\alpha = k \sin \theta$

$$\int \omega d\omega = \int k \sin \theta d\theta$$

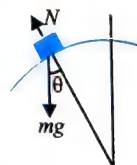
$$\frac{\omega^2}{2} = -[k \cos \theta]_0^\theta \Rightarrow \frac{\omega^2}{2} = -k[\cos \theta - 1]$$

$$\omega^2 = 2k(1 - \cos \theta)$$

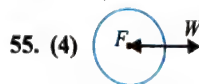
$$a_c = \omega^2 R = 2kR(1 - \cos \theta)$$

54. (2) $mg \cos \theta - N = \frac{mv^2}{R}$

$$N = mg \cos \theta - \frac{mv^2}{R}$$



Hence N decrease as θ increases.



When ship is at rest

$$F - W = \frac{m(R\omega)^2}{R} \quad \dots (i)$$

When ship begins to move

$$F - W' = \frac{m(v + R\omega)^2}{R} \quad \dots (ii)$$

Using Eq. (i) and applying binomial approximation

$$W - W' = 2mv\omega \Rightarrow W' = W$$

56. (4) Case-I: When they rotate in same sense

$$2m\pi = 2\omega t$$

$$\begin{aligned}\frac{3\pi}{2} + 2n\pi &= \omega t \\ 2m\pi &= 2\left(\frac{3\pi}{2} + 2n\pi\right) \\ 2m &= 3\pi + 4n \\ m &= \frac{3}{2} + 2n \\ \Rightarrow m - 2n &= \frac{3}{2}\end{aligned}$$

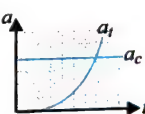
Not possible as m and n are integers.

Case-2: When they rotate in opposite sense

$$\begin{aligned}2m\pi &= 2\omega t \\ \frac{\pi}{2} + 2n\pi &= \omega t \\ 2m\pi &= 2\left(\frac{\pi}{2} + 2n\pi\right) \\ 2m\pi &= \pi + 4n\pi \\ 2m - 4n &= 1\end{aligned}$$

Not possible as m and n are integer.

57. (4) $v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow x = \frac{\alpha^2 t^2}{4}$



$$a_t = v \frac{dv}{dx} = \alpha\sqrt{x} \cdot \frac{\alpha}{2\sqrt{x}} = \frac{\alpha^2}{2} = \text{constant}$$

$$a_c = \frac{v^2}{r} = \frac{\alpha^2 x}{r} = \frac{\alpha^2}{r} \times \frac{\alpha^2 t^2}{4}$$

$$\Rightarrow a_c \propto t^2$$

Multiple Correct Answers Type

1. (1), (3), (4)

Angular speed is constant. Hence, linear speed is also constant, i.e., magnitude of velocity is constant, but direction is changing.

2. (1), (2), (3), (4)

$$v = 2t, a_c = \frac{v^2}{r} = \frac{(2t)^2}{0.2} = 20t^2 = 20 \times 2^2 = 80 \text{ ms}^{-2}$$

$$a_t = \frac{dv}{dt} = 2 \text{ ms}^{-2}$$

$$\text{Net acceleration, } a = \sqrt{a_c^2 + a_t^2} > 80 \text{ ms}^{-2}$$

3. (2), (3), (4)

Given tangential acceleration

$$\begin{aligned}\frac{dv}{dt} &= 3; v = 3t \\ a_c &= \frac{v^2}{r} = \frac{9t^2}{50} \\ 3 &= \frac{9 \times t^2}{50} \Rightarrow t = \sqrt{\frac{50}{3}} \text{ s}\end{aligned}$$

The angular acceleration of car, $\alpha = \frac{a}{r} = \frac{3}{50} \text{ rad s}^{-2}$

The angle rotated by car,

$$\theta = \frac{1}{2} \alpha t^2 = \frac{1}{2} \times \frac{3}{50} \times \frac{50}{3} \text{ rad} = \frac{1}{2} \text{ rad}$$

Distance travelled by car upto this instant is

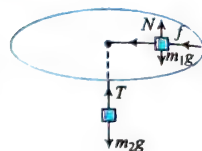
$$s = \theta \cdot R = \frac{1}{2} \times 50 = 25 \text{ m}$$

Net acceleration of the car is

$$a_{\text{total}} = \sqrt{a_r^2 + a_t^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2} \text{ ms}^{-2}$$

4. (1), (3), (4)

FBD: Let the friction be directed radially inward. The forces on m_1 are $f \leftarrow$ (assumed), $T \leftarrow$, $N \uparrow$, $m_1 g \downarrow$. The forces on m_2 are $m_2 g \downarrow$ and $T \uparrow$ as shown in figure.



Force equation:

For m_1 :

$$\Sigma F_y = N - m_1 g = m_1 a_{1y} \quad \dots(i)$$

$$\Sigma F_r = f + T = m_1 a_r \quad \dots(ii)$$

For m_2 :

$$\Sigma F_y = T - m_2 g = m_2 a_{2y} \quad \dots(iii)$$

Law of static friction:

$$f \leq \mu N \quad \dots(iv)$$

Kinematics: $a_{1y} = 0$

$$a_r = \frac{v^2}{R} \quad \dots(vi)$$

$$a_{2y} = 0 \quad \dots(vii)$$

Solving the above equations, we have

$$f = \frac{m_1 v^2}{R} - m_2 g = m_1 \omega^2 R - m_2 g$$

Now substituting the above value of f and N in Eq. (iv)

$$\text{we have } \left(\frac{m_1 v^2}{R} - m_2 g \right) \leq \mu m_1 g$$

$$\text{This gives } v_{\text{max}} = \sqrt{\left(\frac{m_2 + \mu m_1}{m_1} \right) g R}$$

5. (1), (2), (4)

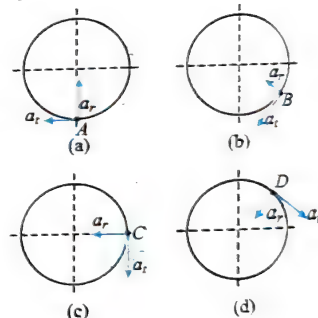
At A: Direction of net acceleration should be vertical up (Choice II).

At B: Net acceleration direction may be anywhere between a_r and a_t .

Hence, possible choices are (I), (IV) and (V).

At C: Possible choice is (IV).

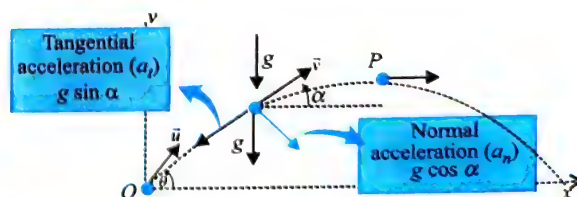
At D: Possible choices are (III) and (IV).



6. (1), (2), (3), (4)

$$\frac{d\vec{v}}{dt} = \text{is total acceleration} = \vec{g}$$

$$\text{Modulus of } \frac{d|\vec{v}|}{dt} \text{ is the tangential acceleration} = g \sin \alpha$$



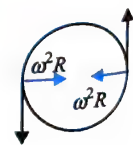
$$\text{Normal acceleration, } a_{\text{normal}} = g \cos \alpha$$

7. (1), (3)

$$\omega = \frac{2\pi}{2} = \pi \text{ rad/s}$$

$$\Delta a_c = 2\omega^2 R = 2\pi^2 R = 6 \Rightarrow R = \frac{3}{\pi^2} \text{ m}$$

$$v = \omega R = \frac{3}{\pi} \text{ m/s}$$



Sense of rotation can not be find out with given information.

$$\Delta v = 2v = \frac{6}{\pi} \text{ m/s}$$

8. (2), (3), (4)

$$T_1 \sin 30^\circ = T_2 \sin 30^\circ + mg \quad \dots(i)$$

$$(T_1 + T_2) \cos 30^\circ = m\omega^2 l (\cos 30^\circ) \quad \dots(ii)$$

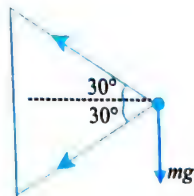
$$T_1 + T_2 = m\omega^2 l$$

$$T_1 - T_2 = 2mg$$

$$2T_2 = m\omega l - 2mg$$

For $T_2 \geq 0$,

$$m\omega^2 l - 2mg \geq 0 \Rightarrow \omega \geq \sqrt{\frac{2g}{l}}$$



9. (1), (3), (4)

Bob moves on a circular path with speed v in the frame of train which is inertial frame so no pseudo force is required.

$$T = \frac{mv^2}{l} \quad \dots(i)$$

The vector sum of velocity of train and velocity of bob w.r.t. train gives net velocity in ground frame. So options (1) and (4) are correct.

10. (1), (3)

A is ground frame so no pseudo force acts. Motion is circular and real force (friction) will provide centripetal acceleration.

B is non inertial frame and object is at rest in this frame. So centrifugal force is balanced by friction.

11. (1), (3)

$$T = m\omega_1^2 l_1, T = m\omega_2^2 l_2$$

$$T \cos \theta = mg \Rightarrow \cos \theta = \frac{mg}{T} < 1$$

$$\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} = \frac{ml}{T} < \frac{l}{g}$$

12. (1), (2), (3)

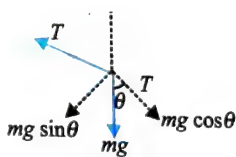
From FBD of bob

$$T = mg \cos \theta = \frac{\sqrt{3}}{2} mg$$

Acceleration of the bob is provided by left out component.

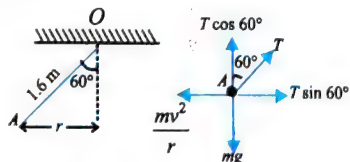
$$ma = mg \sin \theta = \frac{mg}{2} \Rightarrow a = \frac{g}{2}$$

$$\text{On pulley resultant force} = T = \frac{\sqrt{3}}{2} mg$$



13. (1), (2), (3), (4)

For conical pendulum



$$T \sin 60^\circ = \frac{mv^2}{r} \quad \dots(i)$$

$$T \cos 60^\circ = mg \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \tan 60^\circ = \frac{v^2}{rg}$$

$$\text{or } v^2 = rg \tan 60^\circ \quad \dots(iii)$$

Since, in case of conical pendulum, $r = L \sin$

$$\therefore r = 1.6 \sin 60^\circ = 0.8\sqrt{3} \text{ m} \quad \dots(iv)$$

Substituting value of r in Eq. (iii)

$$v^2 = 0.8\sqrt{3} \times 9.8 \times \sqrt{3} = (0.8 \times 9.8 \times 3) \text{ m}^2 \text{ s}^{-2}$$

$$v = 2.8\sqrt{3} \text{ m/s}$$

Time period of revolution is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg \tan 60^\circ}} \quad (\text{using Eq. (iii)})$$

$$= 2\pi \sqrt{\frac{r}{g \tan 60^\circ}} = 2\pi \sqrt{\frac{L \sin 60^\circ}{g \tan 60^\circ}} = 2\pi \sqrt{\frac{L \cos 60^\circ}{g}}$$

$$= 2\pi \sqrt{\frac{0.8}{9.8 \times 2}} = \frac{4\pi}{7} \text{ s}$$

Centripetal acceleration,

$$a_c = \frac{v^2}{r} = \frac{rg \tan 60^\circ}{r} = g \tan 60^\circ = 9.8\sqrt{3} \text{ m/s}^2$$

From Eq. (ii),

$$T = \frac{mg}{\cos 60^\circ} = 2mg$$

Therefore Tension in the string is doubled the weight of the particle.

Hence, options (1), (2), (3) and (4) are correct.

14. (1), (2), (4)

Since $\vec{F} \perp \vec{v}$, motion is circular

$$F = \frac{mv^2}{R} \text{ and } \theta = \frac{S}{R}$$

$$\text{or } \theta = \frac{S}{\left(\frac{mv^2}{F}\right)} = \left(\frac{FS}{mv^2}\right)$$

15. (1), (3)

$$\frac{1}{2} k(2R)^2 + mgR = \frac{1}{2} mv^2 + \frac{1}{2} k2R^2$$

$$v = \sqrt{\frac{2kR^2}{m} + 2gR}$$

$$v = \sqrt{3gR}$$

$$\text{Force, } N = \frac{mv^2}{R} = \frac{2kR^2 + 2mgR}{R} = 2kR + 2mg$$

$$N = 3mg$$

16. (1), (3), (4)

By the conservation of energy

$$mgl + mg2l = \frac{1}{2} mv^2 + \frac{1}{2} m(2v)^2$$

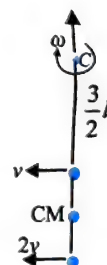
$$3mgl = \frac{5}{2} mv^2$$

$$v = \sqrt{\frac{6gl}{5}}$$

$$\omega = \frac{v}{l} = \sqrt{\frac{6g}{5l}}$$

$$\text{Tension in the rod AB, } T - mg = \frac{m(2v)^2}{2l}$$

$$T = mg + \frac{2m \times 6gl}{l \cdot 5} = \frac{17}{5} mg$$



Tension in the rod BC, $T' - \frac{17}{5}mg - mg = \frac{mv^2}{l}$

$$T' = \frac{17}{5}mg + mg + \frac{m6gl}{5l} = \frac{28}{5}mg$$

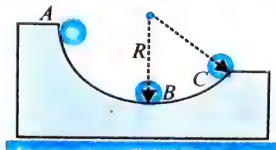
Hinge reaction $N - 2mg = 2mR\omega^2$

$$N = 2mg + 2m \frac{3l}{2} \times \frac{6g}{5l} = \frac{28}{5}mg$$

17. (1), (2), (4)

The ball will acquire maximum velocity when gravity will do maximum work i.e., at B

By momentum conservation, $2mv + mv = 0$



$$\Rightarrow v = -v/2 \text{ (velocities considered from ground frame)}$$

Work energy theorem: $mgR = \frac{1}{2}mv^2 + \frac{1}{2}2mv^2$

$$\Rightarrow v = \sqrt{\frac{4gR}{3}} \quad \therefore \text{Option (1) is correct}$$

Let velocity at point of leave off is $v_x \hat{i} + v_y \hat{j}$ from ground, so at leave off point,

$$mv_x + 2mv' = 0$$

$$\Rightarrow v' = -\frac{v_x}{2} \quad (v' \text{ is velocity of wedge at leave off point})$$

Work energy theorem:

$$mg \left(R - \frac{R}{2} \right) = \frac{1}{2}2mv'^2 + \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2$$

$$\Rightarrow \frac{gR}{2} = \frac{3}{4}v_x^2 + \frac{1}{2}v_y^2 \quad \dots(i)$$

Constraint at leave off point:

$$v' \cos 30^\circ = v_y \cos 60^\circ - v_x \cos 30^\circ \Rightarrow v_x = \frac{2v_y}{3\sqrt{3}}$$

Put v_x in Eq (i), we get $v_y = \sqrt{\frac{9gR}{11}}$

Maximum height attained from leave off point $= \frac{v_y^2}{2g} = \frac{9R}{22}$

$$\text{So height from ground} = \frac{R}{2} + \frac{9R}{22} = \frac{20R}{22} = \frac{10R}{11}$$

(Option (2) is correct)

Maximum velocity of wedge $= \frac{v}{2} = \frac{1}{2} \sqrt{\frac{4gR}{3}} = \sqrt{\frac{gR}{3}}$

[Option (4) is correct]

18. (1), (3)

$$T \cos 30^\circ + N \sin 30^\circ = mg$$

$$\Rightarrow \sqrt{3}T + N = 2mg \quad \dots(i)$$

$$T \sin 30^\circ - N \cos 30^\circ = \frac{mv^2}{\sqrt{3}/2}$$

$$\sqrt{3}T - 3N = 4mv^2 \quad \dots(ii)$$

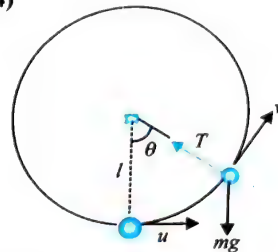
From Eqs. (i) and (ii) $\Rightarrow N = \frac{2mg - 4mv^2}{4}$

$$T = \frac{6mg + 4mv^2}{4\sqrt{3}}$$

For $N > 0 \Rightarrow v < \sqrt{5} \text{ m/s}$

At $v = 2 \text{ m/s}$, $T = \frac{38}{\sqrt{3}} \text{ N}$ and $N = 2 \text{ N}$.

19. (2), (3), (4)



By energy conversion

$$\frac{1}{2}mu^2 = mgl(1 - \cos \theta) + \frac{1}{2}mv^2$$

$$\Rightarrow a_r = \frac{v^2}{l} = \frac{u^2}{l} - 2g + 2g \cos \theta$$

$$a_t = g \sin \theta \Rightarrow a_{t, \max} = g \text{ and } a_{t, \min} = 0$$

$$a_{r, \max} = \frac{u^2}{l}, a_{t, \min} = \frac{u^2}{l} - 4g$$

At $\theta = 90^\circ$

$$a_r = \frac{u^2}{l} - 2g \Rightarrow a_{r, \min} < \frac{u^2}{l} - 2g < a_{r, \max}$$

$$a_t = g = a_{t, \max}$$

When $a_r = a_t$: $\frac{u^2}{l} - 2g + 2g \cos \theta = g \sin \theta$

$$\frac{u^2}{gl} - 2 = \sin \theta - 2 \cos \theta \quad \dots(i)$$

Since ball performs complete circle,

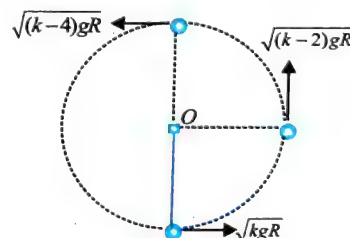
$$u^2 \geq 5gl: \frac{u^2}{gl} - 2 \geq 3$$

LHS of eq. (i) ≥ 3

But maximum value of $\sin \theta - 2 \cos \theta$ is $\sqrt{1+2^2} = \sqrt{5}$. So for no θ , $a_r = a_t$.

20. (2), (3)

Maximum speed at lowest position and minimum speed at highest position.



Option (2): $\frac{v_{\max}}{v_{\min}} = \sqrt{\frac{k}{k-4}} = \lambda$

$$\Rightarrow v_{\text{top}} = \sqrt{(k-4)gR} = \sqrt{\left(\frac{4}{\lambda^2-1}\right)gR}$$

Option (3): $T_{\max} = \frac{Mv_{\text{bottom}}^2}{R} + Mg = Mg(k+1)$

$$T_{\min} = -Mg + \frac{Mv_{\text{top}}^2}{R} = (k-5)Mg = \left(\frac{5-\lambda^2}{\lambda^2-1}\right)Mg$$

$$\therefore \frac{T_{\max}}{T_{\min}} = \left(\frac{5\lambda^2 - 1}{5 - \lambda^2} \right)$$

Option (4): $T_{\text{Horizontal}} = \frac{mv^2}{R} = \left(\frac{2\lambda^2 + 2}{\lambda^2 - 1} \right) Mg$

$$\therefore \frac{T}{Mg} = 2 \left(\frac{\lambda^2 + 1}{\lambda^2 - 1} \right)$$

Linked Comprehension Type

For Problems 1–3

1. (1) Since surface is almost flat at position 1. Acceleration has not radial component is only tangential.
2. (3) At position 2, no tangential component, only radial component exists i.e. upwards.
3. (1) After loosing contact from ramp, the body falls freely i.e. acceleration is downwards.

For Problems 4 and 5

4. (3) $f = m\omega^2 R$

$$= m \frac{g}{2R} \cdot R = \frac{mg}{2} \leftarrow$$

5. (4) If coin is gently placed on the rotating plate, initially the coin will slide with respect to plate and friction will be kinetic in nature.

$$f = \mu N = \mu mg = \frac{3}{4} mg$$

For Problems 6–8

6. (4) At the lowest point: $T_L = mg \sin 37^\circ + \frac{mv_r^2}{l}$

$$\Rightarrow 274 = 3 \times 10 \times \frac{3}{5} + \frac{3v_L^2}{0.75} \Rightarrow v_L = 8 \text{ ms}^{-1}$$

7. (1) Let the speed at the highest point be v_H .

$$\frac{1}{2} mv_H^2 + mg2l \sin 37^\circ = \frac{1}{2} mv_L^2 \Rightarrow v_H = \sqrt{46} \text{ ms}^{-1}$$

8. (1) $T_H + mg \sin 37^\circ = \frac{mv_H^2}{l} \Rightarrow T_H = 166 \text{ N}$

For Problems 9 and 10

9. (4) Let the velocity at the top point is v_C . Then for projectile motion from C to A,

$$\text{Range} = v_C \sqrt{\frac{2(\text{height})}{g}} \Rightarrow 3R = v_C \sqrt{\frac{2 \times 2R}{g}}$$

$$\Rightarrow v_C = \frac{3}{2} \sqrt{Rg}$$

To find velocity at B, apply conservation of energy, i.e.,

$$\frac{1}{2} mv_B^2 = mg2R + \frac{1}{2} mv_C^2 \Rightarrow v_B = \frac{5}{2} \sqrt{Rg}$$

10. (1) Minimum velocity at C: $v_C = \sqrt{gR}$

$$\text{Range } v_C \sqrt{\frac{2 \times 2R}{g}} = 2R$$

This is the minimum value of x .

For Problems 11–13

11. (3)

$$v_1 = \frac{2}{\sqrt{5}} \sqrt{5gR} = 2\sqrt{gR}$$

$$\text{Now, } v_2^2 = v^2 - 2g(R + R \sin \theta) \\ = 2gR - 2gR \sin \theta$$

$$N + mg \sin \theta = \frac{mv_2^2}{R}$$

$$\text{Put } N = 0$$

$$\Rightarrow mg \sin \theta = m2g(1 - \sin \theta)$$

$$\Rightarrow \sin \theta = \frac{2}{3}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{2}{3} \right)$$

12. (1) $v^2 = v_1^2 - 2gR \Rightarrow v^2 = 4gR - 2gR$

$$\Rightarrow v = \sqrt{2gR}$$

$$a_c = \frac{v^2}{R} = 2g$$

$$a_t = g$$

$$\tan \alpha = \frac{a_t}{a_c} = \frac{g}{2g} = \frac{1}{2} \Rightarrow \alpha = \tan^{-1} \left(\frac{1}{2} \right)$$

13. (4) Maximum contact force will occur at the lowest point A.

For Problems 14 and 15

14. (4) $\vec{F}_{\text{net}} = \vec{N} + \vec{W} + \vec{f} = m\vec{a}_r$

$$\Rightarrow |\vec{N} + \vec{W} + \vec{f}| = m|\vec{a}_r|$$

But centripetal acceleration $\vec{a}_r$ has constant magnitude.

$$\Rightarrow |\vec{N} + \vec{W} + \vec{f}| = \text{constant.}$$

15. (3) $\vec{W} + \vec{N} + \vec{f} = m\vec{a}_r \Rightarrow \vec{W} + \vec{f} = m\vec{a}_r - \vec{N}$

In the above equation, LHS & RHS are always directed towards or away from the centre.

For Problems 16 and 17

16. (4) $\frac{1}{2} mu^2 = mgh$

$$h = \frac{u^2}{2g} = \frac{5/2gl}{2g} = \frac{5}{4}l$$

$$h = l(1 - \cos \theta)$$

$$\frac{5}{4}l = l(1 - \cos \theta)$$

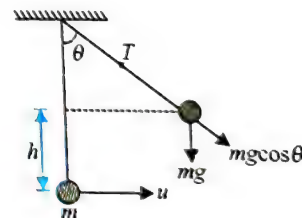
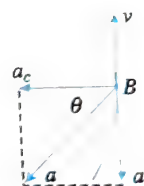
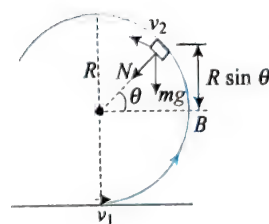
$$\cos \theta = -\frac{1}{4}$$

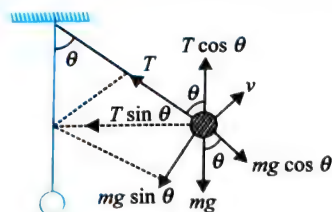
$$\theta = \pi - \cos^{-1} \left(\frac{1}{4} \right)$$

17. (2) If acceleration is horizontal

$$T \cos \theta = mg \Rightarrow T = \frac{mg}{\cos \theta} \quad \dots(i)$$

$$T - mg \cos \theta = \frac{mv^2}{l} \quad \dots(ii)$$





$$v^2 = u^2 - 2gl(1 - \cos\theta)$$

From Eqs. (i), (ii) and (iii),

$$\Rightarrow \frac{mg}{\cos\theta} - mg \cos\theta = \frac{m}{l} \left[\frac{mgl}{2} - 2gl(1 - \cos\theta) \right]$$

$$\frac{1}{\cos\theta} - \cos\theta = \frac{5}{2} - 2 + 2\cos\theta$$

$$\frac{1}{\cos\theta} = 3\cos\theta + \frac{1}{2}$$

$$2 = 6\cos^2\theta + \cos\theta$$

$$\Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

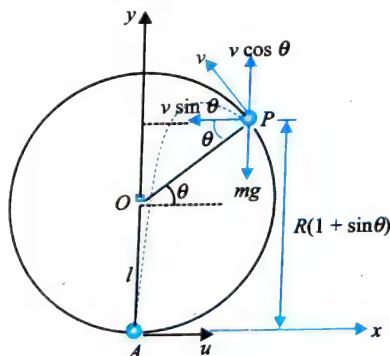
Substituting value of θ in Eq. (i),

$$T = 2mg$$

For Problems 18 and 19

18. (3), 19. (2)

$$y - y_0 = u_y t + \frac{1}{2} a_y t^2$$



$$0 - (R + R \sin\theta) = v \cos\theta t - \frac{1}{2} g t^2 \quad \dots(i)$$

$$mg \sin\theta + N = \frac{mv^2}{R} \quad \dots(ii)$$

When loses contact $N = 0$

$$t = \frac{R \cos\theta}{v \sin\theta} \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get $\theta = 30^\circ$

For Problems 20 and 21

20. (1) The car moves at speed greater than the optimum speed. So it has outward sliding tendency.

21. (1) To retard the car, a tangential force in backward direction is required and to compensate outward slipping tendency, a tangential force down the plane is required.

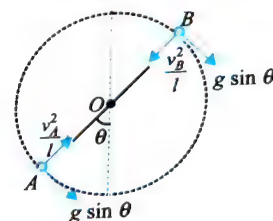
These two are the components of the force of static friction.

For Problems 22 and 23

22. (3), (4); 23. (1)

The difference in KE at positions A and B is

$$K_A - K_B = \frac{1}{2} m v_A^2 - \frac{1}{2} m v_B^2 = mg(2l \cos\theta) = 2mgl \cos\theta \quad \dots(i)$$



$$T_A = \frac{mv_A^2}{l} + mg \cos\theta \quad \dots(ii)$$

$$T_B = \frac{mv_B^2}{l} - mg \cos\theta \quad \dots(iii)$$

From Eqs. (ii) and (iii)

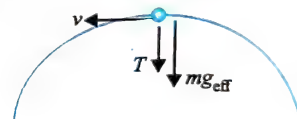
$$T_A - T_B = 6mg \cos\theta$$

The component of accelerations of ball at A and B are as shown in figure.

$$\begin{aligned} \therefore |\vec{a}_A + \vec{a}_B| &= \sqrt{(2g \sin\theta)^2 + \left(\frac{v_A^2}{l} - \frac{v_B^2}{l}\right)^2} \\ &= \sqrt{4g^2 \sin^2\theta + 16g^2 \cos^2\theta} = g\sqrt{4 + 12\cos^2\theta} \end{aligned}$$

For Problems 24 and 25

24. (1) Here $g_{\text{eff}} = g + a_1$ (downwards)



$$\frac{mv^2}{l} = T + mg_{\text{eff}}$$

For minimum u_1 , $T = 0$

$$v = \sqrt{(g + a_1)l}$$

$$\therefore u_1 = \sqrt{5(g + a_1)l} = \sqrt{\frac{15gl}{2}}$$

25. (1) $g_{\text{eff}} = \sqrt{g^2 + a_2^2} = \sqrt{2}g$

$$\frac{mv^2}{l} = T + mg_{\text{eff}}$$

for $T = 0$, $v^2 = \sqrt{2}gl$

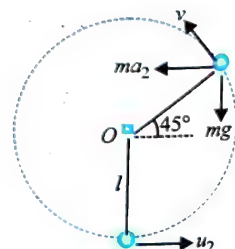
$$\frac{1}{2} m u_2^2 = \frac{1}{2} m v^2 + mg_{\text{eff}} \left(l + \frac{l}{\sqrt{2}} \right)$$

$$u_2^2 = v^2 + 2\sqrt{2}gl \left(1 + \frac{1}{\sqrt{2}} \right);$$

$$u_2^2 = v^2 + 2gl(\sqrt{2} + 1)$$

$$u_2^2 = \sqrt{2}gl + 2gl(\sqrt{2} + 1);$$

$$u_2 = \sqrt{(2 + 3\sqrt{2})gl}$$



Problems 26 and 27

26. (2), (4)

$$A \rightarrow B, mg(4) + \frac{1}{2}k(2^2 - 0) - 11 = \frac{1}{2}mv_B^2$$

$$B \rightarrow C, mg(3) - 4 = \frac{1}{2}m(v_C^2 - v_B^2)$$

27. (1), (4)

$$N_B = \frac{mv_B^2}{R}$$

$$N_C - mg = \frac{mv_C^2}{R}$$

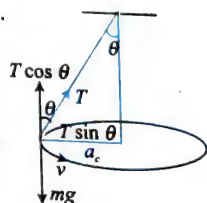
Matrix Match Type

1. i $\rightarrow$ b, c, d; ii $\rightarrow$ a, d; iii $\rightarrow$ b, d; iv $\rightarrow$ c

a. In vertical direction, there is no acceleration.

$$\text{So } T \cos \theta = mg$$

$T \sin \theta$ will provide centripetal acceleration. No tangential acceleration is present, so speed remains same, however velocity has on changing as direction of velocity change.

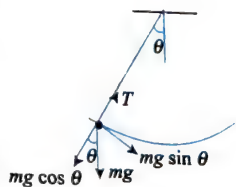


b. At extreme position, velocity is zero.

So a_c is zero, hence net acceleration is zero along string.

$$\text{So } T = mg \cos \theta$$

$mg \sin \theta$ will provide tangential acceleration, so velocity and speed both change.



c. $T \cos \theta = mg$, $T \sin \theta = ma$

As there is acceleration in car, so velocity of car as well as ball changes.

d. Here, as velocity is constant, so string remains vertical

$$T = mg$$

Speed of ball remains same.

2. i $\rightarrow$ b, d; ii $\rightarrow$ a; b; iii $\rightarrow$ a; iv $\rightarrow$ b, c, d

a. $U = 5$ m/s, let maximum height attained by bead is h , then

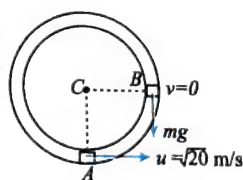
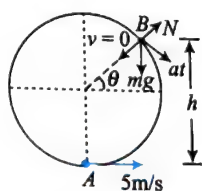
$$h = \frac{u^2}{2g} = \frac{5^2}{2 \times 10} = 1.25 \text{ m}$$

B is top most point of its trajectory where velocity is zero.

$$N = mg \sin \theta$$

N is radially outward acceleration is not zero at B . It is along tangential direction as shown;

$$a_t = \frac{mg \cos \theta}{m} = g \cos \theta$$



b. Maximum height attained:

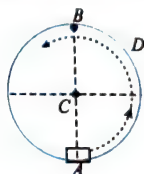
$$h = \frac{u^2}{2g} = \frac{(\sqrt{20})^2}{2 \times 10} = 1 \text{ m}$$

This is equal to radius.

B is the top most point of its trajectory. Here only force acting is mg , here acceleration is g downwards. Normal force is zero.

c. $u = 6 \text{ ms}^{-1}$, $\sqrt{2gR} < u < \sqrt{5gR}$

Block will leave contact at some point D and move in parabolic path afterwards. B is top most point of its trajectory where normal force is zero and acceleration is g downwards.

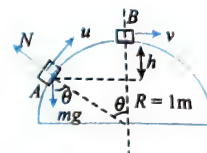


$$\text{d. } \cos = \frac{2}{3}, u = \sqrt{\frac{20}{3}} \text{ ms}^{-1}$$

$$mg \cos \theta - N = \frac{mu^2}{R}$$

$$\Rightarrow N = 0$$

$T(A)$, normal force is zero, but as it goes up normal force increase at B , v comes out to be zero. $N = mg$ at top.



3. i $\rightarrow$ a, c, d; ii $\rightarrow$ a, d; iii $\rightarrow$ a, b, d; iv $\rightarrow$ a, c, d

For the particle to just complete the circular motion

$$v_B = \sqrt{5gR}$$

$$\text{For this, } mgh = \frac{1}{2}mv_B^2 \Rightarrow h = \frac{5}{2}R = 2.5R$$

In this case, velocity at C will be $\sqrt{3gR}$

Force exerted by the particle on the track at point C is

$$\frac{m(\sqrt{3gR})^2}{R} = 3mg$$

and force exerted by the particle on the track at point A will be zero in this case.

For the force at A to be more than its weight,

$$mg + N = \frac{mv_A^2}{R}$$

$$N = \frac{mv_A^2}{R} - mg > mg$$

$$v_A > \sqrt{2gR}$$

Applying conservation of energy between O and A , we get

$$mgh = mg2R + \frac{1}{2}mv_A^2$$

If $v_A > \sqrt{2gR}$, then $h > 3R$

4. (1)

Point A: As the car is moving with constant velocity, hence at this point friction should be zero. Normal reaction can not be zero at this point. Hence (i $\rightarrow$ a)

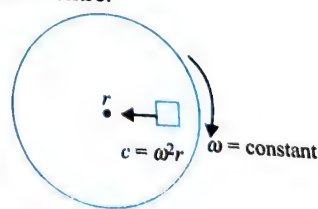
Point B and D: For maintaining constant speed, net force acting on the car should be zero. Hence friction should be up the slope. While moving from A to C and C to E , the component of weight of car is maximum at B and D , hence friction should be maximum at B and D . Hence (ii $\rightarrow$ c, e) and (iv $\rightarrow$ c, e)

Point C: As no component of the external force acting on the car is horizontal direction, hence friction should be zero. At this point the weight may provide required centripetal force, hence normal reaction at this point may be zero. It means (iii $\rightarrow$ a, b)

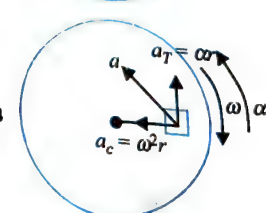
5. (3)

At position P and R , frictional force is max. and normal reaction is mg . At position S and Q , frictional force is zero and normal reaction points towards centre.

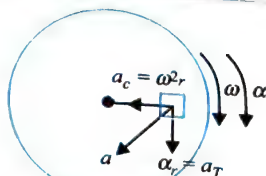
6. (4) Case A



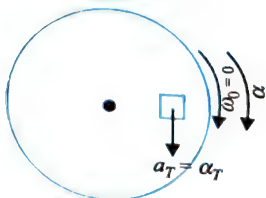
Case B



Case C



Case D

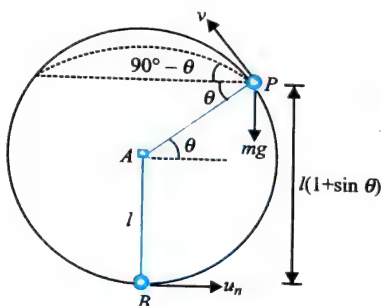


7. (2) $f_{\text{ground}} = \mu N = \mu mg = 2 \text{ N}$

$$f_{\text{hoop}} = \mu N' = \mu \left(\frac{mv^2}{R} \right) = 5 \text{ N}$$

8. (3) For (i): $mgh = \frac{1}{2} m (\sqrt{3gl})^2 \Rightarrow h = \frac{3l}{2}$

For (ii): $mgh = \frac{1}{2} m (\sqrt{4gl})^2 \Rightarrow h = 2l$



For (iii): In this case bob just complete the circle so $h = 2l$

For (iv): As $\sqrt{2gl} < u_0 < \sqrt{5gl}$, the string will slack.

Assuming that string slacks when it makes an angle θ with horizontal.

At P, $T = 0$. So $mg \sin \theta = \frac{mv^2}{l}$

from energy conservation

$$\frac{1}{2} mu_0^2 = \frac{1}{2} mv^2 + mgl(1 + \sin \theta)$$

$$\Rightarrow \sin \theta = \frac{1}{3} \text{ \& } v^2 = \frac{gl}{3}$$

Maximum height attained from starting point

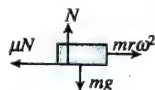
$$= l(1 + \sin \theta) + \frac{v^2 \sin^2(90^\circ - \theta)}{2g} = \frac{4l}{3} + \frac{4l}{27} = \frac{40}{27} l$$

Numerical Value Type

1. (1) $N = mg$, $\mu N = mr\omega^2$

$$\mu mg = m 2 \sin \theta \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{\mu g}{2 \sin \theta}} = \sqrt{\frac{0.1 \times 10}{2 \times \sin 30}} = 1 \text{ rad s}^{-1}$$



2. (5) According to conservation of energy between A and B, loss in gravitational potential energy = gain in elastic potential energy + gain in KE

$$mg \left(\frac{r}{2} + r \right) = \frac{1}{2} k(2r - r)^2 + \frac{1}{2} mv^2 \quad \dots(i)$$

At point B, centripetal force equation is

$$kr - mg = \frac{mv^2}{r} \quad \dots(ii)$$

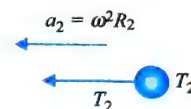
Solving Eqs. (i) and (ii), we get $k = 500 \text{ N m}^{-1}$

3. (1.125)

Both the masses are moving in horizontal plane with same angular speed 10 rad/s. Here forces in radial direction can be tensions only.

For M_2 : $F_{\text{net}} = T_2 = M_2 a_2$

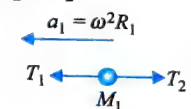
F.B.D. of M_2 : $\Rightarrow T_2 = M_2 \omega^2 R_2 \quad \dots(i)$



For M_1 : $T_1 - T_2 = M_1 a_1$

F.B.D. of M_1 : $\Rightarrow T_1 = M_1 a_1 + T_2$

$T_1 = M_1 \omega^2 R_1 + M_2 \omega^2 R_2 \quad \dots(ii)$



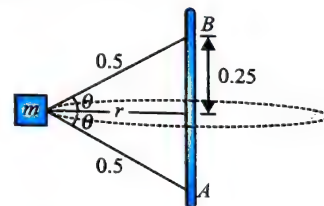
Dividing equation (ii) from (i), we get

$$\therefore \frac{T_1}{T_2} = \frac{M_1 R_1 + M_2 R_2}{M_2 R_2} = \frac{M_1}{M_2} \times \frac{R_1}{R_2} + 1 = \frac{0.25}{1} \times \frac{5}{10} + 1$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{9}{8}$$

Here centripetal force on M_2 is T_2 and on M_1 is $T_1 - T_2$.

4. (9) $\sin \theta = \frac{0.25}{0.50} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$



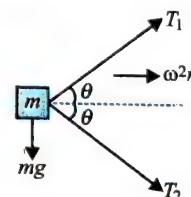
$$r = 0.5 \cos \theta = 0.5 \left(\frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3}}{4}$$

$$(F_{\text{net}})_y = 0 \Rightarrow T_1 \sin \theta - T_2 \sin \theta - mg = 0$$

$$\therefore (T_1 - T_2) \sin \theta = mg \quad \dots(i)$$

$$(F_{\text{net}})_x = m \omega^2 r$$

$$\therefore (T_1 + T_2) \cos \theta = m \omega^2 r \quad \dots(ii)$$



Dividing equations (ii) by (i), we get

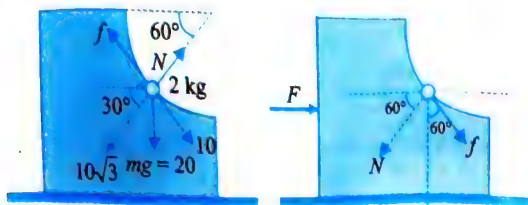
$$\left(\frac{T_1 + T_2}{T_1 - T_2} \right) \cot \theta = \frac{\omega^2 r}{g}$$

$$\frac{T_1 + T_2}{T_1 - T_2} = \frac{\omega^2 r \tan \theta}{g} = \frac{(7)^2 \times (\sqrt{3} \times 4) \times 1 \times \sqrt{3}}{9.8}$$

$$\therefore \frac{T_1 + T_2}{T_1 - T_2} = \frac{5}{4}$$

$$\therefore \frac{T_1}{T_2} = \frac{5+4}{5-4} = 9$$

5. (1) $f = 10 \text{ N}$



$$n - 10\sqrt{3} = \frac{2 \times \left(\frac{1}{2}\right)^2}{\frac{1}{4}}$$

$$N = 10\sqrt{3} + 2$$

$$N + f \sin 60^\circ = N \cos 60^\circ \Rightarrow F = 1 \text{ N}$$

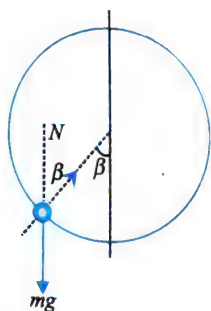
6. (2) Since $\vec{F} \perp \vec{v}$, the particle will move along a circle.

$$\therefore F = \frac{mv^2}{R} \text{ and } \theta = \frac{S}{R}$$

$$\Rightarrow \theta = \frac{FS}{mv^2} = \frac{(10)(10)}{(2)(5)^2} = 2$$

7. (0) $N \cos \beta = mg$

$$N \sin \beta = m\omega^2 R \sin \beta$$



$$\frac{mg}{\cos \beta} = m\omega^2 R$$

$$\cos \beta = \frac{g}{\omega^2 R} = \frac{10}{0.25 \times 10} = 4 \Rightarrow \text{Not possible}$$

$$\Rightarrow \beta = 0$$

8. (48) Total time taken on the circular track,

$$t_1 = 2 \left[\int_0^{\pi/2} \frac{ds}{\left(\frac{-0.25s}{\pi} + 0.25\right)} + \int_0^{\pi/2} \frac{ds}{\left(\frac{0.25s}{\pi} + 0.125\right)} \right]$$

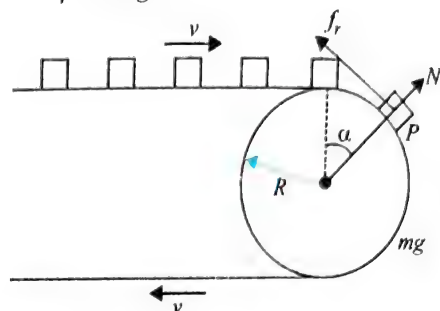
$$\text{Total time, } t = 2 \times \left(\frac{2}{0.25} \right) + t_1$$

$$\Rightarrow t = 48 \text{ s}$$

9. (2) At P, the net force on part along the radial direction provides the centripetal acceleration.

$$mg \cos \alpha - N = \frac{mv^2}{R}$$

$$\cos \alpha - \frac{\sin \alpha}{\mu} = \frac{v^2}{Rg}$$



$$v = 2 \text{ m/s}$$

10. (1) The particle is moving in a horizontal circle, so it is accelerated towards the centre with magnitude v^2/r .

The horizontal force on the particle is due to the spring and equals kl , where l is the elongation and k is the spring constant. Thus,

$$kl = \frac{mv^2}{r} = m\omega^2 r = m\omega^2 (l_0 + l)$$

Here ω is the angular velocity, l_0 is the natural length (0.5 m) and $l_0 + l$ is the total length of the spring which is also the radius of the circle along which the particle moves.

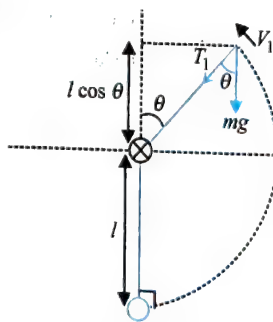
$$\text{Thus, } (k - m\omega^2)l = m\omega^2 l_0$$

$$\text{or, } l = \frac{m\omega^2 l_0}{k - m\omega^2}$$

Putting the values,

$$l = \frac{0.5 \times 4 \times 0.5}{100 - 0.5 \times 4} \text{ m} = \frac{1}{100} \text{ m} = 1 \text{ cm}$$

11. (6)



$$\text{As we know, } v^2 = v_x^2 + v_y^2$$

$$\therefore 2v \frac{dv}{dt} = 2v_x \frac{dv_x}{dt} + 2v_y \frac{dv_y}{dt}$$

$$\therefore \left(\frac{dv}{dt} \right) = \frac{v_x a_x + v_y a_y}{\sqrt{v_x^2 + v_y^2}} = \frac{3 \times 5 + 4 \times \frac{15}{4}}{\sqrt{3^2 + 4^2}} = 6 \text{ m/s}^2$$

$$12. (2) mg \cos \theta + T_1 = \frac{mv_1^2}{l}$$

For leaving circle $T_1 = 0$

$$mv_1^2 = mg l \cos \theta$$

and by energy conservation

$$0 + \frac{1}{2} m (\sqrt{3gl})^2 = \frac{1}{2} mv_1^2 + mg(l + l \cos \theta)$$

$$\frac{1}{2}m(3gl) = \frac{1}{2}mv_1^2 + mgl(1 + \cos\theta)$$

$$\frac{3mgl}{2} = \frac{mgl \cos\theta}{2} + mgl + mgl \cos\theta \quad (\text{by equation (i)})$$

$$\frac{mgl}{2} = \frac{3}{2}mgl \cos\theta$$

$$\cos\theta = \frac{1}{3} \sin\theta = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}$$

$$a_c = \frac{v_1^2}{l} = \frac{g}{l} \cos\theta = g \cos\theta$$

$$a_t = g \sin\theta$$

$$\text{Then } \frac{a_c}{a_t} = \frac{g \cos\theta}{g \sin\theta} = \frac{1/3}{g \sin\theta}$$

$$\Rightarrow \frac{a_c}{a_t} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} = \frac{1}{y\sqrt{2}} \Rightarrow y = 2$$

$$13. (6) T_{\max} - mg = \frac{mu^2}{l}$$

$$T_{\max} + mg = \frac{mu^2}{l} = \frac{m}{l}(u^2 - 4gl)$$

$$T_{\max} = mg + \frac{mu^2}{l}$$

$$T_{\min} = \frac{mu^2}{l} - 5mg$$

$$\therefore \text{Difference} = 6mg$$

Archives

JEE Main

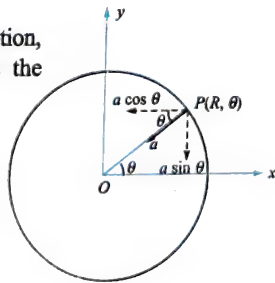
Single Correct Answer Type

1. (3) For a particle in uniform circular motion, acceleration, $a = v^2/R$ is towards the centre.

From figure, we have

$$\vec{a} = -a \cos\theta \hat{i} - a \sin\theta \hat{j}$$

$$\Rightarrow \vec{a} = -\frac{v^2}{R} \cos\theta \hat{i} - \frac{v^2}{R} \sin\theta \hat{j}$$



2. (4) Position time relation of the particle, $s = t^3 + 3$

$$\text{Speed of the particle, } v = \frac{ds}{dt} = \frac{d}{dt}(t^3 + 3) = 3t^2$$

$$\text{Tangential acceleration, } a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t$$

At time, $t = 2$ s

$$\text{Speed of the particle, } v = 3(2)^2 = 12 \text{ m/s}$$

$$\text{Tangential acceleration, } a_t = 6(2) = 12 \text{ m/s}^2$$

Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}^2$$

$$\begin{aligned} \text{Net acceleration, } a &= \sqrt{(a_c)^2 + (a_t)^2} \\ &= \sqrt{(7.2)^2 + (12)^2} \approx 14 \text{ m/s}^2 \end{aligned}$$

3. (3) Centripetal acceleration, $a_c = \omega^2 r$

$$\text{Angular velocity, } \omega = \frac{2\pi}{T}$$

The cars make complete circle in same time.

$$\text{As } T_1 = T_2 \Rightarrow \omega_1 = \omega_2$$

$$\therefore \frac{a_{c1}}{a_{c2}} = \frac{r_1}{r_2}$$

4. (4) Given: Force $\propto \frac{1}{R^n}$

$$\text{Also Force} = m\omega^2 R$$

$$\text{Hence } m\omega^2 R \propto \frac{1}{R^n} \Rightarrow \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$$

$$\text{Time period, } T = \frac{2\pi}{\omega} \text{ or } T \propto \frac{1}{\omega}$$

$$T \propto R^{\frac{n+1}{2}}$$

$$5. (4) F = -\frac{dU}{dr}$$

$$= -\frac{d\left(-\frac{K}{2r^2}\right)}{dr} = \frac{K}{2} \frac{dr^{-2}}{dr} = \frac{K}{2} (-2r^{-3}) = -\frac{K}{r^3}$$

Since it is performing circular motion $F = \frac{mv^2}{r} = \frac{K}{r^3}$

$$mv^2 = \frac{K}{r^2} \Rightarrow \text{K.E.} = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

$$\text{Total energy} = P_E + K_E = -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero}$$

JEE Advanced

Single Correct Answer Type

1. (4)

$$mgR(1 - \cos\theta) = \frac{1}{2}mv^2$$

$$\therefore \frac{mv^2}{R} = 2mg(1 - \cos\theta)$$

$$\therefore N = mg \cos\theta - \frac{mv^2}{R}$$

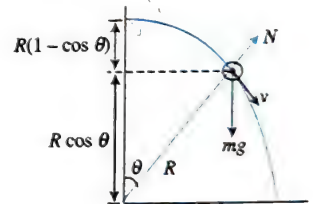
$$= mg \cos\theta - 2mg(1 - \cos\theta)$$

$$= mg \cos\theta + 2mg \cos\theta - 2mg$$

$$= 3mg \cos\theta - 2mg = 3mg \left(\cos\theta - \frac{2}{3} \right)$$

If $\cos\theta > \frac{2}{3}$, N is positive.

Therefore, wire will exert force radially outward. So bead will exert force radially inward initially and radially outward later.



Linked Comprehension Type

For Problems 1 and 2

1. (2) Using work-energy theorem, we get

$$mgR \sin 30^\circ + W_f = \frac{1}{2}mv^2$$

$$200 - 150 = \frac{v^2}{2} \Rightarrow v = 10 \text{ m/s}$$

2. (1)

$$N - mg \cos 60^\circ = \frac{mv^2}{R}$$

$$N = 5 + \frac{5}{2} = 7.5 \text{ N}$$

For Problems 3 and 4

3. (4) $m r \omega^2 = m a$

$$a = r \omega^2$$

$$\frac{v dv}{dr} = r \omega^2$$

$$\int_0^v v dv = \omega^2 \int_{R/2}^r r dr$$

$$v = \omega \sqrt{r^2 - \frac{R^2}{4}}$$

$$\int_{R/2}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

$$\text{Assume } r = \frac{R}{2} \sec \theta$$

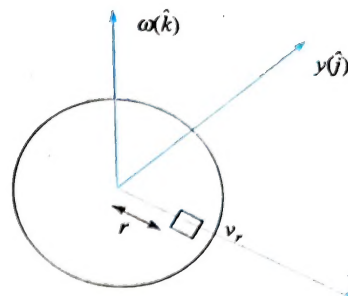
$$dr = \frac{R}{2} \sec \theta \tan \theta d\theta$$

$$\int \frac{\frac{R}{2} \sec \theta \tan \theta d\theta}{\sqrt{\frac{R^2}{4} \tan^2 \theta}} = \int_0^t \omega dt$$

$$\omega t = \ln \left[\frac{2r}{R} + \frac{\sqrt{4r^2 - R^2}}{R} \right]$$

$$r = \frac{R}{4} [e^{\omega t} + e^{-\omega t}]$$

4. (3)



$$\vec{F}_{\text{rot}} = \vec{F}_{\text{in}} + 2m(v_{\text{rot}} \hat{i}) \times \omega \hat{k} + m(\omega \hat{k} \times r \hat{i}) \times \omega \hat{k}$$

$$m r \omega^2 \hat{i} = \vec{F}_{\text{in}} + 2m v_{\text{rot}} \omega (-\hat{j}) + m \omega^2 r \hat{i}$$

$$\vec{F}_{\text{in}} = 2m v_r \omega \hat{j}$$

... (1)

$$r = \frac{R}{4} [e^{\omega t} + e^{-\omega t}]$$

$$\frac{dr}{dt} = v_r = \frac{R}{4} [\omega e^{\omega t} - \omega e^{-\omega t}]$$

$$\vec{F}_{\text{in}} = 2m \frac{R \omega}{4} [e^{\omega t} - e^{-\omega t}] \omega \hat{j}$$

$$\vec{F}_{\text{in}} = \frac{m R \omega^2}{2} [e^{\omega t} - e^{-\omega t}] \hat{j}$$

Also reaction is due to disc surface, then

$$\vec{F}_{\text{reaction}} = \frac{m R \omega^2}{2} [e^{\omega t} - e^{-\omega t}] \hat{j} + m g \hat{k}$$

Numerical Value Type

1. (5)

The initial speed of 1st bob (suspended by a string of length l_1) is $\sqrt{5gl_1}$ The speed of this bob at highest point will be $\sqrt{gl_1}$

When this bob collides with the other bob, their speeds are interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

1

APPENDIX

Chapterwise Solved January 2019 JEE Main Questions (All Sets)

Chapter 1 Dimensions and Measurement

1. The pitch and the number of divisions, on the circular scale, for a given screw gauge are 0.5 mm and 100 respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line. The readings of the main scale and the circular scale, for a thin sheet, are 5.5 mm and 48 respectively, the thickness of this sheet is

- (1) 5.755 mm (2) 5.725 mm
(3) 5.740 mm (4) 5.950 mm

2. Expression for time in terms of G (universal gravitational constant), h (Planck constant) and c (speed of light) is proportional to

- (1) $\sqrt{\frac{Gh}{c^3}}$ (2) $\sqrt{\frac{hc^3}{G}}$
(3) $\sqrt{\frac{c^3}{Gh}}$ (4) $\sqrt{\frac{Gh}{c^5}}$

3. The density of a material in SI units is 128 kg m^{-3} . In certain units in which the unit of length is 25 cm and the unit of mass is 5 g, the numerical value of density of the material is

- (1) 410 (2) 640
(3) 16 (4) 40

4. The diameter and height of a cylinder are measured by a meter scale to be $12.6 \pm 0.1 \text{ cm}$ and $34.2 \pm 0.1 \text{ cm}$, respectively. What will be the value of its volume in appropriate significant figures?

- (1) $4260 \pm 80 \text{ cm}^3$ (2) $4300 \pm 80 \text{ cm}^3$
(3) $4264.4 \pm 81.0 \text{ cm}^3$ (4) $4264 \pm 81 \text{ cm}^3$

5. The force of interaction between two atoms is given by

$$F = \alpha \beta \exp\left(-\frac{x^2}{akT}\right); \text{ where } x \text{ is the distance, } k \text{ is the}$$

Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is

- (1) $M^2 L^2 T^{-2}$ (2) $M^2 L T^{-4}$
(3) $M^0 L^2 T^{-4}$ (4) $M L T^{-2}$

6. If speed (V), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be

- (1) $V^{-2} A^2 F^2$ (2) $V^4 A^2 F$
(3) $V^4 A^{-2} F$ (4) $V^{-2} A^2 F^{-2}$

7. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure $5 \mu\text{m}$ diameter of wire is

- (1) 50 (2) 100
(3) 200 (4) 500

8. Let L , R , C and V represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{L}{RCV}$ in SI units will be

- (1) $[LTA]$ (2) $[LA^{-2}]$
(3) $[A^{-1}]$ (4) $[LT^2]$

Chapter 3 Vectors

9. The position coordinates of a particle moving in a 3-D coordinate system is given by

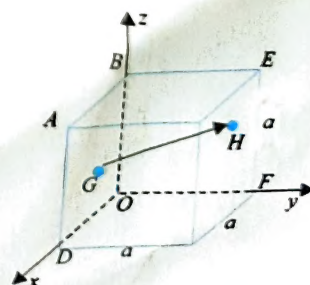
$$x = a \cos \omega t$$

$$y = a \sin \omega t \text{ and } z = a \omega t$$

The speed of the particle is

- (1) $a\omega$ (2) $\sqrt{3} a\omega$
(3) $\sqrt{2} a\omega$ (4) $2a\omega$

10. In the cube of side a shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be



- (1) $\frac{1}{2}a(\hat{i} - \hat{k})$ (2) $\frac{1}{2}a(\hat{j} - \hat{i})$
 (3) $\frac{1}{2}a(\hat{k} - \hat{i})$ (4) $\frac{1}{2}a(\hat{j} - \hat{k})$

11. Two forces P and Q of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle is
 (1) 30° (2) 60°
 (3) 90° (4) 120°

12. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is ' n ' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is

- (1) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (2) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$
 (3) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (4) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

13. A particle is moving along a circular path with a constant speed of 10 ms^{-1} . What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

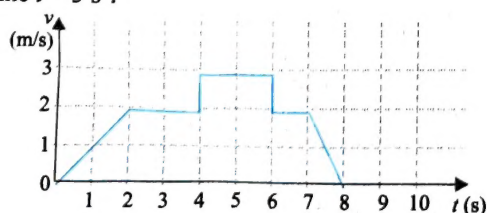
- (1) zero (2) 10 m/s
 (3) $10\sqrt{3} \text{ m/s}$ (4) $10\sqrt{2} \text{ m/s}$

Chapter 4 Kinematics I

14. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed ' v ' more than that of car B . Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then ' v ' is equal to

- (1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}t$
 (3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}t$

15. A particle starts from the origin at time $t = 0$ and moves along the positive x -axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5 \text{ s}$?



- (1) 6 m (2) 9 m
 (3) 3 m (4) 10 m

16. A particle moves from the point $(2.0\hat{i} + 4.0\hat{j}) \text{ m}$ at $t = 0$, with an initial velocity $(5.0\hat{i} + 4.0\hat{j}) \text{ ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i} + 4.0\hat{j}) \text{ ms}^{-2}$. What is the distance of the particle from the origin at time 2 s ?

- (1) $20\sqrt{2} \text{ m}$ (2) $10\sqrt{2} \text{ m}$
 (3) 5 m (4) 15 m

17. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when: (i) they are moving in the same direction, and (ii) in the opposite directions is

- (1) $\frac{5}{2}$ (2) $\frac{25}{11}$
 (3) $\frac{3}{2}$ (4) $\frac{11}{5}$

Chapter 5 Kinematics II

18. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

- (1) $xy = \text{constant}$ (2) $y^2 = x^2 + \text{constant}$
 (3) $y = x^2 + \text{constant}$ (4) $y^2 = x + \text{constant}$

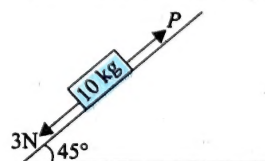
Chapter 6 Newton's Laws of Motion (Without Friction)

19. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is ($g = 10 \text{ ms}^{-2}$)

- (1) 200 N (2) 100 N
 (3) 140 N (4) 70 N

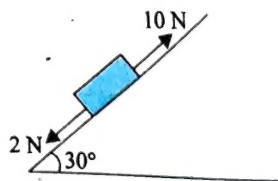
Chapter 7 Newton's Laws of Motion (With Friction)

20. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P , such that the block does not move downward? (take $g = 10 \text{ ms}^{-2}$)



- (1) 32 N (2) 25 N
 (3) 23 N (4) 18 N

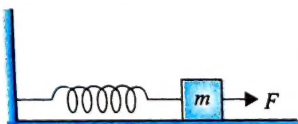
21. A block, kept on a rough inclined plane, as shown in the figure, remains at rest up to a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is [Take $g = 10 \text{ m/s}^2$]



- (1) $\frac{2}{3}$ (2) $\frac{\sqrt{3}}{2}$
(3) $\frac{\sqrt{3}}{4}$ (4) $\frac{1}{2}$

Chapter 8 Work, Energy and Power

22. A block of mass m , lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k . The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F , the maximum speed of the block is

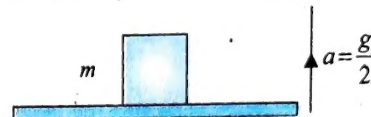


- (1) $\frac{\pi F}{\sqrt{mk}}$ (2) $\frac{2F}{\sqrt{mk}}$
(3) $\frac{F}{\sqrt{mk}}$ (4) $\frac{F}{\pi\sqrt{mk}}$

23. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

- (1) 850 J (2) 900 J
(3) 950 J (4) 875 J

24. A block of mass m is kept on a platform which starts from rest with constant acceleration $g/2$ upward, as shown in figure. Work done by normal reaction on block in time t is



- (1) 0 (2) $\frac{3mg^2t^2}{8}$
(3) $-\frac{mg^2t^2}{8}$ (4) $\frac{mg^2t^2}{8}$

25. A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement?

- (1) 15 J (2) 10 J
(3) 12 J (4) 9 J

Chapter 9 Circular Motion

26. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1 \text{ s}$ is R . Neglecting air resistance and taking acceleration due to gravity $g = 10 \text{ ms}^{-2}$, the value of R is

- (1) 2.5 m (2) 10.3 m
(3) 2.8 m (4) 5.1 m

Answers Key

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (2) | 2. (4) | 3. (4) | 4. (1) | 5. (2) | 16. (1) | 17. (4) | 18. (2) | 19. (2) | 20. (1) |
| 6. (2) | 7. (3) | 8. (3) | 9. (3) | 10. (2) | 21. (2) | 22. (3) | 23. (2) | 24. (2) | 25. (1) |
| 11. (4) | 12. (3) | 13. (2) | 14. (4) | 15. (2) | 26. (3) | | | | |